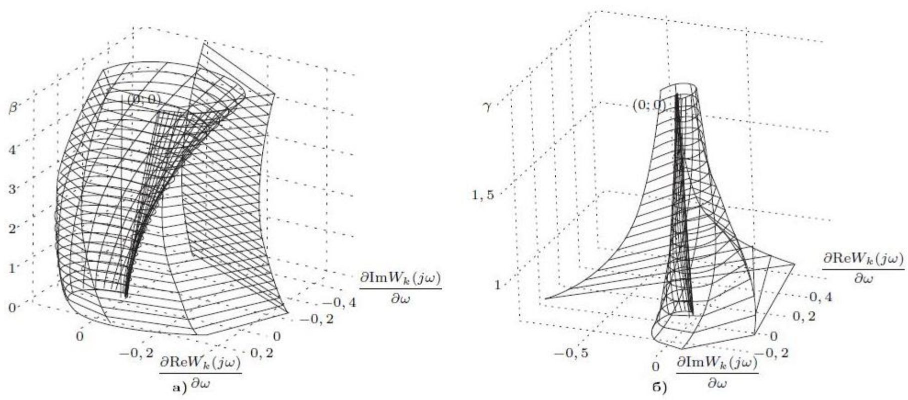
С.А. Прохоров, И.М. Куликовских

ОСНОВНЫЕ ОРТОГОНАЛЬНЫЕ ФУНКЦИИ И ИХ ПРИЛОЖЕНИЯ

Часть I. Ортогональные функции экспоненциального типа



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В первой части предлагаемой монографии рассматриваются ортогональные функции экспоненциального типа.

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ЧАСТЬ I. ОРТОГОНАЛЬНЫЕ ФУНКЦИИ ЭКСПОНЕНЦИАЛЬНОГО ТИПА

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Предисловие

Вашему вниманию предлагается первая часть монографии по основным ортогональным функциям и их приложениям. Эта работа по-своему уникальна, потому как ориентирована, в первую очередь, на конкретного читателя - прикладного математика и программиста - для создания адекватных моделей, оптимальных алгоритмов и написания исходного кода с минимальными вычислительными и ресурсными затратами при решении частных задач. Тем не менее математический аппарат, положенный в основу работы, - теория ортогональных многочленов и рядов Фурье - имеет большой теоретический интерес. Поэтому авторы надеются, что данная работа будет интересна более широкой аудитории.

Под основными ортогональными функциями экспоненциального типа, отраженными в названии работы, на данном этапе, понимаются классические обобщенные многочлены Лагерра $L_k^{(\alpha)}(\tau,\gamma)$, многочлены Якоби $P_k^{(\alpha,\beta)}(\tau,\gamma)$, заданные на интервале $\tau \in [0,\infty)$ с использованием соответствующей замены экспоненциального типа и введением варьируемого параметра масштаба γ. Математическим аспектам определения функций экспоненциального типа и их свойствам значительно больше внимания уделено в тексте монографии в соответствующих разделах. Главным образом ортогональные функции экспоненциального типа предназначены для приближения функций $f(\tau)$, заданной на положительной полуоси $\tau \in [0,\infty)$, для которой справедливо

$$\arg\left(\max(f(\tau))\right) = 0; \lim_{\tau\to\infty} f(\tau) \to 0; \int_0^\infty \Big(f(\tau)\Big)^2 \mu(\tau,\gamma)\mathrm{d}\tau < \infty.$$

Вопросы, касающиеся построения моделей $f(\tau)$ и их разложения в ряды Фурье, а также разработки соответствующих алгоритмов оптимизации при решении задач приближения, будут рассмотрены в последующих главах.

Структура предлагаемой части монографии разработана с учетом дальнейшего приложения к построению математических моделей через разложение в ряды Фурье. В целом первая часть работы охватывает следующие вопросы:

- аналитическое представление во временной области;
- основные и расширенные свойства во временной области;
- основные и расширенные соотношения ортогональности во временной области;
- фазовое представление ортогональных функций;
- интегральное представление ортогональных функций;
- аналитическое представление в частотной области;

- основные и расширенные свойства в частотной области;
- основные и расширенные соотношения ортогональности в частотной области;
- рекуррентные соотношения;
- соотношения взаимосвязи базисных функций;
- обобщенные характеристики ортогональных функций;
- соотношения неопределенности.

Каждый из приведенных разделов содержит следующие этапы:

- определение;
- последовательность нумерованных формул:
  - частные случаи 0-5 порядков;
  - графическая интерпретация частных случаев при заданных параметрах.

На сегодняшний день насчитывается большое число справочников, посвященных описанию специальных функций и ортогональных многочленов. Среди наиболее распространенных следующие: I.S. Gradshteyn, I.M. Ryzhik «Table of Integrals, Series, and Products» (2007); Y.A. Brychkov «Handbook of Special Functions: Derivatives, Integrals, Series and Other Formulas»(2008); NIST Handbook of mathematical functions (2010); М. Абрамовиц, И. Стиган «Справочник по специальным функциям»(1979); А.П. Прудников, Ю.А. Брычков, О.И. Маричев «Интегралы и ряды»(т. 2,3) (1983).

В свою очередь авторы хотели бы обратить внимание в предлагаемой монографии на класс анализируемых функций и на специфику конечных формул, что несомненно определяет их новизну. Более того, в рамках данной работы введен ряд новых понятий и определений, которые имеют значительный практический интерес и расширяют теорию ортогональных многочленов в целом.

На данный момент неоднократно проводилась апробация предлагаемых в монографии формул для построения математических моделей как для создания масштабных проектов - систем «Data Mining», где требуется одновременная обработка больших потоков данных, так и для создания актуальных на данном этапе развития информационных технологий - мобильных приложений на базе таких платформ как iOS, Android, и т.д., основными требованиями которых являются сниженные ресурсные затраты, высокая скорость выполнения операций и низкие требования к объему используемой памяти.

Структура монографии, способ представления материала, фактический материал и графические интерпретации являются исключительно авторскими.

Авторы выражают большую признательность к.ф.-м.н. Л.П. Усольцеву за ценные замечания и внимательное отношение к работе на различных этапах ее формирования.

С.А. Прохоров
И.М. Куликовских

Содержание

Часть I

ОРТОГОНАЛЬНЫЕ ФУНКЦИИ ЭКСПОНЕНЦИАЛЬНОГО ТИПА

Глава 1

Аналитические представления во временной области

Определение.

Классические ортогональные многочлены $\psi_k(x)$ являются решением дифференциального уравнения [2, 4, 13, 18]

$$(ax^2+bx+c)\psi_k''(x)+(dx+e)\psi_k'(x)-k\big(d+(k-1)a\big)\psi_k(x)=0$$

и при заданных параметрах данного уравнения имеем три основных класса многочленов: Якоби, обобщенные Лагерра и Эрмита. Остановимся на рассмотрении ортогональных многочленов Якоби $P_k^{(\alpha,\beta)}(x)$ и обобщенных многочленов Лагерра $L_k^{(\alpha)}(x)$ [2, 4, 13, 16, 17, 19].

Если $a=-1$, $b=0$, $c=1$, $d=-\alpha-\beta-2$ и $e=-\alpha+\beta$, и имеем ортогональные многочлены Якоби [13, 16]

$$P_k^{(\alpha,\beta)}(x)=\frac{(-1)^k}{k!2^k}(1-x)^{-\alpha}(1+x)^{-\beta}\frac{d^k\big((1-x)^{\alpha+k}(1+x)^{\beta+k}\big)}{dx^k}$$

с весовой функцией $\mu^{\{P_k^{(\alpha,\beta)}(x)\}}(x)=(1-x)^\alpha(1+x)^\beta$, удовлетворяющие условию

$$\int_{-1}^{1}P_k^{(\alpha,\beta)}(x)P_n^{(\alpha,\beta)}(x)\mu^{\{P_k^{(\alpha,\beta)}(x)\}}(x)dx=\frac{2^{\alpha+\beta+1}\Gamma(\alpha+k+1)\Gamma(\beta+k+1)}{k!(\alpha+\beta+2k+1)\Gamma(\alpha+\beta+k+1)}\delta_{k,n},$$

где $\delta_{k,n}$ – символ Кроненкера. При $k=n$ последнее соотношение имеет вид $\left\|P_k^{(\alpha,\beta)}\right\|^2$ и называется нормой ортогональных функций Якоби.

Если $a=0$, $b=1$, $c=0$, $d=-1$ и $e=\alpha+1$,имеем обобщенные ортогональные многочлены Лагерра [13, 16]

$$L_k^{(\alpha)}(x)=(-1)^k x^{-\alpha}e^x\frac{d^k\big(x^{\alpha+k}e^{-x}\big)^k}{dx^k}$$

с весовой функцией $\mu^{\{L_k^{(\alpha)}(x)\}}(x)=x^\alpha e^{-x}$, удовлетворяющие условию

$$\int_0^\infty L_k^{(\alpha)}(x)L_n^{(\alpha)}(x)\mu^{\{L_k^{(\alpha)}(x)\}}(x)dx=k!\Gamma(\alpha+k+1)\delta_{k,n}.$$

При $k=n$ вышеприведенное соотношение обозначается $\left\|L_k^{(\alpha)}\right\|^2$ и называется нормой обобщенных ортогональных функций Лагерра.

В последующем ортогональные многочлены $\psi_k(x)$,заданные на интервале $[0,\infty)$ с использованием замены экспоненциального типа, определим как ортогональные функции экспоненциального типа $\psi_k(\tau,\gamma)$.

Замены переменных аргумента $x = f(\tau,\gamma)$, позволяющие определить функции Якоби и обобщенные функции Лагерра [3, 9] представлены в таблице ниже.

Название многочлена	Обозначение $\psi_k(\tau,\gamma)$	Замена аргумента $x = f(\tau,\gamma)$
Обобщенные Лагерра	$L_k^{(\alpha)}(\tau,\gamma)$	$x = \gamma\tau$
Якоби	$P_k^{(\alpha,\beta)}(\tau,\gamma)$	$x = 1 - 2\exp(-c\gamma\tau)$

1.1 Аналитические соотношения для ортогональных функций

[1.1] $L_k(\tau,\gamma) = \sum_{s=0}^{k} \binom{k}{s} \frac{(-\gamma\tau)^s}{s!} \exp\Big(-\frac{\gamma\tau}{2}\Big).$

Частные случаи для функций 0-5 порядков:

$L_0(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big);$

$L_1(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(1-\gamma\tau);$

$L_2(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 - 4\gamma\tau + 2)/2;$

$L_3(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(6 - 18\gamma\tau + 9\gamma^2\tau^2 - \gamma^3\tau^3)/6;$

$L_4(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 - 16\gamma^3\tau^3 + 72\gamma^2\tau^2 - 96\gamma\tau + 24)/24;$

$L_5(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(120 - 600\gamma\tau + 600\gamma^2\tau^2 - 200\gamma^3\tau^3 + 25 \times$
$\times\, \gamma^4\tau^4 - \gamma^5\tau^5)/120.$

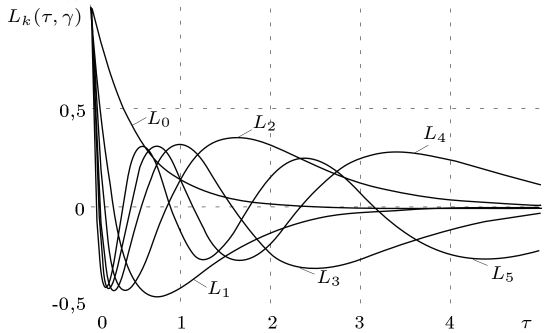


Рис. 1.1. Вид ортогональных функций Лагерра 0-5 порядков; $\gamma = 4$

[1.2] $L_k^{(1)}(\tau,\gamma) = \sum_{s=0}^{k} \binom{k+1}{k-s} \frac{(-\gamma\tau)^s}{s!} \exp\Big(-\frac{\gamma\tau}{2}\Big).$

Частные случаи для функций 0-5 порядков:

$L_0^{(1)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big);$

$L_1^{(1)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(2-\gamma\tau);$

$L_2^{(1)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 - 6\gamma\tau + 6)/2;$

$L_3^{(1)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(24 - 36\gamma\tau + 12\gamma^2\tau^2 - \gamma^3\tau^3)/6;$

$L_4^{(1)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 - 20\gamma^3\tau^3 + 120\gamma^2\tau^2 - 240\gamma\tau +$
$+ 120)/24;$

$L_5^{(1)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(720 - 1800\gamma\tau + 1200\gamma^2\tau^2 - 300\gamma^3\tau^3 +$
$+ 30\gamma^4\tau^4 - \gamma^5\tau^5)/120.$

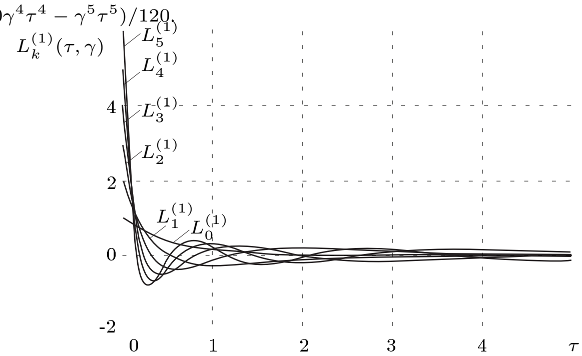


Рис. 1.2. Вид ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 4$, $\alpha = 1$

[1.3] $L_k^{(2)}(\tau,\gamma) = \sum_{s=0}^{k} \binom{k+2}{k-s} \frac{(-\gamma\tau)^s}{s!} \exp\Big(-\frac{\gamma\tau}{2}\Big).$

Частные случаи для функций 0-5 порядков:

$L_0^{(2)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big);$

$L_1^{(2)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(3-\gamma\tau);$

$L_2^{(2)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 - 8\gamma\tau + 12)/2;$

$L_3^{(2)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(60 - 60\gamma\tau + 15\gamma^2\tau^2 - \gamma^3\tau^3)/6;$

$L_4^{(2)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 - 24\gamma^3\tau^3 + 180\gamma^2\tau^2 - 480\gamma\tau +$
$+ 360)/24;$

$L_5^{(2)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(2520 - 4200\gamma\tau + 2100\gamma^2\tau^2 - 420\gamma^3\tau^3 +$
$+ 35\gamma^4\tau^4 - \gamma^5\tau^5)/120.$

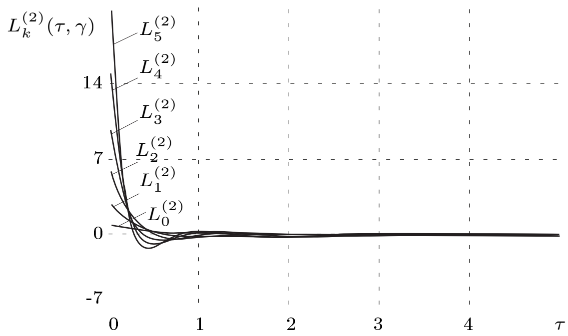


Рис. 1.3. Вид ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 4$, $\alpha = 2$

[1.4] $$L_k^{(\alpha)}(\tau,\gamma) = \sum_{s=0}^{k} \binom{k+\alpha}{k-s} \frac{(-\gamma\tau)^s}{s!} \exp\Big(-\frac{\gamma\tau}{2}\Big).$$

Частные случаи для функций 0-5 порядков:

$L_0^{(\alpha)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big);$

$L_1^{(\alpha)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)(\alpha+1-\gamma\tau);$

$L_2^{(\alpha)}(\tau,\gamma) = \frac{1}{2}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^2\tau^2 - 2(\alpha+2)\gamma\tau + (\alpha+1)(\alpha+2)\big);$

$L_3^{(\alpha)}(\tau,\gamma) = \frac{1}{6}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big((\alpha+1)(\alpha+2)(\alpha+3) - 3(\alpha+2)\,\times$
$\times\,(\alpha+3)\gamma\tau + 3(\alpha+3)\gamma^2\tau^2 - \gamma^3\tau^3\big);$

$L_4^{(\alpha)}(\tau,\gamma) = \frac{1}{24}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^4\tau^4 - 4(\alpha+4)\gamma^3\tau^3 + 6(\alpha+3)\,\times$
$\times\,(\alpha+4)\gamma^2\tau^2 - 4(\alpha+2)(\alpha+3)(\alpha+4)\gamma\tau + (\alpha+1)(\alpha+2)\,\times$
$\times\,(\alpha+3)(\alpha+4)\big);$

$L_5^{(\alpha)}(\tau,\gamma) = \frac{1}{120}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big((\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4)\,\times$
$\times\,(\alpha+5) - 5(\alpha+2)(\alpha+3)(\alpha+4)(\alpha+5)\gamma\tau + 10(\alpha+3)(\alpha+4)\,\times$
$\times\,(\alpha+5)\gamma^2\tau^2 - 10(\alpha+4)(\alpha+5)\gamma^3\tau^3 + 5(\alpha+5)\gamma^4\tau^4 - \gamma^5\tau^5\big).$

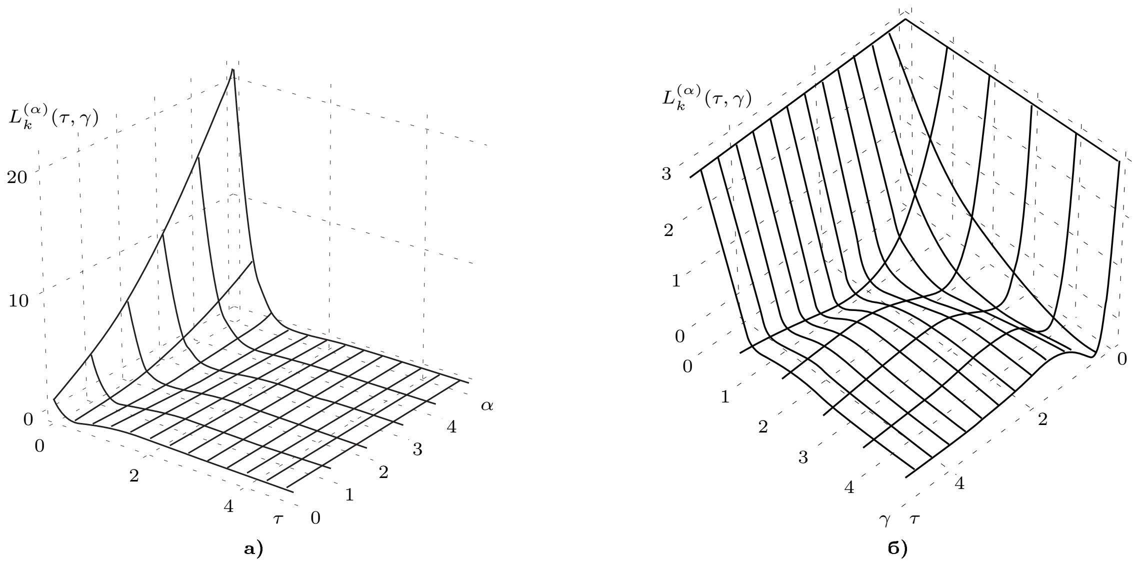


Рис. 1.4. Вид ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 4$, $\alpha \in [0;5]$; б) $\gamma \in (0;5]$, $\alpha = 1$

[1.5] $$P_k^{(-1/2,0)}(\tau,\gamma) = \sum_{s=0}^{k} \binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s \times \exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big).$$

Частные случаи для функций 0-5 порядков:

$P_0^{(-1/2,0)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big);$

$P_1^{(-1/2,0)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)\big(1 - 3\exp(-2\gamma\tau)\big)/2;$

$P_2^{(-1/2,0)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)\big(3 - 30\exp(-2\gamma\tau) + 35\,\times$
$\times\,\exp(-4\gamma\tau)\big)/8;$

$P_3^{(-1/2,0)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)\big(5 - 105\exp(-2\gamma\tau) + 315\,\times$
$\times\,\exp(-4\gamma\tau) - 231\exp(-6\gamma\tau)\big)/16;$

$P_4^{(-1/2,0)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)\big(35 - 1260\exp(-2\gamma\tau) + 6930\,\times$
$\times\,\exp(-4\gamma\tau) - 12012\exp(-6\gamma\tau) + 6435\exp(-8\gamma\tau)\big)/128;$

$P_5^{(-1/2,0)}(\tau,\gamma) = \exp\Big(-\frac{\gamma\tau}{2}\Big)\big(63 - 3465\exp(-2\gamma\tau) + 15015\,\times$
$\times\,\exp(-4\gamma\tau) - 45045\exp(-6\gamma\tau) + 109395\exp(-8\gamma\tau) - 46189\,\times$
$\times\,\exp(-10\gamma\tau)\big)/256.$

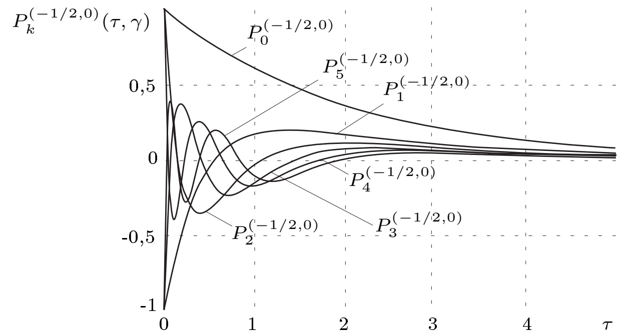


Рис. 1.5. Вид ортогональных функций Якоби 0-5 порядков; $\gamma = 1,\ c = 2,\ \alpha = -1/2,\ \beta = 0$

$$\textbf{[1.6]}\quad Leg_k(\tau,\gamma) = \sum_{s=0}^{k} \binom{k}{s}\binom{k+s}{s}(-1)^s \exp\bigl(-(2s+1)\gamma\tau\bigr).$$

Частные случаи для функций 0-5 порядков:

$Leg_0(\tau,\gamma) = \exp(-\gamma\tau)$;

$Leg_1(\tau,\gamma) = \exp(-\gamma\tau)\bigl(1 - 2\exp(-2\gamma\tau)\bigr)$;

$Leg_2(\tau,\gamma) = \exp(-\gamma\tau)\bigl(1 - 6\exp(-2\gamma\tau) + 6\exp(-4\gamma\tau)\bigr)$;

$Leg_3(\tau,\gamma) = \exp(-\gamma\tau)\bigl(1 - 12\exp(-2\gamma\tau) + 30\exp(-4\gamma\tau) -$
$- 20\exp(-6\gamma\tau)\bigr)$;

$Leg_4(\tau,\gamma) = \exp(-\gamma\tau)\bigl(1 - 20\exp(-2\gamma\tau) + 90\exp(-4\gamma\tau) -$
$- 140\exp(-6\gamma\tau) + 70\exp(-8\gamma\tau)\bigr)$;

$Leg_5(\tau,\gamma) = \exp(-\gamma\tau)\bigl(1 - 30\exp(-2\gamma\tau) + 210\exp(-4\gamma\tau) -$
$- 560\exp(-6\gamma\tau) + 630\exp(-8\gamma\tau) - 252\exp(-10\gamma\tau)\bigr)$.

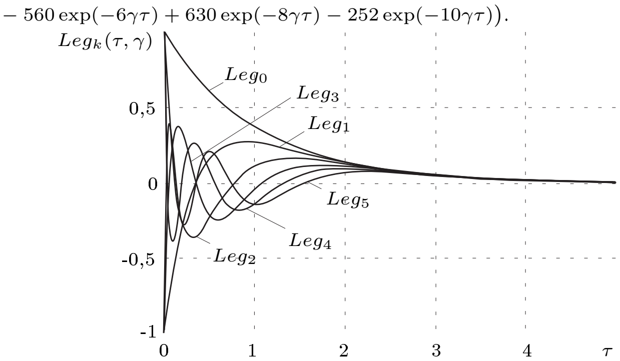


Рис. 1.6. Вид ортогональных функций Лежандра 0-5 порядков; $\gamma = 1,\ c = 2$

$$\textbf{[1.7]}\quad P_k^{(1/2,0)}(\tau,\gamma) = \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s \times \exp\Bigl(-\frac{(4s+3)}{2}\gamma\tau\Bigr).$$

Частные случаи для функций 0-5 порядков:

$P_0^{(1/2,0)}(\tau,\gamma) = \exp\Bigl(-\frac{3\gamma\tau}{2}\Bigr)$;

$P_1^{(1/2,0)}(\tau,\gamma) = \exp\Bigl(-\frac{3\gamma\tau}{2}\Bigr)\bigl(3 - 5\exp(-2\gamma\tau)\bigr)/2$;

$P_2^{(1/2,0)}(\tau,\gamma) = \exp\Bigl(-\frac{3\gamma\tau}{2}\Bigr)\bigl(15 - 70\exp(-2\gamma\tau) + 63 \times$
$\times \exp(-4\gamma\tau)\bigr)/8$;

$P_3^{(1/2,0)}(\tau,\gamma) = \exp\Bigl(-\frac{3\gamma\tau}{2}\Bigr)\bigl(35 - 315\exp(-2\gamma\tau) + 693 \times$
$\times \exp(-4\gamma\tau) - 429\exp(-6\gamma\tau)\bigr)/16$;

$P_4^{(1/2,0)}(\tau,\gamma) = \exp\Bigl(-\frac{3\gamma\tau}{2}\Bigr)\bigl(315 - 4620\exp(-2\gamma\tau) + 18018 \times$
$\times \exp(-4\gamma\tau) - 25740\exp(-6\gamma\tau) + 12155\exp(-8\gamma\tau)\bigr)/128$;

$P_5^{(1/2,0)}(\tau,\gamma) = \exp\Bigl(-\frac{3\gamma\tau}{2}\Bigr)\bigl(693 - 15015\exp(-2\gamma\tau) + 45045 \times$
$\times \exp(-4\gamma\tau) - 218790\exp(-6\gamma\tau) + 230945\exp(-8\gamma\tau) - 88179 \times$
$\times \exp(-10\gamma\tau)\bigr)/256$.

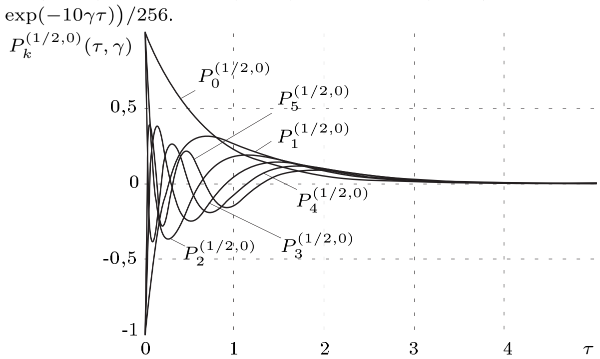


Рис. 1.7. Вид ортогональных функций Якоби 0-5 порядков; $\gamma = 1,\ c = 2,\ \alpha = 1/2,\ \beta = 0$

$$\textbf{[1.8]}\quad P_k^{(1,0)}(\tau,\gamma) = \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1}{s+1}(-1)^s \times \exp\bigl(-(s+1)\gamma\tau\bigr).$$

Частные случаи для функций 0-5 порядков:

$P_0^{(1,0)}(\tau,\gamma) = \exp(-\gamma\tau)$;

$P_1^{(1,0)}(\tau,\gamma) = \exp(-\gamma\tau)\bigl(2 - 3\exp(-\gamma\tau)\bigr)$;

$P_2^{(1,0)}(\tau,\gamma) = \exp(-\gamma\tau)\bigl(3 - 12\exp(-\gamma\tau) + 10\exp(-2\gamma\tau)\bigr)$;

$P_3^{(1,0)}(\tau,\gamma) = \exp(-\gamma\tau)\bigl(4 - 30\exp(-\gamma\tau) + 60\exp(-2\gamma\tau) -$
$- 35\exp(-3\gamma\tau)\bigr)$;

$P_4^{(1,0)}(\tau,\gamma) = \exp(-\gamma\tau)\bigl(5 - 60\exp(-\gamma\tau) + 210\exp(-2\gamma\tau) -$
$- 280\exp(-3\gamma\tau) + 126\exp(-4\gamma\tau)\bigr)$;

$P_5^{(1,0)}(\tau,\gamma) = \exp(-\gamma\tau)\bigl(6 - 105\exp(-\gamma\tau) + 560\exp(-2\gamma\tau) -$
$- 1260\exp(-3\gamma\tau) + 1260\exp(-4\gamma\tau) - 462\exp(-5\gamma\tau)\bigr)$.

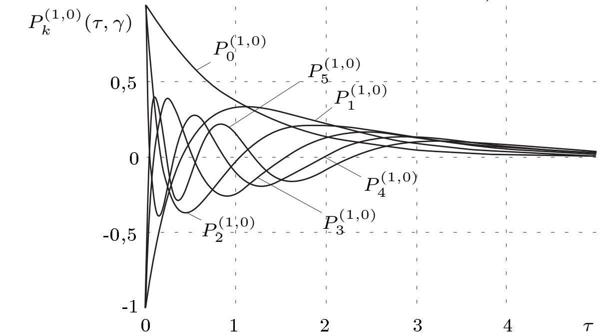


Рис. 1.8. Вид ортогональных функций Якоби 0-5 порядков; $\gamma = 1,\ c = 1,\ \alpha = 1,\ \beta = 0$

[1.9] $$P_k^{(2,0)}(\tau,\gamma)=\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s\times\exp\bigl(-(2s+3)\gamma\tau\bigr).$$

Частные случаи для функций 0-5 порядков:

$P_0^{(2,0)}(\tau,\gamma)=\exp(-3\gamma\tau)$;

$P_1^{(2,0)}(\tau,\gamma)=\exp(-3\gamma\tau)\bigl(3-4\exp(-2\gamma\tau)\bigr)$;

$P_2^{(2,0)}(\tau,\gamma)=\exp(-3\gamma\tau)\bigl(6-20\exp(-2\gamma\tau)+15\exp(-4\gamma\tau)\bigr)$;

$P_3^{(2,0)}(\tau,\gamma)=\exp(-3\gamma\tau)\bigl(10-60\exp(-2\gamma\tau)+105\times$
$\times\exp(-4\gamma\tau)-56\exp(-6\gamma\tau)\bigr)$;

$P_4^{(2,0)}(\tau,\gamma)=\exp(-\gamma\tau)\bigl(15-140\exp(-2\gamma\tau)+420\times$
$\times\exp(-4\gamma\tau)-504\exp(-6\gamma\tau)+210\exp(-8\gamma\tau)\bigr)$;

$P_5^{(2,0)}(\tau,\gamma)=\exp(-\gamma\tau)\bigl(21-280\exp(-2\gamma\tau)+1260\times$
$\times\exp(-4\gamma\tau)-2520\exp(-6\gamma\tau)+2310\exp(-8\gamma\tau)-792\times$
$\times\exp(-10\gamma\tau)\bigr)$.

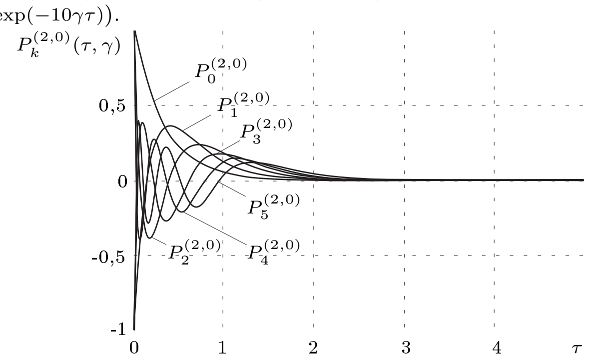


Рис. 1.9. Вид ортогональных функций Якоби 0-5 порядков; $\gamma=1$, $c=2$, $\alpha=2$, $\beta=0$

[1.10] $$P_k^{(\alpha,0)}(\tau,\gamma)=\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s\times\exp\bigl(-(2s+\alpha+1)c\gamma\tau/2\bigr).$$

Частные случаи для функций 0-5 порядков:

$P_0^{(\alpha,0)}(\tau,\gamma)=\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)$;

$P_1^{(\alpha,0)}(\tau,\gamma)=\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl(\alpha+1-(\alpha+2)\exp(-c\gamma\tau)\bigr)$;

$P_2^{(\alpha,0)}(\tau,\gamma)=\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)(\alpha+2)-2(\alpha+2)\times$
$\times(\alpha+3)\exp(-c\gamma\tau)+(\alpha+3)(\alpha+4)\exp(-2c\gamma\tau)\bigr)/2$;

$P_3^{(\alpha,0)}(\tau,\gamma)=\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)(\alpha+2)(\alpha+3)-$
$-3(\alpha+2)(\alpha+3)(\alpha+4)\exp(-c\gamma\tau)+3(\alpha+3)(\alpha+4)(\alpha+5)\times$
$\times\exp(-2c\gamma\tau)-(\alpha+4)(\alpha+5)(\alpha+6)\exp(-3c\gamma\tau)\bigr)/6$;

$P_4^{(\alpha,0)}(\tau,\gamma)=\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)(\alpha+2)(\alpha+3)\times$
$\times(\alpha+4)-4(\alpha+2)(\alpha+3)(\alpha+4)(\alpha+5)\exp(-c\gamma\tau)+6(\alpha+3)\times$
$\times(\alpha+4)(\alpha+5)(\alpha+6)\exp(-2c\gamma\tau)-4(\alpha+4)(\alpha+5)(\alpha+6)\times$
$\times(\alpha+7)\exp(-3c\gamma\tau)+(\alpha+5)(\alpha+6)(\alpha+7)(\alpha+8)\times$
$\times\exp(-4c\gamma\tau)\bigr)/24$;

$P_5^{(\alpha,0)}(\tau,\gamma)=\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)(\alpha+2)(\alpha+3)\times$
$\times(\alpha+4)(\alpha+5)-5(\alpha+2)(\alpha+3)(\alpha+4)(\alpha+5)(\alpha+6)\times$
$\times\exp(-c\gamma\tau)+10(\alpha+3)(\alpha+4)(\alpha+5)(\alpha+6)(\alpha+7)\times$
$\times\exp(-2c\gamma\tau)-10(\alpha+4)(\alpha+5)(\alpha+6)(\alpha+7)(\alpha+8)\times$
$\times\exp(-3c\gamma\tau)+5(\alpha+5)(\alpha+6)(\alpha+7)(\alpha+8)(\alpha+9)\times$
$\times\exp(-4c\gamma\tau)-(\alpha+6)(\alpha+7)(\alpha+8)(\alpha+9)(\alpha+10)\times$
$\times\exp(-5c\gamma\tau)\bigr)/120$.

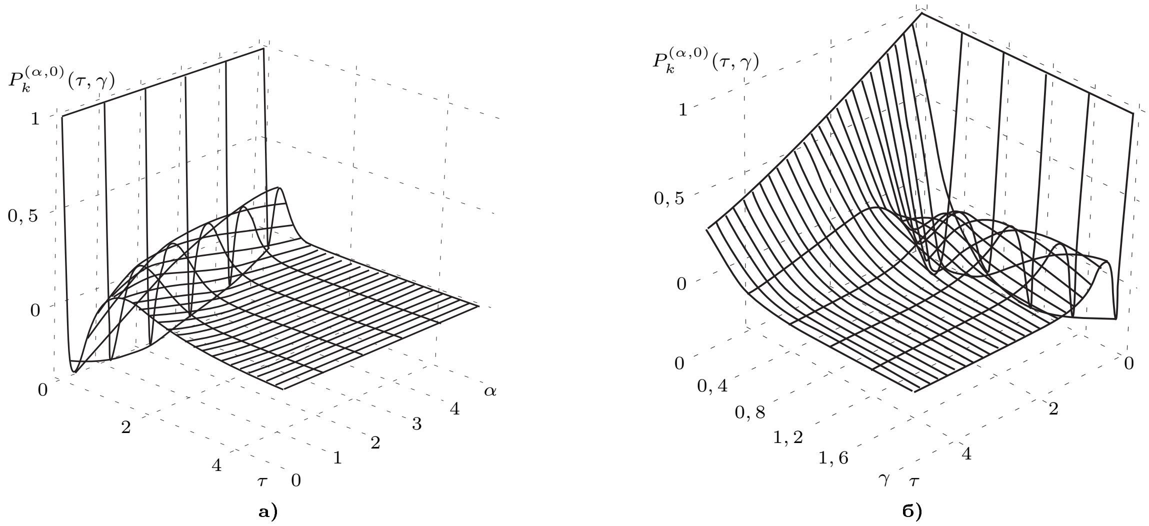


Рис. 1.10. Вид ортогональных функций Якоби 2-ого порядка: а) $\gamma=1$, $c=2$, $\alpha\in[0;5]$, $\beta=0$; б) $\gamma\in(0;2]$, $c=2$, $\alpha=1$, $\beta=0$

[1.11] $$P_k^{(0,1)}(\tau,\gamma)=\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s\times\exp\bigl(-(2s+1)\gamma\tau\bigr).$$

Частные случаи для функций 0-5 порядков:

$P_0^{(0,1)}(\tau,\gamma)=\exp(-\gamma\tau)$;

$P_1^{(0,1)}(\tau,\gamma)=\exp(-\gamma\tau)(1-3\exp(-2\gamma\tau))$;

$P_2^{(0,1)}(\tau,\gamma) = \exp(-\gamma\tau)(1 - 8\exp(-2\gamma\tau) + 10\exp(-4\gamma\tau));$

$P_3^{(0,1)}(\tau,\gamma) = \exp(-\gamma\tau)(1 - 15\exp(-2\gamma\tau) + 45\exp(-4\gamma\tau) -$
$- 35\exp(-6\gamma\tau));$

$P_4^{(0,1)}(\tau,\gamma) = \exp(-\gamma\tau)(1 - 24\exp(-2\gamma\tau) + 126\exp(-4\gamma\tau) -$
$- 224\exp(-6\gamma\tau) + 126\exp(-8\gamma\tau));$

$P_5^{(0,1)}(\tau,\gamma) = \exp(-\gamma\tau)(1 - 35\exp(-2\gamma\tau) + 280\exp(-4\gamma\tau) -$
$- 840\exp(-6\gamma\tau) + 1050\exp(-8\gamma\tau) - 462\exp(-10\gamma\tau).$

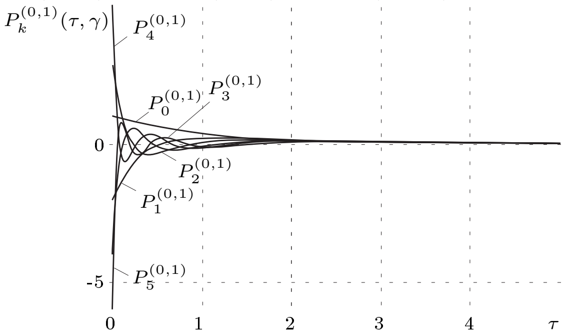


Рис. 1.11. Вид ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.12] $$P_k^{(0,2)}(\tau,\gamma) = \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+2}{s}(-1)^s \times \exp\bigl(-(2s+1)\gamma\tau\bigr).$$

Частные случаи для функций 0-5 порядков:

$P_0^{(0,2)}(\tau,\gamma) = \exp(-\gamma\tau);$

$P_1^{(0,2)}(\tau,\gamma) = \exp(-\gamma\tau)(1 - 4\exp(-2\gamma\tau));$

$P_2^{(0,2)}(\tau,\gamma) = \exp(-\gamma\tau)(1 - 10\exp(-2\gamma\tau) + 15\exp(-4\gamma\tau));$

$P_3^{(0,2)}(\tau,\gamma) = \exp(-\gamma\tau)(1 - 18\exp(-2\gamma\tau) + 63\exp(-4\gamma\tau) -$
$- 56\exp(-6\gamma\tau));$

$P_4^{(0,2)}(\tau,\gamma) = \exp(-\gamma\tau)(1 - 28\exp(-2\gamma\tau) + 168\exp(-4\gamma\tau) -$
$- 336\exp(-6\gamma\tau) + 210\exp(-8\gamma\tau));$

$P_5^{(0,2)}(\tau,\gamma) = \exp(-\gamma\tau)(1 - 40\exp(-2\gamma\tau) + 360\exp(-4\gamma\tau) -$
$- 1200\exp(-6\gamma\tau) + 1650\exp(-8\gamma\tau) - 792\exp(-10\gamma\tau).$

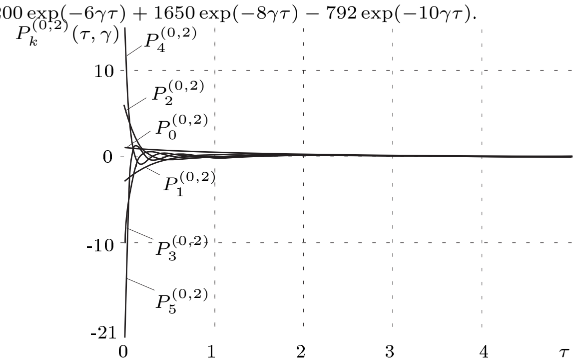


Рис. 1.12. Вид ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.13] $$P_k^{(0,\beta)}(\tau,\gamma) = \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+\beta}{s}(-1)^s \times \exp\bigl(-(2s+1)c\gamma\tau/2\bigr).$$

Частные случаи для функций 0-5 порядков:

$P_0^{(0,\beta)}(\tau,\gamma) = \exp(-c\gamma\tau/2);$

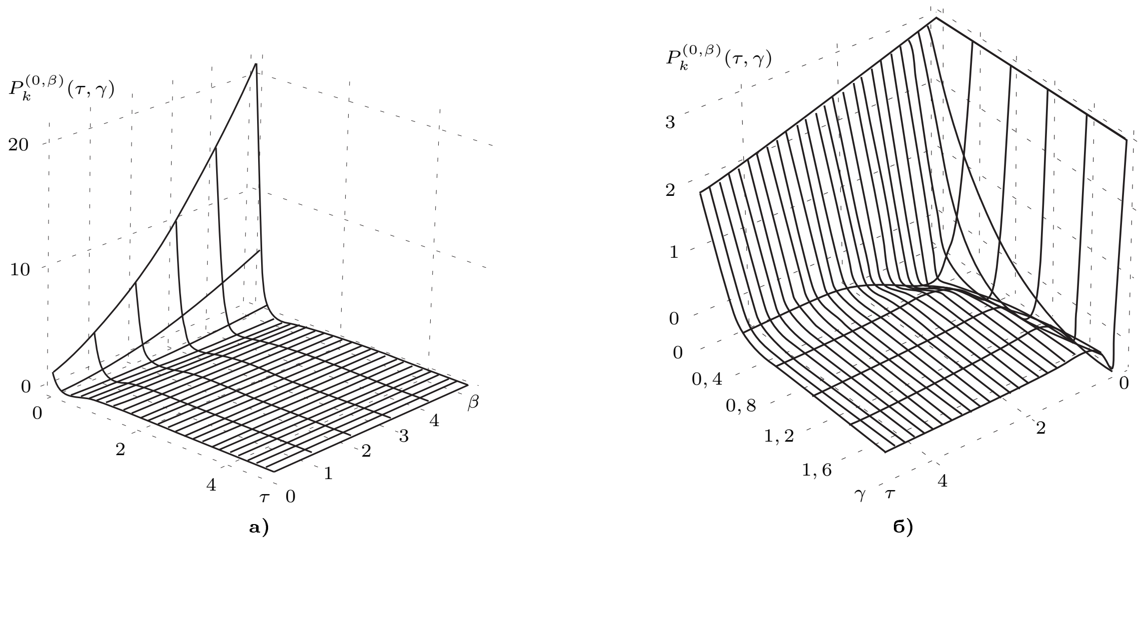
$P_1^{(0,\beta)}(\tau,\gamma) = \exp(-c\gamma\tau/2)\bigl(1 - (\beta+2)\exp(-c\gamma\tau)\bigr);$

$P_2^{(0,\beta)}(\tau,\gamma) = \exp(-c\gamma\tau/2)\bigl(1 - 2(\beta+3)\exp(-c\gamma\tau) + (\beta+3) \times$
$\times (\beta+4)\exp(-2c\gamma\tau)/2\bigr);$

$P_3^{(0,\beta)}(\tau,\gamma) = \exp(-c\gamma\tau/2)\bigl(1 - 3(\beta+4)\exp(-c\gamma\tau) + 3(\beta+4) \times$
$\times (\beta+5)\exp(-2c\gamma\tau)/2 - (\beta+4)(\beta+5)(\beta+6)\exp(-3c\gamma\tau)/6\bigr);$

$P_4^{(0,\beta)}(\tau,\gamma) = \exp(-c\gamma\tau/2)\bigl(1 - 4(\beta+5)\exp(-c\gamma\tau) + 3(\beta+5) \times$
$\times (\beta+6)\exp(-2c\gamma\tau) - 2(\beta+5)(\beta+6)(\beta+7) \times$
$\times \exp(-3c\gamma\tau)/3 + (\beta+5)(\beta+6)(\beta+7)(\beta+8)\exp(-4c\gamma\tau)/24\bigr);$

$P_5^{(0,\beta)}(\tau,\gamma) = \exp(-c\gamma\tau/2)\bigl(1 - 5(\beta+6)\exp(-c\gamma\tau) + 5(\beta+6) \times$
$\times (\beta+7)\exp(-2c\gamma\tau) - 5(\beta+6)(\beta+7)(\beta+8)\exp(-3c\gamma\tau)/3 +$
$+ 5(\beta+6)(\beta+7)(\beta+8)(\beta+9)\exp(-4c\gamma\tau)/24 - (\beta+6)(\beta+7) \times$
$\times (\beta+8)(\beta+9)(\beta+10)\exp(-5c\gamma\tau)/120\bigr).$



а) **б)**

Рис. 1.13. Вид ортогональных функций Якоби 2-ого порядка: а) $\gamma = 1$, $c = 2$, $\beta \in [0; 5]$, $\alpha = 0$; б) $\gamma \in (0; 2]$, $c = 2$, $\alpha = 0$, $\beta = 1$

1.2 Аналитические соотношения для производных ортогональных функций

[1.14]
$$\frac{\partial L_k(\tau,\gamma)}{\partial \tau} = -\gamma\left(\sum_{s=1}^{k}\binom{k}{s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{2}\sum_{s=0}^{k}\binom{k}{s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\Big(-\frac{\gamma\tau}{2}\Big).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial L_0(\tau,\gamma)}{\partial \tau} = -\frac{\gamma}{2}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$

$$\frac{\partial L_1(\tau,\gamma)}{\partial \tau} = \frac{\gamma}{2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau - 3);$$

$$\frac{\partial L_2(\tau,\gamma)}{\partial \tau} = -\frac{\gamma}{4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 - 8\gamma\tau + 10);$$

$$\frac{\partial L_3(\tau,\gamma)}{\partial \tau} = \frac{\gamma}{12}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 - 15\gamma^2\tau^2 + 54\gamma\tau - 42);$$

$$\frac{\partial L_4(\tau,\gamma)}{\partial \tau} = -\frac{\gamma}{48}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 - 24\gamma^3\tau^3 + 168\gamma^2\tau^2 - 384\gamma\tau + 216);$$

$$\frac{\partial L_5(\tau,\gamma)}{\partial \tau} = \frac{\gamma}{240}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 - 35\gamma^4\tau^4 + 400\gamma^3\tau^3 - 1800\gamma^2\tau^2 + 3000\gamma\tau - 1320).$$

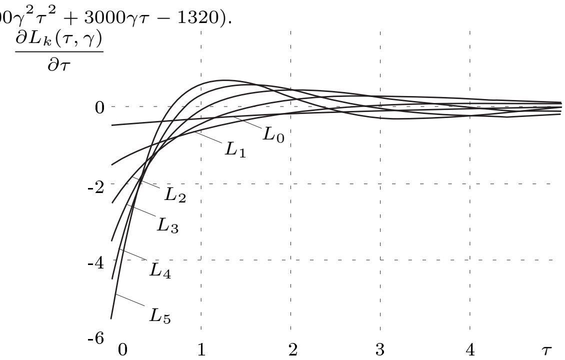


Рис. 1.14. Вид 1-ой производной ортогональных функций Лагерра 0-5 порядков; $\gamma = 1$

[1.15]
$$\frac{\partial^2 L_k(\tau,\gamma)}{\partial \tau^2} = \gamma^2\left(\sum_{s=2}^{k}\binom{k}{s}\frac{(-\gamma\tau)^{s-2}}{(s-2)!} + \sum_{s=1}^{k}\binom{k}{s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{4}\sum_{s=0}^{k}\binom{k}{s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\Big(-\frac{\gamma\tau}{2}\Big).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 L_0(\tau,\gamma)}{\partial \tau^2} = \frac{\gamma^2}{4}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$

$$\frac{\partial^2 L_1(\tau,\gamma)}{\partial \tau^2} = -\frac{\gamma^2}{4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau - 5);$$

$$\frac{\partial^2 L_2(\tau,\gamma)}{\partial \tau^2} = \frac{\gamma^2}{8}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 - 12\gamma\tau + 26);$$

$$\frac{\partial^2 L_3(\tau,\gamma)}{\partial \tau^2} = -\frac{\gamma^2}{24}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 - 21\gamma^2\tau^2 + 114\gamma\tau - 150);$$

$$\frac{\partial^2 L_4(\tau,\gamma)}{\partial \tau^2} = \frac{\gamma^2}{96}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 - 32\gamma^3\tau^3 + 312\gamma^2\tau^2 - 1056\gamma\tau + 984);$$

$$\frac{\partial^2 L_5(\tau,\gamma)}{\partial \tau^2} = -\frac{\gamma^2}{480}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 - 45\gamma^4\tau^4 + 680\gamma^3\tau^3 - 4200\gamma^2\tau^2 + 10200\gamma\tau - 7320).$$

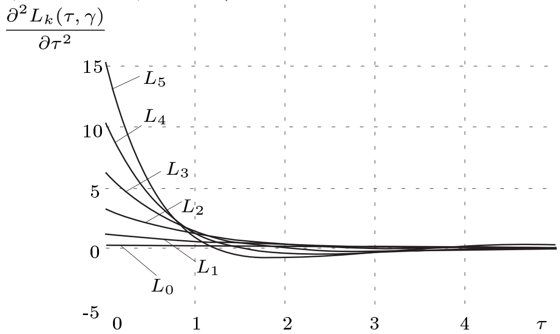


Рис. 1.15. Вид 2-ой производной ортогональных функций Лагерра 0-5 порядков; $\gamma = 1$

[1.16]
$$\frac{\partial^3 L_k(\tau,\gamma)}{\partial \tau^3} = -\gamma^3\left(\sum_{s=3}^{k}\binom{k}{s}\frac{(-\gamma\tau)^{s-3}}{(s-3)!} + \frac{3}{2}\sum_{s=2}^{k}\binom{k}{s}\frac{(-\gamma\tau)^{s-2}}{(s-2)!} + \frac{3}{4}\sum_{s=1}^{k}\binom{k}{s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{8}\sum_{s=0}^{k}\binom{k}{s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\Big(-\frac{\gamma\tau}{2}\Big).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 L_0(\tau,\gamma)}{\partial \tau^3} = -\frac{\gamma^3}{8}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$

$$\frac{\partial^3 L_1(\tau,\gamma)}{\partial \tau^3} = \frac{\gamma^3}{8}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau - 7);$$

$$\frac{\partial^3 L_2(\tau,\gamma)}{\partial \tau^3} = -\frac{\gamma^3}{16}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 - 16\gamma\tau + 50);$$

$$\frac{\partial^3 L_3(\tau,\gamma)}{\partial \tau^3} = \frac{\gamma^3}{48}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 - 27\gamma^2\tau^2 + 198\gamma\tau - 378);$$

$$\frac{\partial^3 L_4(\tau,\gamma)}{\partial \tau^3} = -\frac{\gamma^3}{192}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 - 40\gamma^3\tau^3 + 504\gamma^2\tau^2 - 2304\gamma\tau + 3096);$$

$$\frac{\partial^3 L_5(\tau,\gamma)}{\partial \tau^3} = \frac{\gamma^3}{960}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 - 55\gamma^4\tau^4 + 1040\gamma^3\tau^3 - 8280\gamma^2\tau^2 + 27000\gamma\tau - 27720).$$

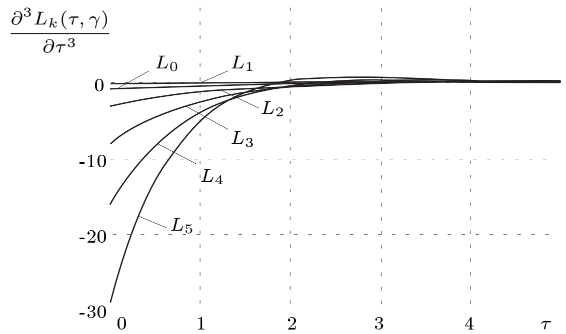


Рис. 1.16. Вид 3-ой производной ортогональных функций Лагерра 0-5 порядков; $\gamma = 1$

[1.17]
$$\frac{\partial^n L_k(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\sum_{s=0}^{k}\binom{k}{s}\begin{cases}\dfrac{(-\gamma\tau)^{s-n+j}}{(s-n+j)!}, & \text{если } s-n+j\geq 0;\\ 0, & \text{иначе.}\end{cases}$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n L_0(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\begin{cases}\dfrac{(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\geq 0;\\ 0, & \text{иначе.}\end{cases};$$

$$\frac{\partial^n L_1(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_2(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$+\begin{cases}\dfrac{2(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_3(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$+\begin{cases}\dfrac{3(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{3(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_4(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$+\begin{cases}\dfrac{4(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{6(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{4(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{4-n+j}}{(4-n+j)!}, & \text{если } 4-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_5(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$+\begin{cases}\dfrac{5(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{10(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{10(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{5(-\gamma\tau)^{4-n+j}}{(4-n+j)!}, & \text{если } 4-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{5-n+j}}{(5-n+j)!}, & \text{если } 5-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}\right).$$

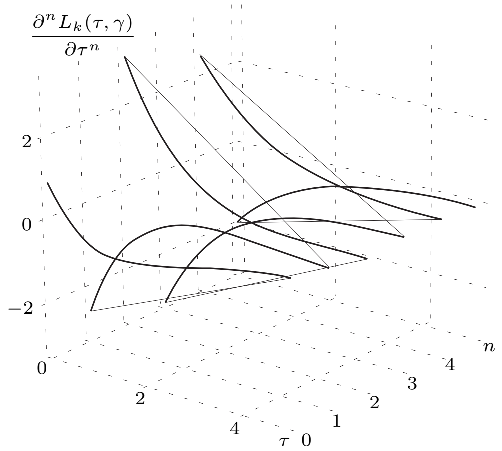


Рис. 1.17. Вид n-ой производной ортогональных функций Лагерра 2-ого порядка; $n = 0..5$, $\gamma = 1$

[1.18]
$$\frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial \tau} = -\gamma\left(\sum_{s=1}^{k}\binom{k+1}{k-s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{2}\sum_{s=0}^{k}\binom{k+1}{k-s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial L_0^{(1)}(\tau,\gamma)}{\partial \tau} = -\frac{\gamma}{2}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr);$$
$$\frac{\partial L_1^{(1)}(\tau,\gamma)}{\partial \tau} = \frac{\gamma}{2}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma\tau-4);$$
$$\frac{\partial L_2^{(1)}(\tau,\gamma)}{\partial \tau} = -\frac{\gamma}{4}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^2\tau^2-10\gamma\tau+18);$$
$$\frac{\partial L_3^{(1)}(\tau,\gamma)}{\partial \tau} = \frac{\gamma}{12}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^3\tau^3-18\gamma^2\tau^2+84\gamma\tau-96);$$
$$\frac{\partial L_4^{(1)}(\tau,\gamma)}{\partial \tau} = -\frac{\gamma}{48}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^4\tau^4-28\gamma^3\tau^3+240\gamma^2\tau^2- -720\gamma\tau+600);$$
$$\frac{\partial L_5^{(1)}(\tau,\gamma)}{\partial \tau} = \frac{\gamma}{240}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^5\tau^5-40\gamma^4\tau^4+540\gamma^3\tau^3- -3000\gamma^2\tau^2+6600\gamma\tau-4320).$$

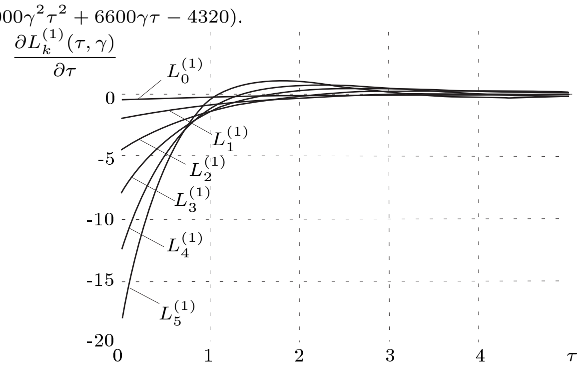


Рис. 1.18. Вид 1-ой производной ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 1$, $\alpha = 1$

[1.19]
$$\frac{\partial^2 L_k^{(1)}(\tau,\gamma)}{\partial \tau^2} = \gamma^2\left(\sum_{s=2}^{k}\binom{k+1}{k-s}\frac{(-\gamma\tau)^{s-2}}{(s-2)!} + \sum_{s=1}^{k}\binom{k+1}{k-s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{4}\sum_{s=0}^{k}\binom{k+1}{k-s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 L_0^{(1)}(\tau,\gamma)}{\partial \tau^2} = \frac{\gamma^2}{4}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr);$$
$$\frac{\partial^2 L_1^{(1)}(\tau,\gamma)}{\partial \tau^2} = -\frac{\gamma^2}{4}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma\tau-6);$$
$$\frac{\partial^2 L_2^{(1)}(\tau,\gamma)}{\partial \tau^2} = \frac{\gamma^2}{8}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^2\tau^2-14\gamma\tau+38);$$
$$\frac{\partial^2 L_3^{(1)}(\tau,\gamma)}{\partial \tau^2} = -\frac{\gamma^2}{24}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^3\tau^3-24\gamma^2\tau^2+156\gamma\tau-264);$$
$$\frac{\partial^2 L_4^{(1)}(\tau,\gamma)}{\partial \tau^2} = \frac{\gamma^2}{96}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^4\tau^4-36\gamma^3\tau^3+408\gamma^2\tau^2- -1680\gamma\tau+2040);$$
$$\frac{\partial^2 L_5^{(1)}(\tau,\gamma)}{\partial \tau^2} = -\frac{\gamma^2}{480}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^5\tau^5-50\gamma^4\tau^4+860\gamma^3\tau^3- -6240\gamma^2\tau^2+18600\gamma\tau-17520).$$

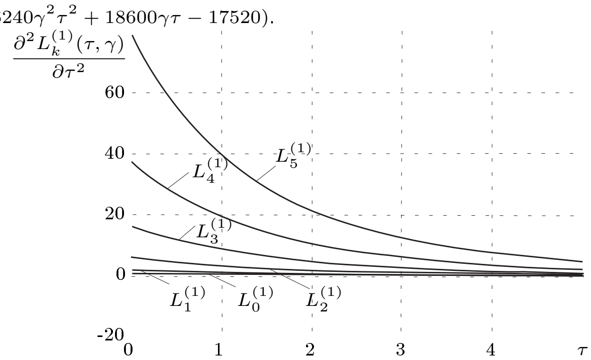


Рис. 1.19. Вид 2-ой производной ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 1$, $\alpha = 1$

[1.20]
$$\frac{\partial^3 L_k^{(1)}(\tau,\gamma)}{\partial \tau^3} = -\gamma^3\left(\sum_{s=3}^{k}\binom{k+1}{k-s}\frac{(-\gamma\tau)^{s-3}}{(s-3)!} + \frac{3}{2}\sum_{s=2}^{k}\binom{k+1}{k-s}\frac{(-\gamma\tau)^{s-2}}{(s-2)!} + \frac{3}{4}\sum_{s=1}^{k}\binom{k+1}{k-s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{8}\sum_{s=0}^{k}\binom{k+1}{k-s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 L_0^{(1)}(\tau,\gamma)}{\partial \tau^3} = -\frac{\gamma^3}{8}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr);$$
$$\frac{\partial^3 L_1^{(1)}(\tau,\gamma)}{\partial \tau^3} = \frac{\gamma^3}{8}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma\tau-8);$$
$$\frac{\partial^3 L_2^{(1)}(\tau,\gamma)}{\partial \tau^3} = -\frac{\gamma^3}{16}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^2\tau^2-18\gamma\tau+66);$$

$$\frac{\partial^3 L_3^{(1)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{48}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^3\tau^3 - 30\gamma^2\tau^2 + 252\gamma\tau - 576);$$

$$\frac{\partial^3 L_4^{(1)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{192}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^4\tau^4 - 44\gamma^3\tau^3 + 624\gamma^2\tau^2 - 3312\gamma\tau + 5400);$$

$$\frac{\partial^3 L_5^{(1)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{960}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma^5\tau^5 - 60\gamma^4\tau^4 + 1260\gamma^3\tau^3 - 11400\gamma^2\tau^2 + 43560\gamma\tau - 54720).$$

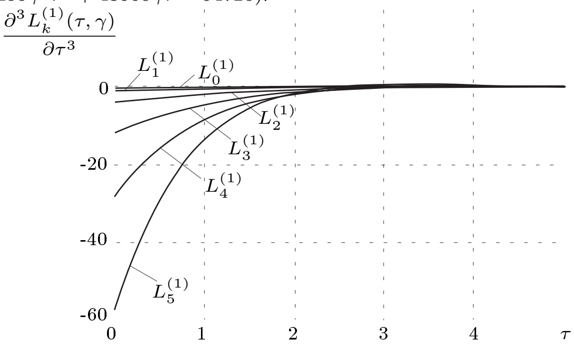


Рис. 1.20. Вид 3-ой производной ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 1$, $\alpha = 1$

[1.21]
$$\frac{\partial^n L_k^{(1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \times\sum_{s=0}^{k}\binom{k+1}{k-s}\begin{cases}\dfrac{(-\gamma\tau)^{s-n+j}}{(s-n+j)!}, & \text{если } s-n+j\ge 0;\\ 0, & \text{иначе.}\end{cases}$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n L_0^{(1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \times\begin{cases}\dfrac{(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе.}\end{cases};$$

$$\frac{\partial^n L_1^{(1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \times\left(\begin{cases}\dfrac{2(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_2^{(1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \times\left(\begin{cases}\dfrac{3(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{3(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_3^{(1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \times\left(\begin{cases}\dfrac{4(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{6(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{4(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_4^{(1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \times\left(\begin{cases}\dfrac{5(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{10(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{10(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{5(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{(-\gamma\tau)^{4-n+j}}{(4-n+j)!}, & \text{если } 4-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_5^{(1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \times\left(\begin{cases}\dfrac{6(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{15(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{20(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{15(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{6(-\gamma\tau)^{4-n+j}}{(4-n+j)!}, & \text{если } 4-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \begin{cases}\dfrac{(-\gamma\tau)^{5-n+j}}{(5-n+j)!}, & \text{если } 5-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right).$$

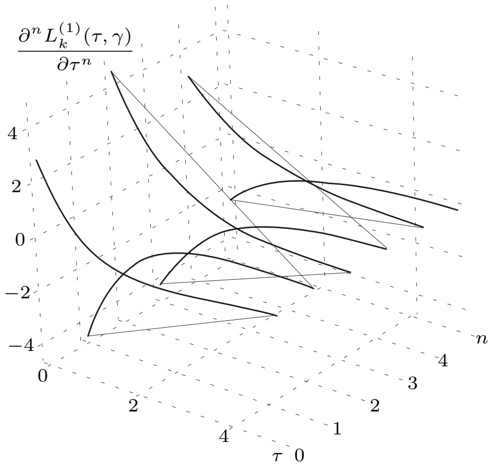


Рис. 1.21. Вид n-ой производной ортогональных функций Сонина-Лагерра 2-ого порядка; $n = 0..5$, $\gamma = 1$, $\alpha = 1$

[1.22]
$$\frac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau} = -\gamma\left(\sum_{s=1}^{k}\binom{k+2}{k-s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{2}\sum_{s=0}^{k}\binom{k+2}{k-s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\Big(-\frac{\gamma\tau}{2}\Big).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial L_0^{(2)}(\tau,\gamma)}{\partial\tau} = -\frac{\gamma}{2}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$
$$\frac{\partial L_1^{(2)}(\tau,\gamma)}{\partial\tau} = \frac{\gamma}{2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau-5);$$
$$\frac{\partial L_2^{(2)}(\tau,\gamma)}{\partial\tau} = -\frac{\gamma}{4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2-12\gamma\tau+28);$$
$$\frac{\partial L_3^{(2)}(\tau,\gamma)}{\partial\tau} = \frac{\gamma}{12}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3-21\gamma^2\tau^2+120\gamma\tau-180);$$
$$\frac{\partial L_4^{(2)}(\tau,\gamma)}{\partial\tau} = -\frac{\gamma}{48}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4-32\gamma^3\tau^3+324\gamma^2\tau^2-$$
$$-1200\gamma\tau+1320);$$
$$\frac{\partial L_5^{(2)}(\tau,\gamma)}{\partial\tau} = \frac{\gamma}{240}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5-45\gamma^4\tau^4+700\gamma^3\tau^3-$$
$$-4620\gamma^2\tau^2+12600\gamma\tau-10920).$$

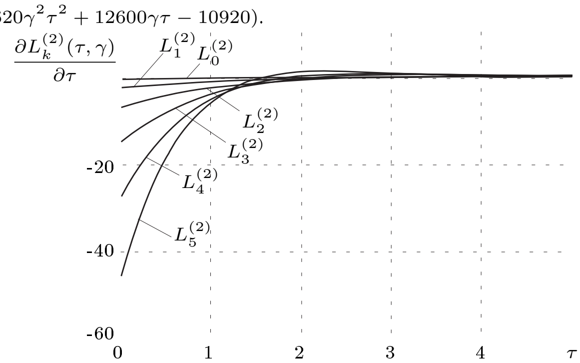


Рис. 1.22. Вид 1-ой производной ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 1$, $\alpha = 2$

[1.23]
$$\frac{\partial^2 L_k^{(2)}(\tau,\gamma)}{\partial\tau^2} = \gamma^2\left(\sum_{s=2}^{k}\binom{k+2}{k-s}\frac{(-\gamma\tau)^{s-2}}{(s-2)!} + \sum_{s=1}^{k}\binom{k+2}{k-s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{4}\sum_{s=0}^{k}\binom{k+2}{k-s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\Big(-\frac{\gamma\tau}{2}\Big).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 L_0^{(2)}(\tau,\gamma)}{\partial\tau^2} = \frac{\gamma^2}{4}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$
$$\frac{\partial^2 L_1^{(2)}(\tau,\gamma)}{\partial\tau^2} = -\frac{\gamma^2}{4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau-7);$$
$$\frac{\partial^2 L_2^{(2)}(\tau,\gamma)}{\partial\tau^2} = \frac{\gamma^2}{8}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2-16\gamma\tau+52);$$
$$\frac{\partial^2 L_3^{(2)}(\tau,\gamma)}{\partial\tau^2} = -\frac{\gamma^2}{24}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3-27\gamma^2\tau^2+204\gamma\tau-420);$$
$$\frac{\partial^2 L_4^{(2)}(\tau,\gamma)}{\partial\tau^2} = \frac{\gamma^2}{96}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4-40\gamma^3\tau^3+516\gamma^2\tau^2-$$
$$-2496\gamma\tau+3720);$$
$$\frac{\partial^2 L_5^{(2)}(\tau,\gamma)}{\partial\tau^2} = -\frac{\gamma^2}{480}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5-55\gamma^4\tau^4+1060\gamma^3\tau^3-$$
$$-8820\gamma^2\tau^2+31080\gamma\tau-36120).$$

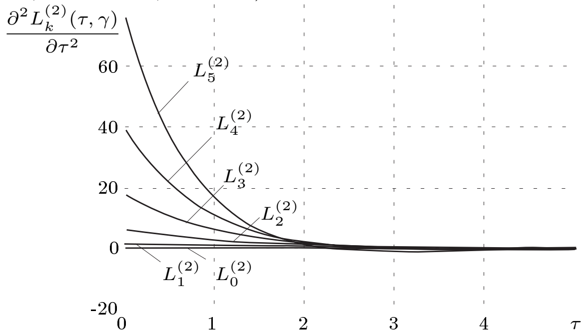


Рис. 1.23. Вид 2-ой производной ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 1$, $\alpha = 2$

[1.24]
$$\frac{\partial^3 L_k^{(2)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\left(\sum_{s=3}^{k}\binom{k+2}{k-s}\frac{(-\gamma\tau)^{s-3}}{(s-3)!} + \frac{3}{2}\sum_{s=2}^{k}\binom{k+2}{k-s}\frac{(-\gamma\tau)^{s-2}}{(s-2)!} + \frac{3}{4}\sum_{s=1}^{k}\binom{k+2}{k-s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{8}\sum_{s=0}^{k}\binom{k+2}{k-s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\Big(-\frac{\gamma\tau}{2}\Big).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 L_0^{(2)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{8}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$
$$\frac{\partial^3 L_1^{(2)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{8}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau-9);$$
$$\frac{\partial^3 L_2^{(2)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{16}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2-20\gamma\tau+84);$$

$$\frac{\partial^3 L_3^{(2)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{48}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 - 33\gamma^2\tau^2 + 312\gamma\tau - 828);$$

$$\frac{\partial^3 L_4^{(2)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{192}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 - 48\gamma^3\tau^3 + 756\gamma^2\tau^2 - $$
$$- 4560\gamma\tau + 8712);$$

$$\frac{\partial^3 L_5^{(2)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{960}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 - 65\gamma^4\tau^4 + 1500\gamma^3\tau^3 - $$
$$- 15180\gamma^2\tau^2 + 66360\gamma\tau - 98280).$$

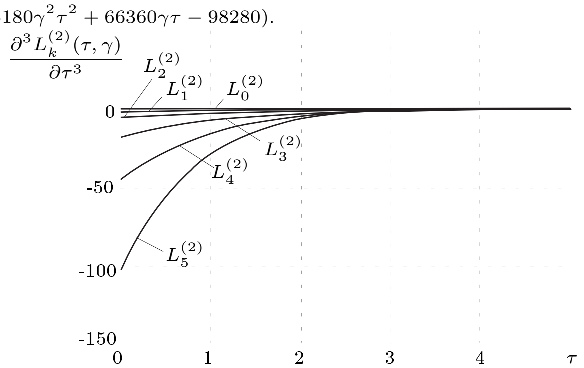


Рис. 1.24. Вид 3-ой производной ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 1$, $\alpha = 2$

[1.25]
$$\frac{\partial^n L_k^{(2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\sum_{s=0}^{k}\binom{k+2}{k-s}\begin{cases}\dfrac{(-\gamma\tau)^{s-n+j}}{(s-n+j)!}, & \text{если } s-n+j\ge 0;\\ 0, & \text{иначе.}\end{cases}$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n L_0^{(2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\begin{cases}\dfrac{(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе.}\end{cases};$$

$$\frac{\partial^n L_1^{(2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{3(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \right.$$
$$\left. + \begin{cases}\dfrac{(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_2^{(2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{6(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \right.$$
$$+ \begin{cases}\dfrac{4(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$\left. + \begin{cases}\dfrac{(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_3^{(2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{10(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \right.$$
$$+ \begin{cases}\dfrac{10(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$+ \begin{cases}\dfrac{5(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$\left. + \begin{cases}\dfrac{(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_4^{(2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{15(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \right.$$
$$+ \begin{cases}\dfrac{20(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$+ \begin{cases}\dfrac{15(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$+ \begin{cases}\dfrac{6(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$\left. + \begin{cases}\dfrac{(-\gamma\tau)^{4-n+j}}{(4-n+j)!}, & \text{если } 4-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_5^{(2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{21(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + \right.$$
$$+ \begin{cases}\dfrac{35(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$+ \begin{cases}\dfrac{35(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$+ \begin{cases}\dfrac{21(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$+ \begin{cases}\dfrac{7(-\gamma\tau)^{4-n+j}}{(4-n+j)!}, & \text{если } 4-n+j\ge 0;\\ 0, & \text{иначе}\end{cases} + $$
$$\left. + \begin{cases}\dfrac{(-\gamma\tau)^{5-n+j}}{(5-n+j)!}, & \text{если } 5-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right).$$

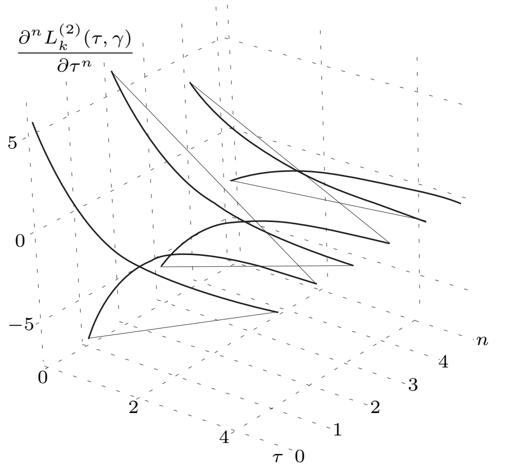


Рис. 1.25. Вид n-ой производной ортогональных функций Сонина-Лагерра 2-ого порядка; $n = 0..5$, $\gamma = 1$, $\alpha = 2$

[1.26]
$$\frac{\partial L_k^{(\alpha)}(\tau,\gamma)}{\partial\tau} = -\gamma\left(\sum_{s=1}^{k}\binom{k+\alpha}{k-s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{2}\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\left(-\frac{\gamma\tau}{2}\right).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial L_0^{(\alpha)}(\tau,\gamma)}{\partial\tau} = -\frac{\gamma}{2}\exp\left(-\frac{\gamma\tau}{2}\right);$$

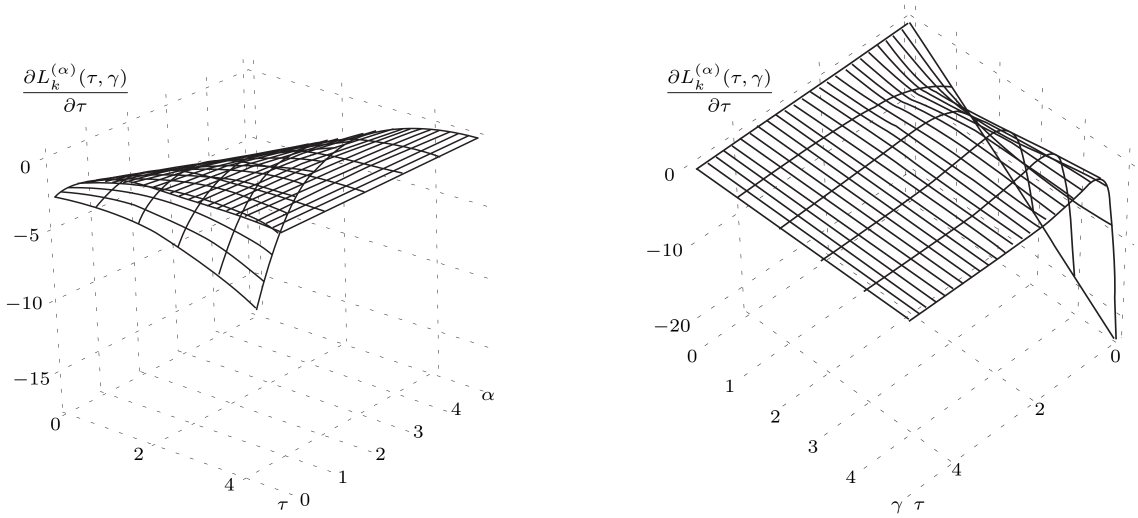
$$\frac{\partial L_1^{(\alpha)}(\tau,\gamma)}{\partial\tau} = \frac{\gamma}{2}\exp\left(-\frac{\gamma\tau}{2}\right)(\gamma\tau-\alpha-3);$$

$$\frac{\partial L_2^{(\alpha)}(\tau,\gamma)}{\partial\tau} = -\frac{\gamma}{4}\exp\left(-\frac{\gamma\tau}{2}\right)\left(\gamma^2\tau^2-2(\alpha+4)\gamma\tau+\alpha^2+10\right);$$

$$\frac{\partial L_3^{(\alpha)}(\tau,\gamma)}{\partial\tau} = \frac{\gamma}{12}\exp\left(-\frac{\gamma\tau}{2}\right)\left(\gamma^3\tau^3-3(\alpha+5)\gamma^2\tau^2+3(\alpha^2+9\alpha+18)\gamma\tau-\alpha^3-12\alpha^2-41\alpha-42\right);$$

$$\frac{\partial L_4^{(\alpha)}(\tau,\gamma)}{\partial\tau} = -\frac{\gamma}{48}\exp\left(-\frac{\gamma\tau}{2}\right)\left(\gamma^4\tau^4-4(\alpha+6)\gamma^3\tau^3+6(\alpha^2+11\alpha+28)\gamma^2\tau^2-4(\alpha^3+15\alpha^2+68\alpha+96)\gamma\tau+\alpha^4+18\alpha^3+107\alpha^2+258\alpha+216\right);$$

$$\frac{\partial L_5^{(\alpha)}(\tau,\gamma)}{\partial\tau} = \frac{\gamma}{240}\exp\left(-\frac{\gamma\tau}{2}\right)\left(\gamma^5\tau^5-5(\alpha+7)\gamma^4\tau^4+10(\alpha^2+13\alpha+40)\gamma^3\tau^3-10(\alpha^3+18\alpha^2+101\alpha+180)\gamma^2\tau^2+5(\alpha^4+22\alpha^3+167\alpha^2+530\alpha+600)\gamma\tau-\alpha^5-25\alpha^4-225\alpha^3-935\alpha^2-1814\alpha-1320\right).$$



а) б)

Рис. 1.26. Вид 1-ой производной ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 1$, $\alpha \in [0;5]$; б) $\gamma \in (0;5]$, $\alpha = 1$

[1.27]
$$\frac{\partial^2 L_k^{(\alpha)}(\tau,\gamma)}{\partial\tau^2} = \gamma^2\left(\sum_{s=2}^{k}\binom{k+\alpha}{k-s}\frac{(-\gamma\tau)^{s-2}}{(s-2)!} + \sum_{s=1}^{k}\binom{k+\alpha}{k-s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!} + \frac{1}{4}\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\frac{(-\gamma\tau)^{s}}{s!}\right)\exp\left(-\frac{\gamma\tau}{2}\right).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 L_0^{(\alpha)}(\tau,\gamma)}{\partial\tau^2} = \frac{\gamma^2}{4}\exp\left(-\frac{\gamma\tau}{2}\right);$$

$$\frac{\partial^2 L_1^{(\alpha)}(\tau,\gamma)}{\partial\tau^2} = -\frac{\gamma^2}{4}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma\tau-\alpha-5\big);$$

$$\frac{\partial^2 L_2^{(\alpha)}(\tau,\gamma)}{\partial\tau^2} = \frac{\gamma^2}{8}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^2\tau^2-2(\alpha+6)\gamma\tau+\alpha^2+11\alpha+ + 26\big);$$

$$\frac{\partial^2 L_3^{(\alpha)}(\tau,\gamma)}{\partial\tau^2} = -\frac{\gamma^2}{24}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^3\tau^3-3(\alpha+7)\gamma^2\tau^2+3(\alpha^2+ + 13\alpha+38)\gamma\tau-\alpha^3-18\alpha^2-95\alpha-150\big);$$

$$\frac{\partial^2 L_4^{(\alpha)}(\tau,\gamma)}{\partial\tau^2} = \frac{\gamma^2}{96}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^4\tau^4-4(\alpha+8)\gamma^3\tau^3+6(\alpha^2+ + 15\alpha+52)\gamma^2\tau^2-4(\alpha^3+21\alpha^2+134\alpha+264)\gamma\tau+\alpha^4+26\alpha^3+ + 227\alpha^2+802\alpha+984\big);$$

$$\frac{\partial^2 L_5^{(\alpha)}(\tau,\gamma)}{\partial\tau^2} = -\frac{\gamma^2}{480}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^5\tau^5-5(\alpha+9)\gamma^4\tau^4+10\times \times(\alpha^2+17\alpha+63)\gamma^3\tau^3-10(\alpha^3+24\alpha^2+179\alpha+420)\gamma^2\tau^2+ + 5(\alpha^4+30\alpha^3+311\alpha^2+1338\alpha+2040)\gamma\tau-\alpha^5-35\alpha^4- - 445\alpha^3-1555\alpha^2-7114\alpha-7320\big).$$

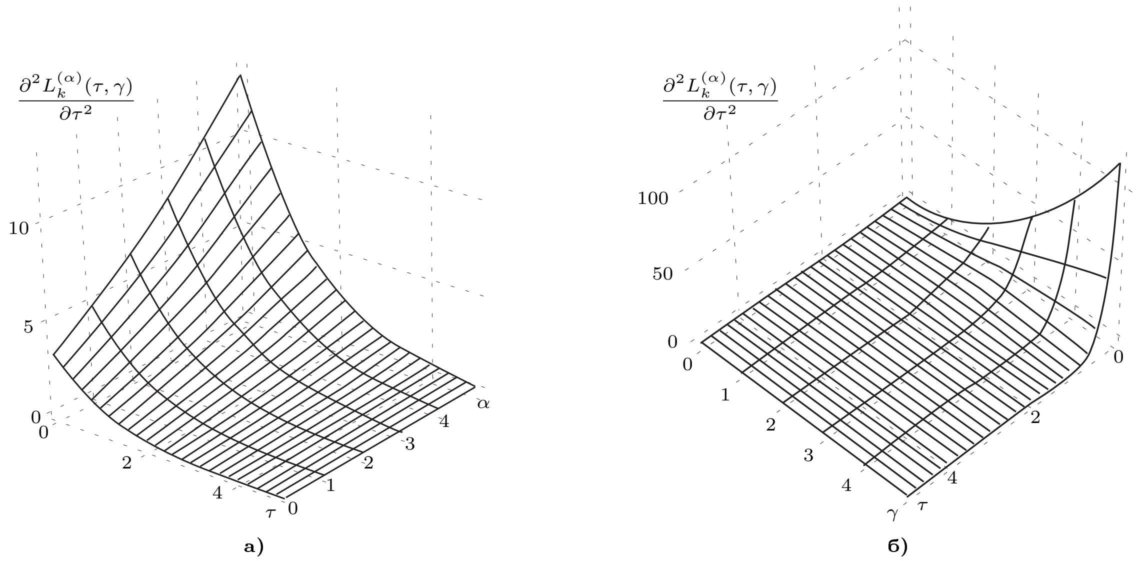


Рис. 1.27. Вид 2-ой производной ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 1$, $\alpha \in [0;5]$; б) $\gamma \in (0;5]$, $\alpha = 1$

[1.28]
$$\frac{\partial^3 L_k^{(\alpha)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\Bigg(\sum_{s=3}^{k}\binom{k+\alpha}{k-s}\frac{(-\gamma\tau)^{s-3}}{(s-3)!}+ +\frac{3}{2}\sum_{s=2}^{k}\binom{k+\alpha}{k-s}\frac{(-\gamma\tau)^{s-2}}{(s-2)!}+\frac{3}{4}\sum_{s=1}^{k}\binom{k+\alpha}{k-s}\frac{(-\gamma\tau)^{s-1}}{(s-1)!}+ +\frac{1}{8}\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\frac{(-\gamma\tau)^{s}}{s!}\Bigg)\exp\Big(-\frac{\gamma\tau}{2}\Big).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 L_0^{(\alpha)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{8}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$

$$\frac{\partial^3 L_1^{(\alpha)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{8}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma\tau-\alpha-7\big);$$

$$\frac{\partial^3 L_2^{(\alpha)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{16}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^2\tau^2-2(\alpha+8)\gamma\tau+\alpha^2+15\alpha+ + 50\big);$$

$$\frac{\partial^3 L_3^{(\alpha)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{48}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^3\tau^3-3(\alpha+9)\gamma^2\tau^2+3(\alpha^2+ + 17\alpha+66)\gamma\tau-\alpha^3-24\alpha^2-173\alpha-378\big);$$

$$\frac{\partial^3 L_4^{(\alpha)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{192}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^4\tau^4-4(\alpha+10)\gamma^3\tau^3+6(\alpha^2+ + 19\alpha+84)\gamma^2\tau^2-4(\alpha^3+27\alpha^2+224\alpha+576)\gamma\tau+\alpha^4+34\alpha^3+ + 395\alpha^2+1874\alpha+3096\big);$$

$$\frac{\partial^3 L_5^{(\alpha)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{960}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^5\tau^5-5(\alpha+11)\gamma^4\tau^4+10\times \times(\alpha^2+21\alpha+104)\gamma^3\tau^3-10(\alpha^3+30\alpha^2+281\alpha+828)\gamma^2\tau^2+ + 5(\alpha^4+38\alpha^3+503\alpha^2+2770\alpha+5400)\gamma\tau-\alpha^5-45\alpha^4-745\alpha^3- - 5715\alpha^2-20494\alpha-27720\big).$$

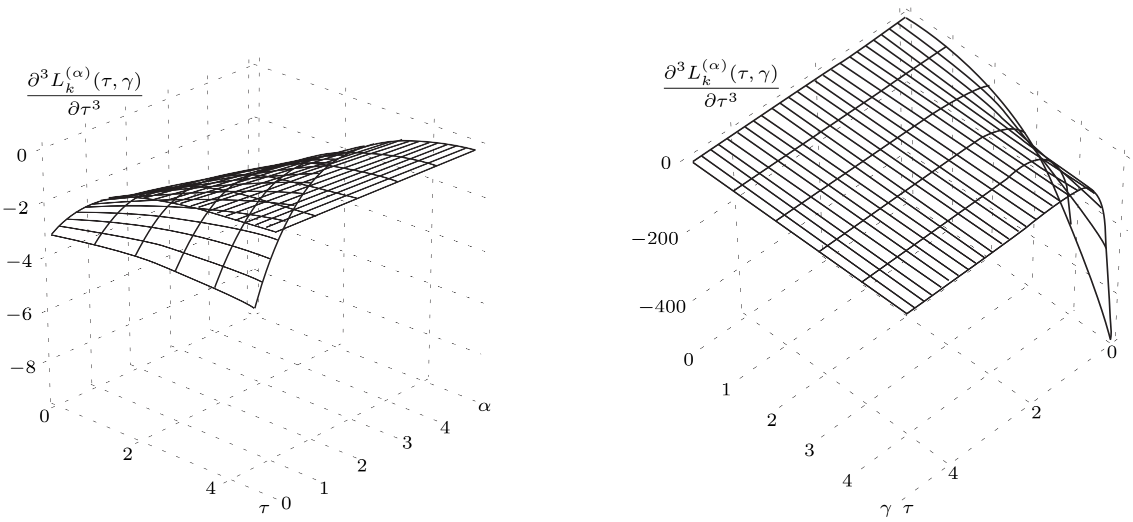


Рис. 1.28. Вид 3-ой производной ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 1$, $\alpha \in [0;5]$; б) $\gamma \in (0;5]$, $\alpha = 1$

[1.29]
$$\frac{\partial^n L_k^{(\alpha)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\begin{cases}\dfrac{(-\gamma\tau)^{s-n+j}}{(s-n+j)!}, & \text{если } s-n+j\ge 0;\\ 0, & \text{иначе.}\end{cases}$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n L_0^{(\alpha)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\begin{cases}\dfrac{(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе.}\end{cases};$$

$$\frac{\partial^n L_1^{(\alpha)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{(\alpha+1)(-\gamma\tau)^{-n+j}}{(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

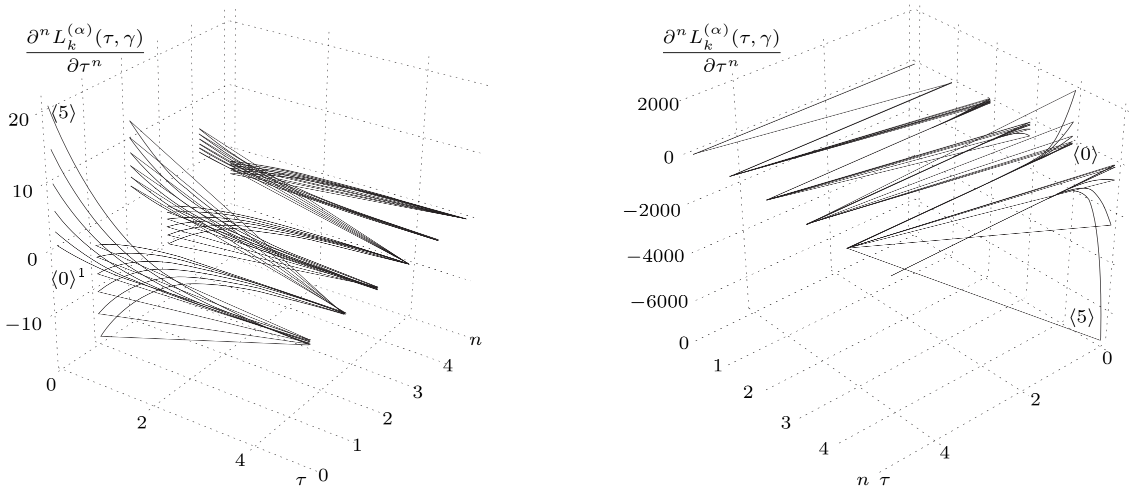
$$\frac{\partial^n L_2^{(\alpha)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{(\alpha+1)(\alpha+2)(-\gamma\tau)^{-n+j}}{2(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$+\begin{cases}\dfrac{(\alpha+2)(-\gamma\tau)^{1-n+j}}{(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_3^{(\alpha)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{(\alpha+3)!(-\gamma\tau)^{-n+j}}{6\alpha!(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$+\begin{cases}\dfrac{(\alpha+2)(\alpha+3)(-\gamma\tau)^{1-n+j}}{2(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{(\alpha+3)(-\gamma\tau)^{2-n+j}}{(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_4^{(\alpha)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$
$$\times\left(\begin{cases}\dfrac{(\alpha+4)!(-\gamma\tau)^{-n+j}}{24\alpha!(-n+j)!}, & \text{если } -n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$+\begin{cases}\dfrac{(\alpha+3)!(-\gamma\tau)^{1-n+j}}{6(\alpha+1)!(1-n+j)!}, & \text{если } 1-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{(\alpha+3)(\alpha+4)(-\gamma\tau)^{2-n+j}}{2(2-n+j)!}, & \text{если } 2-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{(\alpha+4)(-\gamma\tau)^{3-n+j}}{(3-n+j)!}, & \text{если } 3-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}+$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{4-n+j}}{(4-n+j)!}, & \text{если } 4-n+j\ge 0;\\ 0, & \text{иначе}\end{cases}\right);$$

$$\frac{\partial^n L_5^{(\alpha)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times$$

$$\times\left(\begin{cases}\dfrac{(\alpha+5)!(-\gamma\tau)^{-n+j}}{120\alpha!(-n+j)!}, & \text{если } -n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+\right.$$
$$+\begin{cases}\dfrac{(\alpha+5)!(-\gamma\tau)^{1-n+j}}{24(\alpha+1)!(1-n+j)!}, & \text{если } 1-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{(\alpha+5)!(-\gamma\tau)^{2-n+j}}{6(\alpha+2)!(2-n+j)!}, & \text{если } 2-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{(\alpha+4)(\alpha+5)(-\gamma\tau)^{3-n+j}}{2(3-n+j)!}, & \text{если } 3-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$+\begin{cases}\dfrac{(\alpha+5)(-\gamma\tau)^{4-n+j}}{(4-n+j)!}, & \text{если } 4-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}+$$
$$\left.+\begin{cases}\dfrac{(-\gamma\tau)^{5-n+j}}{(5-n+j)!}, & \text{если } 5-n+j\geq 0;\\ 0, & \text{иначе}\end{cases}\right).$$



а) **б)**

Рис. 1.29. Вид n-ой производной ортогональных функций Сонина-Лагерра 2-ого порядка: а) $n=0..5$, $\gamma=1$, $\alpha\in[0;5]$; б) $n=0..5$, $\gamma\in(0;5]$, $\alpha=1$

[1.30]
$$\frac{\partial P_k^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}=-\frac{\gamma}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}\times$$
$$\times(-1)^s(4s+1)\exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial P_0^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}=-\frac{\gamma}{2}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$
$$\frac{\partial P_1^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}=\frac{\gamma}{4}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(15\exp(-2\gamma\tau)-1\big);$$
$$\frac{\partial P_2^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}=-\frac{\gamma}{16}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(315\exp(-4\gamma\tau)-$$
$$-150\exp(-2\gamma\tau)+3\big);$$
$$\frac{\partial P_3^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}=\frac{\gamma}{32}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(3003\exp(-6\gamma\tau)-$$
$$-2835\exp(-4\gamma\tau)+525\exp(-2\gamma\tau)-5\big);$$
$$\frac{\partial P_4^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}=-\frac{\gamma}{256}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(109395\exp(-8\gamma\tau)-$$
$$-156156\exp(-6\gamma\tau)+62370\exp(-4\gamma\tau)-6300\exp(-2\gamma\tau)+35\big);$$
$$\frac{\partial P_5^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}=\frac{\gamma}{512}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(969969\exp(-10\gamma\tau)-$$
$$-1859715\exp(-8\gamma\tau)+1171170\exp(-6\gamma\tau)-270270\times$$
$$\times\exp(-4\gamma\tau)+17325\exp(-2\gamma\tau)-63\big).$$

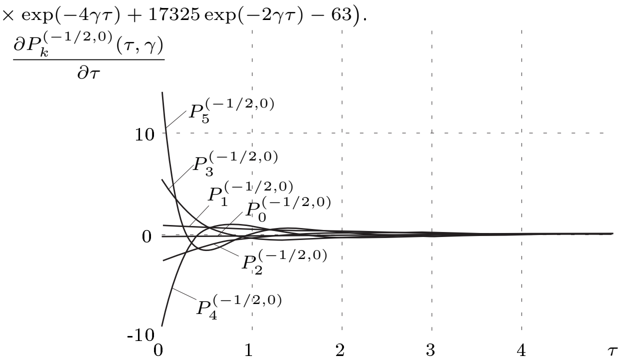


Рис. 1.30. Вид 1-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma=0,25$, $c=2$, $\alpha=-1/2$, $\beta=0$

[1]числовой эквивалент, характеризующий порядок кривой при изменении анализируемого параметра с равномерным шагом в выбранном диапазоне. Например, параметр $\alpha\in[0;5]$: $\langle 0\rangle$ - кривая при значении $\alpha=0$, $\langle 5\rangle$ - кривая при значении $\alpha=5$.

[1.31]
$$\frac{\partial^2 P_k^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^2} = \frac{\gamma^2}{4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}\times$$
$$\times (-1)^s(4s+1)^2\exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$\dfrac{\partial^2 P_0^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^2} = \dfrac{\gamma^2}{4}\exp\Big(-\dfrac{\gamma\tau}{2}\Big);$

$\dfrac{\partial^2 P_1^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^2} = -\dfrac{\gamma^2}{8}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(75\exp(-2\gamma\tau)-1\big);$

$\dfrac{\partial^2 P_2^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^2} = \dfrac{3\gamma^2}{32}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(945\exp(-4\gamma\tau)-$
$-250\exp(-2\gamma\tau)+1\big);$

$\dfrac{\partial^2 P_3^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^2} = -\dfrac{\gamma^2}{64}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(39039\exp(-6\gamma\tau)-$
$-25515\exp(-4\gamma\tau)+2625\exp(-2\gamma\tau)-5\big);$

$\dfrac{\partial^2 P_4^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^2} = \dfrac{\gamma^2}{512}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(1859715\exp(-8\gamma\tau)-$
$-2030028\exp(-6\gamma\tau)+561330\exp(-4\gamma\tau)-31500\exp(-2\gamma\tau)+$
$+35\big);$

$\dfrac{\partial^2 P_5^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^2} = -\dfrac{9\gamma^2}{1024}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(2263261\exp(-10\gamma\tau)-$
$-3512795\exp(-8\gamma\tau)+1691690\exp(-6\gamma\tau)-270270\times$
$\times\exp(-4\gamma\tau)+9625\exp(-2\gamma\tau)-7\big).$

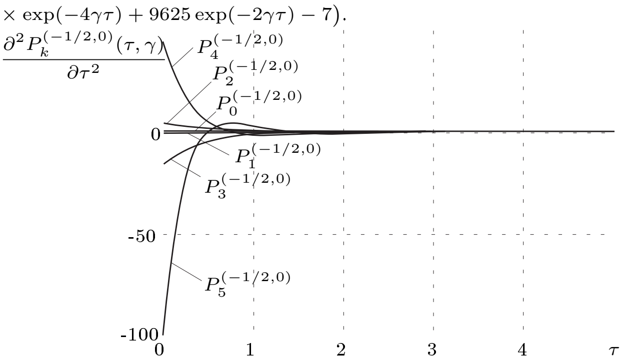


Рис. 1.31. Вид 2-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

[1.32]
$$\frac{\partial^3 P_k^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^3} = -\frac{\gamma^3}{8}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}\times$$
$$\times (-1)^s(4s+1)^3\exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$\dfrac{\partial^3 P_0^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^3} = -\dfrac{\gamma^3}{8}\exp\Big(-\dfrac{\gamma\tau}{2}\Big);$

$\dfrac{\partial^3 P_1^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^3} = \dfrac{\gamma^3}{16}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(375\exp(-2\gamma\tau)-1\big);$

$\dfrac{\partial^3 P_2^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^3} = -\dfrac{\gamma^3}{64}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(25515\exp(-4\gamma\tau)-$
$-1875\exp(-2\gamma\tau)+3\big);$

$\dfrac{\partial^3 P_3^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^3} = \dfrac{\gamma^3}{128}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(507507\exp(-6\gamma\tau)-$
$-229635\exp(-4\gamma\tau)+13125\exp(-2\gamma\tau)-5\big);$

$\dfrac{\partial^3 P_4^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^3} = -\dfrac{\gamma^3}{1024}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(31615155\exp(-8\gamma\tau)-$
$-26390364\exp(-6\gamma\tau)+5051970\exp(-4\gamma\tau)-157500\exp(-2\gamma\tau)+$
$+35\big);$

$\dfrac{\partial^3 P_5^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^3} = \dfrac{9\gamma^3}{2048}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(47528481\exp(-10\gamma\tau)-$
$-59717515\exp(-8\gamma\tau)+21991970\exp(-6\gamma\tau)-2432430\times$
$\times\exp(-4\gamma\tau)+48125\exp(-2\gamma\tau)-7\big).$

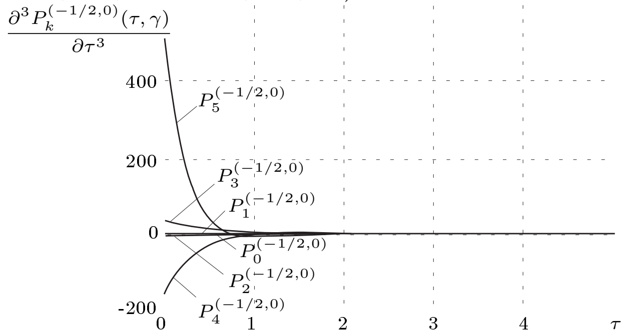


Рис. 1.32. Вид 3-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

[1.33]
$$\frac{\partial^n P_k^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^n} = \Big(-\frac{\gamma}{2}\Big)^n\sum_{s=0}^{k}\binom{k}{s}\times$$
$$\times\binom{k+s-1/2}{s-1/2}(-1)^s(4s+1)^n\exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big).$$

Частные случаи для n-ой производной функций 0-5 порядков:

$\dfrac{\partial^n P_0^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^n} = \Big(-\dfrac{\gamma}{2}\Big)^n\exp\Big(-\dfrac{\gamma\tau}{2}\Big);$

$\dfrac{\partial^n P_1^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^n} = -\dfrac{1}{2}\Big(-\dfrac{\gamma}{2}\Big)^n\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(3\cdot 5^n\times$
$\times\exp(-2\gamma\tau)-1\big);$

$\dfrac{\partial^n P_2^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^n} = \dfrac{1}{8}\Big(-\dfrac{\gamma}{2}\Big)^n\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(35\cdot 9^n\times$
$\times\exp(-4\gamma\tau)-30\cdot 5^n\exp(-2\gamma\tau)+3\big);$

$\dfrac{\partial^n P_3^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^n} = -\dfrac{1}{16}\Big(-\dfrac{\gamma}{2}\Big)^n\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(231\cdot 13^n\times$
$\times\exp(-6\gamma\tau)-315\cdot 9^n\exp(-4\gamma\tau)+105\cdot 5^n\exp(-2\gamma\tau)-5\big);$

$\dfrac{\partial^n P_4^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^n} = \dfrac{1}{128}\Big(-\dfrac{\gamma}{2}\Big)^n\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(6435\cdot 17^n\times$
$\times\exp(-8\gamma\tau)-12012\cdot 13^n\exp(-6\gamma\tau)+6930\cdot 9^n\exp(-4\gamma\tau)-$
$-1260\cdot 5^n\exp(-2\gamma\tau)+35\big);$

$\dfrac{\partial^n P_5^{(-1/2,0)}(\tau,\gamma)}{\partial \tau^n} = -\dfrac{1}{256}\Big(-\dfrac{\gamma}{2}\Big)^n\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(46189\cdot 21^n\times$
$\times\exp(-10\gamma\tau)-109395\cdot 17^n\exp(-8\gamma\tau)+90090\cdot 13^n\times$
$\times\exp(-6\gamma\tau)-30030\cdot 9^n\exp(-4\gamma\tau)+3465\cdot 5^n\exp(-2\gamma\tau)-63\big).$

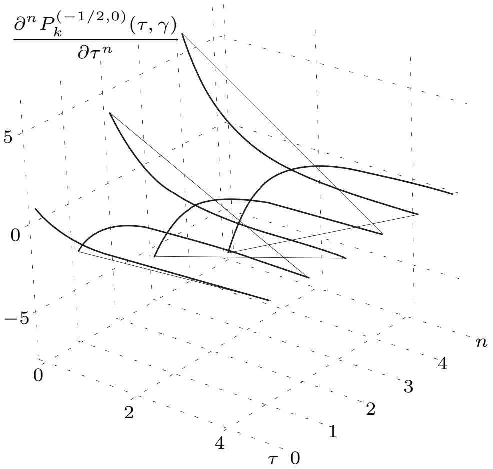


Рис. 1.33. Вид n-ой производной ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 0,25$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

$$\textbf{[1.34]}\quad \frac{\partial Leg_k(\tau,\gamma)}{\partial\tau} = -\gamma\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\times \times(2s+1)\exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial Leg_0(\tau,\gamma)}{\partial\tau} = -\gamma\exp(-\gamma\tau);$$

$$\frac{\partial Leg_1(\tau,\gamma)}{\partial\tau} = \gamma\exp(-\gamma\tau)\big(6\exp(-2\gamma\tau)-1\big);$$

$$\frac{\partial Leg_2(\tau,\gamma)}{\partial\tau} = -\gamma\exp(-\gamma\tau)\big(30\exp(-4\gamma\tau)-18\exp(-2\gamma\tau)+ +1\big);$$

$$\frac{\partial Leg_3(\tau,\gamma)}{\partial\tau} = \gamma\exp(-\gamma\tau)\big(140\exp(-6\gamma\tau)-150\exp(-4\gamma\tau)+ +36\exp(-2\gamma\tau)-1\big);$$

$$\frac{\partial Leg_4(\tau,\gamma)}{\partial\tau} = -\gamma\exp(-\gamma\tau)\big(630\exp(-8\gamma\tau)-980\times \times\exp(-6\gamma\tau)+450\exp(-4\gamma\tau)-60\exp(-2\gamma\tau)+1\big);$$

$$\frac{\partial Leg_5(\tau,\gamma)}{\partial\tau} = \gamma\exp(-\gamma\tau)\big(2772\exp(-10\gamma\tau)-5670\times \times\exp(-8\gamma\tau)+3920\exp(-6\gamma\tau)-1050\exp(-4\gamma\tau)+90\times \times\exp(-2\gamma\tau)-1\big).$$

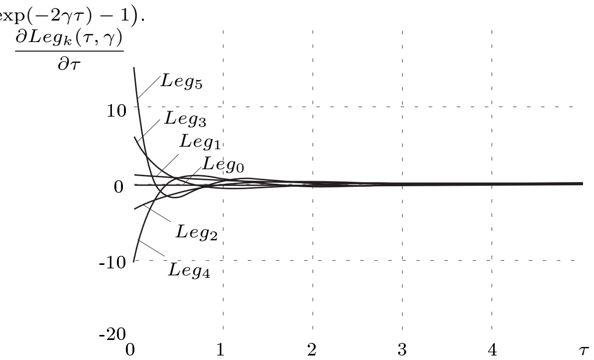


Рис. 1.34. Вид 1-ой производной ортогональных функций Лежандра 0-5 порядков; $\gamma = 0,25$, $c = 2$

$$\textbf{[1.35]}\quad \frac{\partial^2 Leg_k(\tau,\gamma)}{\partial\tau^2} = \gamma^2\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\times \times(2s+1)^2\exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 Leg_0(\tau,\gamma)}{\partial\tau^2} = \gamma^2\exp(-\gamma\tau);$$

$$\frac{\partial^2 Leg_1(\tau,\gamma)}{\partial\tau^2} = -\gamma^2\exp(-\gamma\tau)\big(18\exp(-2\gamma\tau)-1\big);$$

$$\frac{\partial^2 Leg_2(\tau,\gamma)}{\partial\tau^2} = \gamma^2\exp(-\gamma\tau)\big(150\exp(-4\gamma\tau)-54\exp(-2\gamma\tau)+ +1\big);$$

$$\frac{\partial^2 Leg_3(\tau,\gamma)}{\partial\tau^2} = -\gamma^2\exp(-\gamma\tau)\big(980\exp(-6\gamma\tau)-750\times \times\exp(-4\gamma\tau)+108\exp(-2\gamma\tau)-1\big);$$

$$\frac{\partial^2 Leg_4(\tau,\gamma)}{\partial\tau^2} = \gamma^2\exp(-\gamma\tau)\big(5670\exp(-8\gamma\tau)-6860\times \times\exp(-6\gamma\tau)+2250\exp(-4\gamma\tau)-180\exp(-2\gamma\tau)+1\big);$$

$$\frac{\partial^2 Leg_5(\tau,\gamma)}{\partial\tau^2} = -\gamma^2\exp(-\gamma\tau)\big(30492\exp(-10\gamma\tau)-51030\times \times\exp(-8\gamma\tau)+27440\exp(-6\gamma\tau)-5250\exp(-4\gamma\tau)+ +270\exp(-2\gamma\tau)-1\big).$$

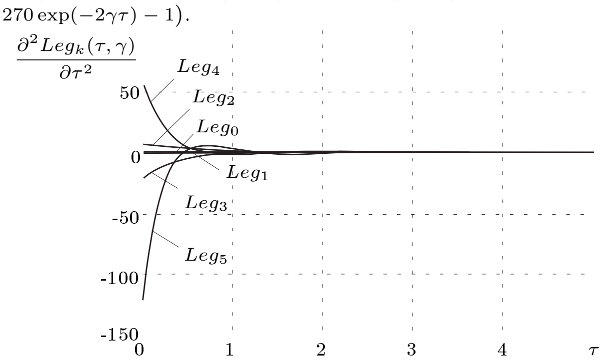


Рис. 1.35. Вид 2-ой производной ортогональных функций Лежандра 0-5 порядков; $\gamma = 0,25$, $c = 2$

$$\textbf{[1.36]}\quad \frac{\partial^3 Leg_k(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\times \times(2s+1)^3\exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 Leg_0(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\exp(-\gamma\tau);$$

$$\frac{\partial^3 Leg_1(\tau,\gamma)}{\partial\tau^3} = \gamma^3\exp(-\gamma\tau)\big(54\exp(-2\gamma\tau)-1\big);$$

$$\frac{\partial^3 Leg_2(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\exp(-\gamma\tau)\big(750\exp(-4\gamma\tau)-162\times \times\exp(-2\gamma\tau)+1\big);$$

$$\frac{\partial^3 Leg_3(\tau,\gamma)}{\partial\tau^3} = \gamma^3\exp(-\gamma\tau)\big(6860\exp(-6\gamma\tau)-3750\times \times\exp(-4\gamma\tau)+324\exp(-2\gamma\tau)-1\big);$$

$$\frac{\partial^3 Leg_4(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\exp(-\gamma\tau)\big(51030\exp(-8\gamma\tau)-48020\times \times\exp(-6\gamma\tau)+11250\exp(-4\gamma\tau)-540\exp(-2\gamma\tau)+1\big);$$

$$\frac{\partial^3 Leg_5(\tau,\gamma)}{\partial\tau^3} = \gamma^3\exp(-\gamma\tau)\big(335412\exp(-10\gamma\tau)-459270\times$$

$\times \exp(-8\gamma\tau) + 192080 \exp(-6\gamma\tau) - 26250 \exp(-4\gamma\tau) + 810 \times$
$\times \exp(-2\gamma\tau) - 1\big).$

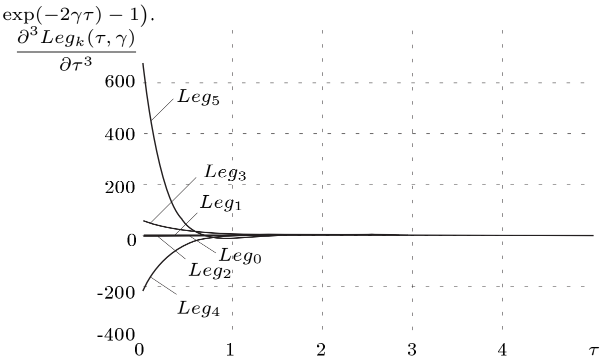


Рис. 1.36. Вид 3-ой производной ортогональных функций Лежандра 0-5 порядков; $\gamma = 0,25$, $c = 2$

[1.37] $$\frac{\partial^n Leg_k(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \sum_{s=0}^{k} \binom{k}{s}\binom{k+s}{s} (-1)^s \times$$
$$\times (2s+1)^n \exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n Leg_0(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp(-\gamma\tau);$$

$$\frac{\partial^n Leg_1(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n \exp(-\gamma\tau)\big(2\cdot 3^n \exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^n Leg_2(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp(-\gamma\tau)\big(6\cdot 5^n \exp(-4\gamma\tau) - 6\cdot 3^n \times$$
$$\times \exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial^n Leg_3(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n \exp(-\gamma\tau)\big(20\cdot 7^n \exp(-6\gamma\tau) -$$
$$- 30\cdot 5^n \exp(-4\gamma\tau) + 12\cdot 3^n \exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^n Leg_4(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp(-\gamma\tau)\big(70\cdot 9^n \exp(-8\gamma\tau) - 140\cdot 7^n \times$$
$$\times \exp(-6\gamma\tau) + 90\cdot 5^n \exp(-4\gamma\tau) - 20\cdot 3^n \exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial^n Leg_5(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n \exp(-\gamma\tau)\big(252\cdot 11^n \exp(-10\gamma\tau) -$$
$$- 630\cdot 9^n \exp(-8\gamma\tau) + 560\cdot 7^n \exp(-6\gamma\tau) - 210\cdot 5^n \exp(-4\gamma\tau) +$$
$$+ 30\cdot 3^n \exp(-2\gamma\tau) - 1\big).$$

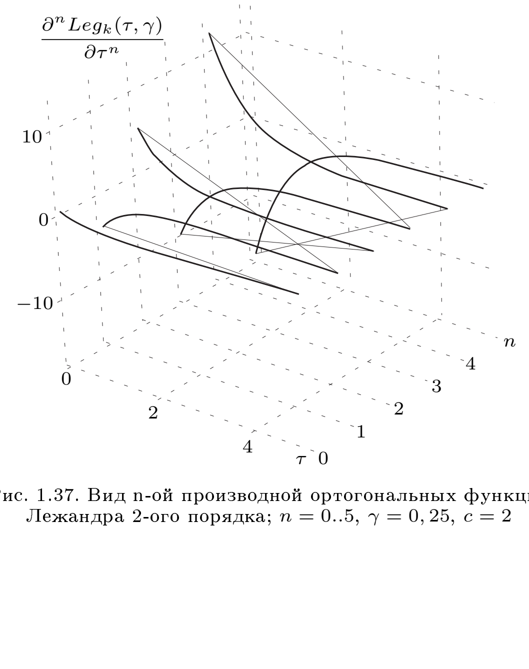


Рис. 1.37. Вид n-ой производной ортогональных функций Лежандра 2-ого порядка; $n = 0..5$, $\gamma = 0,25$, $c = 2$

[1.38] $$\frac{\partial P_k^{(1/2,0)}(\tau,\gamma)}{\partial\tau} = -\frac{\gamma}{2} \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1/2}{s+1/2} \times$$
$$\times (-1)^s (4s+3) \exp\Big(-\frac{(4s+3)}{2}\gamma\tau\Big).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial P_0^{(1/2,0)}(\tau,\gamma)}{\partial\tau} = -\frac{3\gamma}{2} \exp\Big(-\frac{3\gamma\tau}{2}\Big);$$

$$\frac{\partial P_1^{(1/2,0)}(\tau,\gamma)}{\partial\tau} = \frac{\gamma}{4} \exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(35 \exp(-2\gamma\tau) - 9\big);$$

$$\frac{\partial P_2^{(1/2,0)}(\tau,\gamma)}{\partial\tau} = -\frac{\gamma}{16} \exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(693 \exp(-4\gamma\tau) - 490 \times$$
$$\times \exp(-2\gamma\tau) + 45\big);$$

$$\frac{\partial P_3^{(1/2,0)}(\tau,\gamma)}{\partial\tau} = \frac{\gamma}{32} \exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(6435 \exp(-6\gamma\tau) - 7623 \times$$
$$\times \exp(-4\gamma\tau) + 2205 \exp(-2\gamma\tau) - 105\big);$$

$$\frac{\partial P_4^{(1/2,0)}(\tau,\gamma)}{\partial\tau} = -\frac{\gamma}{256} \exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(230945 \exp(-8\gamma\tau) -$$
$$- 386100 \exp(-6\gamma\tau) + 198198 \exp(-4\gamma\tau) - 32340 \exp(-2\gamma\tau) +$$
$$+ 945\big);$$

$$\frac{\partial P_5^{(1/2,0)}(\tau,\gamma)}{\partial\tau} = \frac{\gamma}{512} \exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(2028117 \exp(-10\gamma\tau) -$$
$$- 4387955 \times \exp(-8\gamma\tau) + 3281850 \exp(-6\gamma\tau) - 990990 \times$$
$$\times \exp(-4\gamma\tau) + 105105 \exp(-2\gamma\tau) - 2079\big).$$

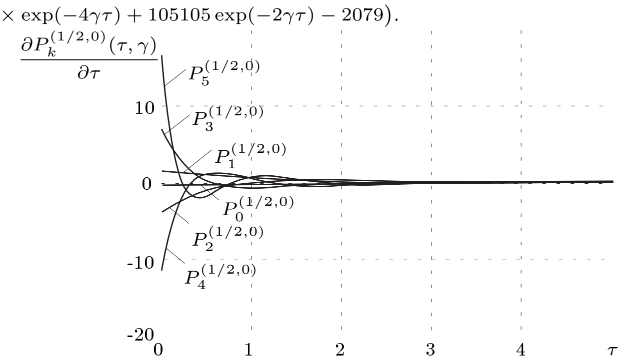


Рис. 1.38. Вид 1-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[1.39] $$\frac{\partial^2 P_k^{(1/2,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{\gamma^2}{4} \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1/2}{s+1/2} \times$$
$$\times (-1)^s (4s+3)^2 \exp\Big(-\frac{(4s+3)}{2}\gamma\tau\Big).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 P_0^{(1/2,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{9\gamma^2}{4} \exp\Big(-\frac{3\gamma\tau}{2}\Big);$$

$$\frac{\partial^2 P_1^{(1/2,0)}(\tau,\gamma)}{\partial\tau^2} = -\frac{\gamma^2}{8} \exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(245 \exp(-2\gamma\tau) - 27\big);$$

$$\frac{\partial^2 P_2^{(1/2,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{\gamma^2}{32} \exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(7623 \exp(-4\gamma\tau) - 3430 \times$$
$$\times \exp(-2\gamma\tau) + 135\big);$$

$$\frac{\partial^2 P_3^{(1/2,0)}(\tau,\gamma)}{\partial\tau^2} = -\frac{\gamma^2}{64} \exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(96525 \exp(-6\gamma\tau) -$$
$$- 83853 \exp(-4\gamma\tau) + 15435 \exp(-2\gamma\tau) - 315\big);$$

$$\frac{\partial^2 P_4^{(1/2,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{\gamma^2}{512}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(4387955\exp(-8\gamma\tau) - 1447875\exp(-6\gamma\tau) + 2180178\exp(-4\gamma\tau) - 226380\exp(-2\gamma\tau) + 2835\big);$$

$$\frac{\partial^2 P_5^{(1/2,0)}(\tau,\gamma)}{\partial\tau^2} = -\frac{9\gamma^2}{1024}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(46646681\exp(-10\gamma\tau) - 83371145\exp(-8\gamma\tau) + 49227750\exp(-6\gamma\tau) - 10900890\times \exp(-4\gamma\tau) + 735735\exp(-2\gamma\tau) - 6237\big).$$

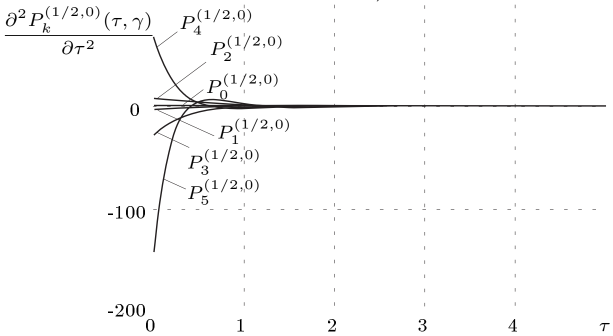


Рис. 1.39. Вид 2-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[1.40]
$$\frac{\partial^3 P_k^{(1/2,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{8}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}\times (-1)^s(4s+3)^3\exp\Big(-\frac{(4s+3)}{2}\gamma\tau\Big).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 P_0^{(1/2,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{27\gamma^3}{8}\exp\Big(-\frac{3\gamma\tau}{2}\Big);$$

$$\frac{\partial^3 P_1^{(1/2,0)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{16}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(1715\exp(-2\gamma\tau) - 81\big);$$

$$\frac{\partial^3 P_2^{(1/2,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{64}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(83853\exp(-4\gamma\tau) - 24010\exp(-2\gamma\tau) + 405\big);$$

$$\frac{\partial^3 P_3^{(1/2,0)}(\tau,\gamma)}{\partial\tau^3} = \frac{\gamma^3}{128}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(1447875\exp(-6\gamma\tau) - 922383\exp(-4\gamma\tau) + 108045\exp(-2\gamma\tau) - 945\big);$$

$$\frac{\partial^3 P_4^{(1/2,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{\gamma^3}{1024}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(83371145\times \exp(-8\gamma\tau) - 86872500\exp(-6\gamma\tau) + 23981958\exp(-4\gamma\tau) - 1584660\exp(-2\gamma\tau) + 8505\big);$$

$$\frac{\partial^3 P_5^{(1/2,0)}(\tau,\gamma)}{\partial\tau^3} = \frac{9\gamma^3}{2048}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(1072873893\times \exp(-10\gamma\tau) - 1584051755\exp(-8\gamma\tau) + 738416250\times \exp(-6\gamma\tau) - 119909790\exp(-4\gamma\tau) + 5150145\exp(-2\gamma\tau) - 18711\big).$$

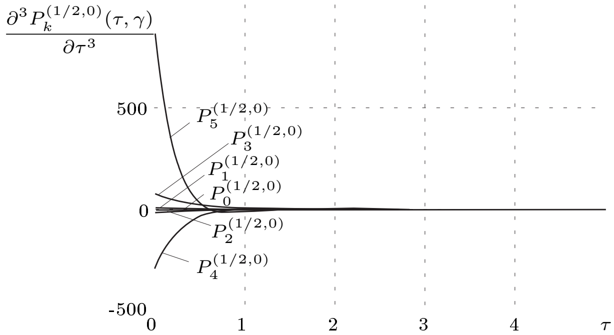


Рис. 1.40. Вид 3-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[1.41]
$$\frac{\partial^n P_k^{(1/2,0)}(\tau,\gamma)}{\partial\tau^n} = \Big(-\frac{\gamma}{2}\Big)^n\sum_{s=0}^{k}\binom{k}{s}\times \binom{k+s+1/2}{s+1/2}(-1)^s(4s+3)^n\exp\Big(-\frac{(4s+3)}{2}\gamma\tau\Big).$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n P_0^{(1/2,0)}(\tau,\gamma)}{\partial\tau^n} = 3^n\Big(-\frac{\gamma}{2}\Big)^n\exp\Big(-\frac{3\gamma\tau}{2}\Big);$$

$$\frac{\partial^n P_1^{(1/2,0)}(\tau,\gamma)}{\partial\tau^n} = -\frac{1}{2}\Big(-\frac{\gamma}{2}\Big)^n\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(5\cdot 7^n\exp(-2\gamma\tau) - 3\cdot 3^n\big);$$

$$\frac{\partial^n P_2^{(1/2,0)}(\tau,\gamma)}{\partial\tau^n} = \frac{1}{8}\Big(-\frac{\gamma}{2}\Big)^n\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(63\cdot 11^n\exp(-4\gamma\tau) - 70\cdot 7^n\exp(-2\gamma\tau) + 15\cdot 3^n\big);$$

$$\frac{\partial^n P_3^{(1/2,0)}(\tau,\gamma)}{\partial\tau^n} = -\frac{1}{16}\Big(-\frac{\gamma}{2}\Big)^n\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(429\cdot 15^n\times \exp(-6\gamma\tau) - 693\cdot 11^n\exp(-4\gamma\tau) + 315\cdot 7^n\exp(-2\gamma\tau) - 35\cdot 3^n\big);$$

$$\frac{\partial^n P_4^{(1/2,0)}(\tau,\gamma)}{\partial\tau^n} = \frac{1}{128}\Big(-\frac{\gamma}{2}\Big)^n\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(12155\cdot 19^n\times \exp(-8\gamma\tau) - 25740\cdot 15^n\exp(-6\gamma\tau) + 18018\cdot 11^n\exp(-4\gamma\tau) - 4620\cdot 7^n\exp(-2\gamma\tau) + 315\cdot 3^n\big);$$

$$\frac{\partial^n P_5^{(1/2,0)}(\tau,\gamma)}{\partial\tau^n} = -\frac{1}{256}\Big(-\frac{\gamma}{2}\Big)^n\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(88179\cdot 23^n\times \exp(-10\gamma\tau) - 230945\cdot 19^n\exp(-8\gamma\tau) + 218790\cdot 15^n\times \exp(-6\gamma\tau) - 90090\cdot 11^n\exp(-4\gamma\tau) + 15015\cdot 7^n\exp(-2\gamma\tau) - 693\cdot 3^n\big).$$

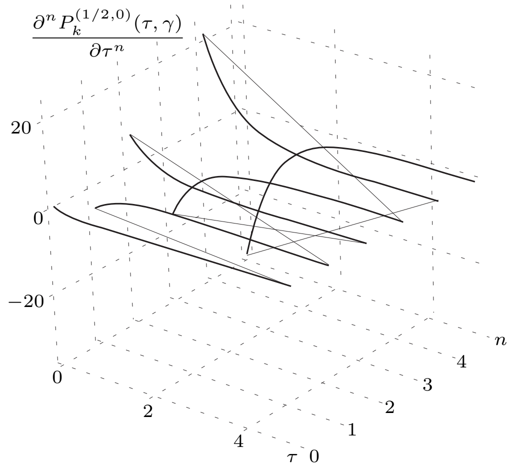


Рис. 1.41. Вид n-ой производной ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 0,25$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[1.42]
$$\frac{\partial P_k^{(1,0)}(\tau,\gamma)}{\partial \tau} = -\gamma \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1}{s+1}(-1)^s \times$$
$$\times (s+1)\exp\bigl(-(s+1)\gamma\tau\bigr).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial P_0^{(1,0)}(\tau,\gamma)}{\partial \tau} = -\gamma \exp(-\gamma\tau);$$
$$\frac{\partial P_1^{(1,0)}(\tau,\gamma)}{\partial \tau} = 2\gamma \exp(-\gamma\tau)\bigl(3\exp(-\gamma\tau)-1\bigr);$$
$$\frac{\partial P_2^{(1,0)}(\tau,\gamma)}{\partial \tau} = -3\gamma \exp(-\gamma\tau)\bigl(10\exp(-2\gamma\tau)-8\exp(-\gamma\tau)+$$
$$+1\bigr);$$
$$\frac{\partial P_3^{(1,0)}(\tau,\gamma)}{\partial \tau} = 4\gamma \exp(-\gamma\tau)\bigl(35\exp(-3\gamma\tau)-45\exp(-2\gamma\tau)+$$
$$+15\exp(-\gamma\tau)-1\bigr);$$
$$\frac{\partial P_4^{(1,0)}(\tau,\gamma)}{\partial \tau} = -5\gamma \exp(-\gamma\tau)\bigl(126\exp(-4\gamma\tau)-224\times$$
$$\times\exp(-3\gamma\tau)+126\exp(-2\gamma\tau)-24\exp(-\gamma\tau)+1\bigr);$$
$$\frac{\partial P_5^{(1,0)}(\tau,\gamma)}{\partial \tau} = 6\gamma \exp(-\gamma\tau)\bigl(462\exp(-5\gamma\tau)-1050\times$$
$$\times\exp(-4\gamma\tau)+840\exp(-3\gamma\tau)-280\exp(-2\gamma\tau)+210\times$$
$$\times\exp(-\gamma\tau)-1\bigr).$$

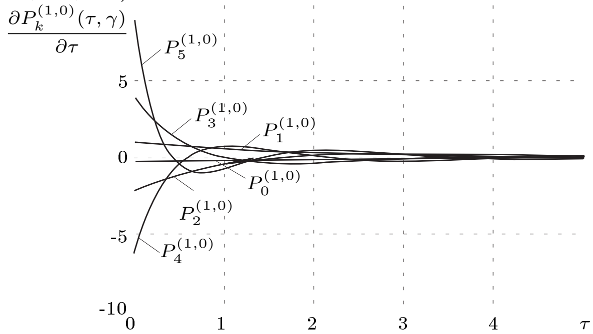


Рис. 1.42. Вид 1-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 1$, $\alpha = 1$, $\beta = 0$

[1.43]
$$\frac{\partial^2 P_k^{(1,0)}(\tau,\gamma)}{\partial \tau^2} = \gamma^2 \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1}{s+1}(-1)^s \times$$
$$\times (s+1)^2\exp\bigl(-(s+1)\gamma\tau\bigr).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 P_0^{(1,0)}(\tau,\gamma)}{\partial \tau^2} = \gamma^2 \exp(-\gamma\tau);$$
$$\frac{\partial^2 P_1^{(1,0)}(\tau,\gamma)}{\partial \tau^2} = -2\gamma^2 \exp(-\gamma\tau)\bigl(6\exp(-\gamma\tau)-1\bigr);$$
$$\frac{\partial^2 P_2^{(1,0)}(\tau,\gamma)}{\partial \tau^2} = 3\gamma^2 \exp(-\gamma\tau)\bigl(30\exp(-2\gamma\tau)-16\exp(-\gamma\tau)+$$
$$+1\bigr);$$
$$\frac{\partial^2 P_3^{(1,0)}(\tau,\gamma)}{\partial \tau^2} = -4\gamma^2 \exp(-\gamma\tau)\bigl(140\exp(-3\gamma\tau)-135\times$$
$$\times\exp(-2\gamma\tau)+30\exp(-\gamma\tau)-1\bigr);$$
$$\frac{\partial^2 P_4^{(1,0)}(\tau,\gamma)}{\partial \tau^2} = 5\gamma^2 \exp(-\gamma\tau)\bigl(630\exp(-4\gamma\tau)-896\times$$
$$\times\exp(-3\gamma\tau)+378\exp(-2\gamma\tau)-48\exp(-\gamma\tau)+1\bigr);$$
$$\frac{\partial^2 P_5^{(1,0)}(\tau,\gamma)}{\partial \tau^2} = -6\gamma^2 \exp(-\gamma\tau)\bigl(2772\exp(-5\gamma\tau)-5250\times$$
$$\times\exp(-4\gamma\tau)+3360\exp(-3\gamma\tau)-840\exp(-2\gamma\tau)+70\times$$
$$\times\exp(-\gamma\tau)-1\bigr).$$

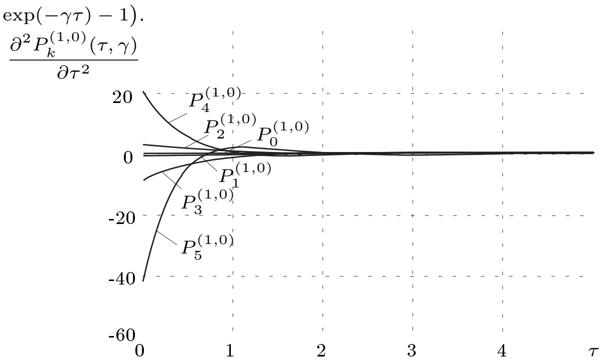


Рис. 1.43. Вид 2-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 1$, $\alpha = 1$, $\beta = 0$

[1.44]
$$\frac{\partial^3 P_k^{(1,0)}(\tau,\gamma)}{\partial \tau^3} = -\gamma^3 \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1}{s+1}(-1)^s \times$$
$$\times (s+1)^3\exp\bigl(-(s+1)\gamma\tau\bigr).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 P_0^{(1,0)}(\tau,\gamma)}{\partial \tau^3} = -\gamma^3 \exp(-\gamma\tau);$$
$$\frac{\partial^3 P_1^{(1,0)}(\tau,\gamma)}{\partial \tau^3} = 2\gamma^3 \exp(-\gamma\tau)\bigl(12\exp(-\gamma\tau)-1\bigr);$$
$$\frac{\partial^3 P_2^{(1,0)}(\tau,\gamma)}{\partial \tau^3} = -3\gamma^3 \exp(-\gamma\tau)\bigl(90\exp(-2\gamma\tau)-32\times$$
$$\times\exp(-\gamma\tau)+1\bigr);$$
$$\frac{\partial^3 P_3^{(1,0)}(\tau,\gamma)}{\partial \tau^3} = 4\gamma^3 \exp(-\gamma\tau)\bigl(560\exp(-3\gamma\tau)-405\times$$
$$\times\exp(-2\gamma\tau)+60\exp(-\gamma\tau)-1\bigr);$$
$$\frac{\partial^3 P_4^{(1,0)}(\tau,\gamma)}{\partial \tau^3} = -5\gamma^3 \exp(-\gamma\tau)\bigl(3150\exp(-4\gamma\tau)-3584\times$$
$$\times\exp(-3\gamma\tau)+1134\exp(-2\gamma\tau)-96\exp(-\gamma\tau)+1\bigr);$$

$$\frac{\partial^3 P_5^{(1,0)}(\tau,\gamma)}{\partial\tau^3} = 6\gamma^3\exp(-\gamma\tau)\big(16632\exp(-5\gamma\tau) - 26250 \times$$
$$\times \exp(-4\gamma\tau) + 13440\exp(-3\gamma\tau) - 2520\exp(-2\gamma\tau) + 140 \times$$
$$\times \exp(-\gamma\tau) - 1\big).$$

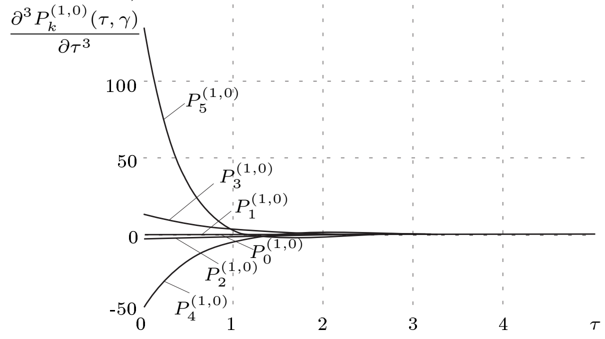


Рис. 1.44. Вид 3-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 1$, $\alpha = 1$, $\beta = 0$

[1.45]
$$\frac{\partial^n P_k^{(1,0)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1}{s+1} \times$$
$$\times (-1)^s (s+1)^n \exp\big(-(s+1)\gamma\tau\big).$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n P_0^{(1,0)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp(-\gamma\tau);$$
$$\frac{\partial^n P_1^{(1,0)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n\exp(-\gamma\tau)\big(3\cdot 2^n\exp(-\gamma\tau) - 2\big);$$
$$\frac{\partial^n P_2^{(1,0)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp(-\gamma\tau)\big(10\cdot 3^n\exp(-2\gamma\tau) - 12\cdot 2^n \times$$
$$\times \exp(-\gamma\tau) + 3\big);$$
$$\frac{\partial^n P_3^{(1,0)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n\exp(-\gamma\tau)\big(35\cdot 4^n\exp(-3\gamma\tau) -$$
$$- 60\cdot 3^n\exp(-2\gamma\tau) + 30\cdot 2^n\exp(-\gamma\tau) - 4\big);$$
$$\frac{\partial^n P_4^{(1,0)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp(-\gamma\tau)\big(126\cdot 5^n\exp(-4\gamma\tau) -$$
$$- 280\cdot 4^n\exp(-3\gamma\tau) + 210\cdot 3^n\exp(-2\gamma\tau) - 60\cdot 2^n\exp(-\gamma\tau) +$$
$$+ 5\big);$$
$$\frac{\partial^n P_5^{(1,0)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n\exp(-\gamma\tau)\big(462\cdot 6^n\exp(-5\gamma\tau) -$$
$$- 1260\cdot 5^n\exp(-4\gamma\tau) + 1260\cdot 4^n\exp(-3\gamma\tau) - 560\cdot 3^n \times$$
$$\times \exp(-2\gamma\tau) + 105\cdot 2^n\exp(-\gamma\tau) - 1\big).$$

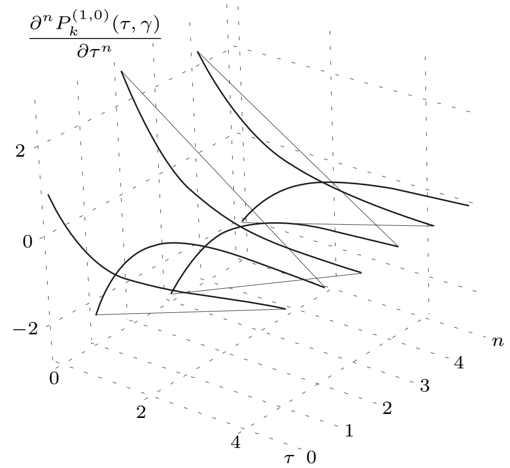


Рис. 1.45. Вид n-ой производной ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 0,25$, $c = 1$, $\alpha = 1$, $\beta = 0$

[1.46]
$$\frac{\partial P_k^{(2,0)}(\tau,\gamma)}{\partial\tau} = -\gamma \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+2}{s+2}(-1)^s \times$$
$$\times (2s+3)\exp\big(-(2s+3)\gamma\tau\big).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial P_0^{(2,0)}(\tau,\gamma)}{\partial\tau} = -3\gamma\exp(-3\gamma\tau);$$
$$\frac{\partial P_1^{(2,0)}(\tau,\gamma)}{\partial\tau} = \gamma\exp(-3\gamma\tau)\big(20\exp(-2\gamma\tau) - 9\big);$$
$$\frac{\partial P_2^{(2,0)}(\tau,\gamma)}{\partial\tau} = -\gamma\exp(-3\gamma\tau)\big(105\exp(-4\gamma\tau) - 100 \times$$
$$\times \exp(-2\gamma\tau) + 18\big);$$
$$\frac{\partial P_3^{(2,0)}(\tau,\gamma)}{\partial\tau} = \gamma\exp(-3\gamma\tau)\big(504\exp(-6\gamma\tau) - 735 \times$$
$$\times \exp(-4\gamma\tau) + 300\exp(-2\gamma\tau) - 30\big);$$
$$\frac{\partial P_4^{(2,0)}(\tau,\gamma)}{\partial\tau} = -\gamma\exp(-3\gamma\tau)\big(2310\exp(-8\gamma\tau) - 4536 \times$$
$$\times \exp(-6\gamma\tau) + 2940\exp(-4\gamma\tau) - 700\exp(-2\gamma\tau) + 45\big);$$
$$\frac{\partial P_5^{(2,0)}(\tau,\gamma)}{\partial\tau} = \gamma\exp(-3\gamma\tau)\big(10296\exp(-10\gamma\tau) - 25410 \times$$
$$\times \exp(-8\gamma\tau) + 22680\exp(-6\gamma\tau) - 8820\exp(-4\gamma\tau) + 1400 \times$$
$$\times \exp(-2\gamma\tau) - 63\big).$$

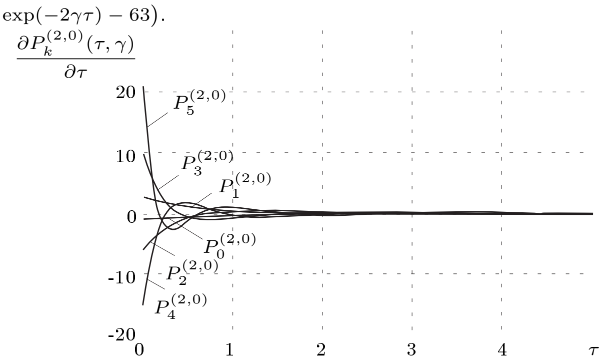


Рис. 1.46. Вид 1-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 2$, $\beta = 0$

[1.47] $$\frac{\partial^2 P_k^{(2,0)}(\tau,\gamma)}{\partial\tau^2} = \gamma^2 \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+2}{s+2}(-1)^s \times \times (2s+3)^2 \exp\bigl(-(2s+3)\gamma\tau\bigr).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 P_0^{(2,0)}(\tau,\gamma)}{\partial\tau^2} = 9\gamma^2 \exp(-3\gamma\tau);$$

$$\frac{\partial^2 P_1^{(2,0)}(\tau,\gamma)}{\partial\tau^2} = -\gamma^2 \exp(-3\gamma\tau)\bigl(100\exp(-2\gamma\tau) - 27\bigr);$$

$$\frac{\partial^2 P_2^{(2,0)}(\tau,\gamma)}{\partial\tau^2} = \gamma^2 \exp(-3\gamma\tau)\bigl(735\exp(-4\gamma\tau) - 500 \times \exp(-2\gamma\tau) + 54\bigr);$$

$$\frac{\partial^2 P_3^{(2,0)}(\tau,\gamma)}{\partial\tau^2} = -\gamma^2 \exp(-3\gamma\tau)\bigl(4536\exp(-6\gamma\tau) - 5145 \times \exp(-4\gamma\tau) + 1500\exp(-2\gamma\tau) - 90\bigr);$$

$$\frac{\partial^2 P_4^{(2,0)}(\tau,\gamma)}{\partial\tau^2} = \gamma^2 \exp(-3\gamma\tau)\bigl(25410\exp(-8\gamma\tau) - 40824 \times \exp(-6\gamma\tau) + 20580\exp(-4\gamma\tau) - 3500\exp(-2\gamma\tau) + 135\bigr);$$

$$\frac{\partial^2 P_5^{(2,0)}(\tau,\gamma)}{\partial\tau^2} = -\gamma^2 \exp(-3\gamma\tau)\bigl(133848\exp(-10\gamma\tau) - 279510\exp(-8\gamma\tau) + 204120\exp(-6\gamma\tau) - 61740 \times \exp(-4\gamma\tau) + 7000\exp(-2\gamma\tau) - 189\bigr).$$

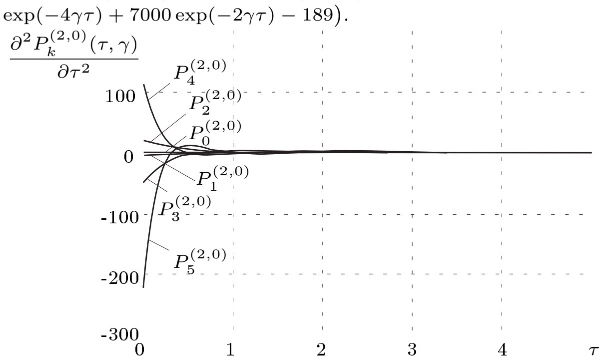


Рис. 1.47. Вид 2-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 2$, $\beta = 0$

[1.48] $$\frac{\partial^3 P_k^{(2,0)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3 \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+2}{s+2}(-1)^s \times \times (2s+3)^3 \exp\bigl(-(2s+3)\gamma\tau\bigr).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 P_0^{(2,0)}(\tau,\gamma)}{\partial\tau^3} = -27\gamma^3 \exp(-3\gamma\tau);$$

$$\frac{\partial^3 P_1^{(2,0)}(\tau,\gamma)}{\partial\tau^3} = \gamma^3 \exp(-3\gamma\tau)\bigl(500\exp(-2\gamma\tau) - 81\bigr);$$

$$\frac{\partial^3 P_2^{(2,0)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3 \exp(-3\gamma\tau)\bigl(5145\exp(-4\gamma\tau) - 2500 \times \exp(-2\gamma\tau) + 162\bigr);$$

$$\frac{\partial^3 P_3^{(2,0)}(\tau,\gamma)}{\partial\tau^3} = \gamma^3 \exp(-3\gamma\tau)\bigl(40824\exp(-6\gamma\tau) - 36015 \times \exp(-4\gamma\tau) + 7500\exp(-2\gamma\tau) - 270\bigr);$$

$$\frac{\partial^3 P_4^{(2,0)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3 \exp(-3\gamma\tau)\bigl(279510\exp(-8\gamma\tau) - 367416\exp(-6\gamma\tau) + 144060\exp(-4\gamma\tau) - 17500\exp(-2\gamma\tau) + 405\bigr);$$

$$\frac{\partial^3 P_5^{(2,0)}(\tau,\gamma)}{\partial\tau^3} = \gamma^3 \exp(-3\gamma\tau)\bigl(1740024\exp(-10\gamma\tau) - 3074610\exp(-8\gamma\tau) + 1837080\exp(-6\gamma\tau) - 432180 \times \exp(-4\gamma\tau) + 35000\exp(-2\gamma\tau) - 567\bigr).$$

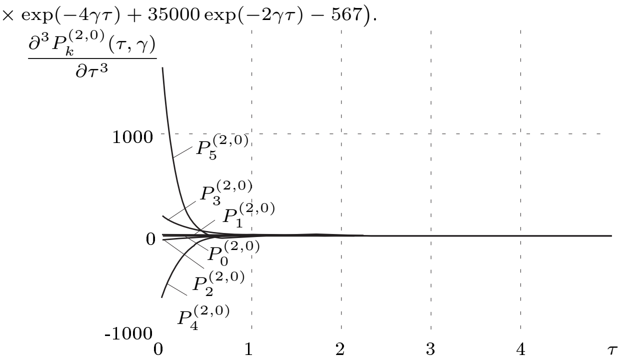


Рис. 1.48. Вид 3-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 2$, $\beta = 0$

[1.49] $$\frac{\partial^n P_k^{(2,0)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+2}{s+2} \times \times (-1)^s (2s+3)^n \exp\bigl(-(2s+3)\gamma\tau\bigr).$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n P_0^{(2,0)}(\tau,\gamma)}{\partial\tau^n} = (-3\gamma)^n \exp(-3\gamma\tau);$$

$$\frac{\partial^n P_1^{(2,0)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n \exp(-3\gamma\tau)\bigl(4\cdot 5^n\exp(-2\gamma\tau) - 3\cdot 3^n\bigr);$$

$$\frac{\partial^n P_2^{(2,0)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp(-3\gamma\tau)\bigl(15\cdot 7^n\exp(-4\gamma\tau) - 20\cdot 5^n\exp(-2\gamma\tau) + 6\cdot 3^n\bigr);$$

$$\frac{\partial^n P_3^{(2,0)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n \exp(-3\gamma\tau)\bigl(56\cdot 9^n\exp(-6\gamma\tau) - 105\cdot 7^n\exp(-4\gamma\tau) + 60\cdot 5^n\exp(-2\gamma\tau) - 10\cdot 3^n\bigr);$$

$$\frac{\partial^n P_4^{(2,0)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp(-3\gamma\tau)\bigl(210\cdot 11^n\exp(-8\gamma\tau) - 504\cdot 9^n\exp(-6\gamma\tau) + 420\cdot 7^n\exp(-4\gamma\tau) - 140\cdot 5^n\exp(-2\gamma\tau) + 15\cdot 3^n\bigr);$$

$$\frac{\partial^n P_5^{(2,0)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n \exp(-3\gamma\tau)\bigl(792\cdot 13^n\exp(-10\gamma\tau) - 2310\cdot 11^n\exp(-8\gamma\tau) + 2520\cdot 9^n\exp(-6\gamma\tau) - 1260\cdot 7^n \times \exp(-4\gamma\tau) + 280\cdot 5^n\exp(-2\gamma\tau) - 21\cdot 3^n\bigr).$$

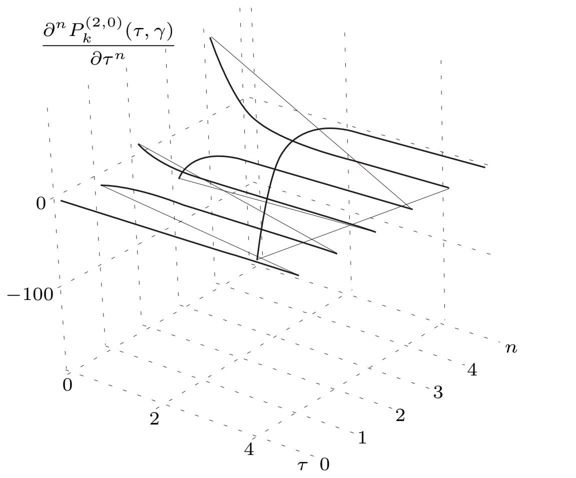


Рис. 1.49. Вид n-ой производной ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 0,25$, $c = 2$, $\alpha = 2$, $\beta = 0$

[1.50]
$$\frac{\partial P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s \times$$
$$\times\,(2s+\alpha+1)\exp\bigl(-(2s+\alpha+1)c\gamma\tau/2\bigr).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial P_0^{(\alpha,0)}(\tau,\gamma)}{\partial\tau} = -\frac{(\alpha+1)c\gamma}{2}\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr);$$

$$\frac{\partial P_1^{(\alpha,0)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{2}\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)^2-(\alpha+2)\times$$
$$\times\,(\alpha+3)\exp(-c\gamma\tau)\bigr);$$

$$\frac{\partial P_2^{(\alpha,0)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{4}\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)^2(\alpha+2)-$$
$$-\,2(\alpha+2)(\alpha+3)^2\exp(-c\gamma\tau)+(\alpha+3)(\alpha+4)(\alpha+5)\exp(-2c\gamma\tau)\bigr);$$

$$\frac{\partial P_3^{(\alpha,0)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{12}\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)^2(\alpha+2)\times$$
$$\times\,(\alpha+3)-3(\alpha+2)(\alpha+3)^2(\alpha+4)\exp(-c\gamma\tau)+3(\alpha+3)(\alpha+4)\times$$
$$\times\,(\alpha+5)^2\exp(-2c\gamma\tau)-(\alpha+4)(\alpha+5)(\alpha+6)(\alpha+7)\exp(-3c\gamma\tau)\bigr);$$

$$\frac{\partial P_4^{(\alpha,0)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{48}\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)^2(\alpha+2)\times$$
$$\times\,(\alpha+3)(\alpha+4)-4(\alpha+2)(\alpha+3)^2(\alpha+4)(\alpha+5)\exp(-c\gamma\tau)+$$
$$+\,6(\alpha+3)(\alpha+4)(\alpha+5)^2(\alpha+6)\exp(-2c\gamma\tau)-4(\alpha+4)\times$$
$$\times\,(\alpha+5)(\alpha+6)(\alpha+7)^2\exp(-3c\gamma\tau)+(\alpha+5)(\alpha+6)(\alpha+7)\times$$
$$\times\,(\alpha+8)(\alpha+9)\exp(-4c\gamma\tau)\bigr);$$

$$\frac{\partial P_5^{(\alpha,0)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{240}\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)^2\times$$
$$\times\,(\alpha+2)(\alpha+3)(\alpha+4)(\alpha+5)-5(\alpha+2)(\alpha+3)^2(\alpha+4)\times$$
$$\times\,(\alpha+5)(\alpha+6)\exp(-c\gamma\tau)+10(\alpha+3)(\alpha+4)(\alpha+5)^2\times$$
$$\times\,(\alpha+6)(\alpha+7)\exp(-2c\gamma\tau)-10(\alpha+4)(\alpha+5)(\alpha+6)\times$$
$$\times\,(\alpha+7)^2(\alpha+8)\exp(-3c\gamma\tau)+5(\alpha+5)(\alpha+6)(\alpha+7)(\alpha+8)\times$$
$$\times\,(\alpha+9)^2\exp(-4c\gamma\tau)-(\alpha+6)(\alpha+7)(\alpha+8)(\alpha+9)\times$$
$$\times\,(\alpha+10)(\alpha+11)\exp(-5c\gamma\tau)\bigr).$$

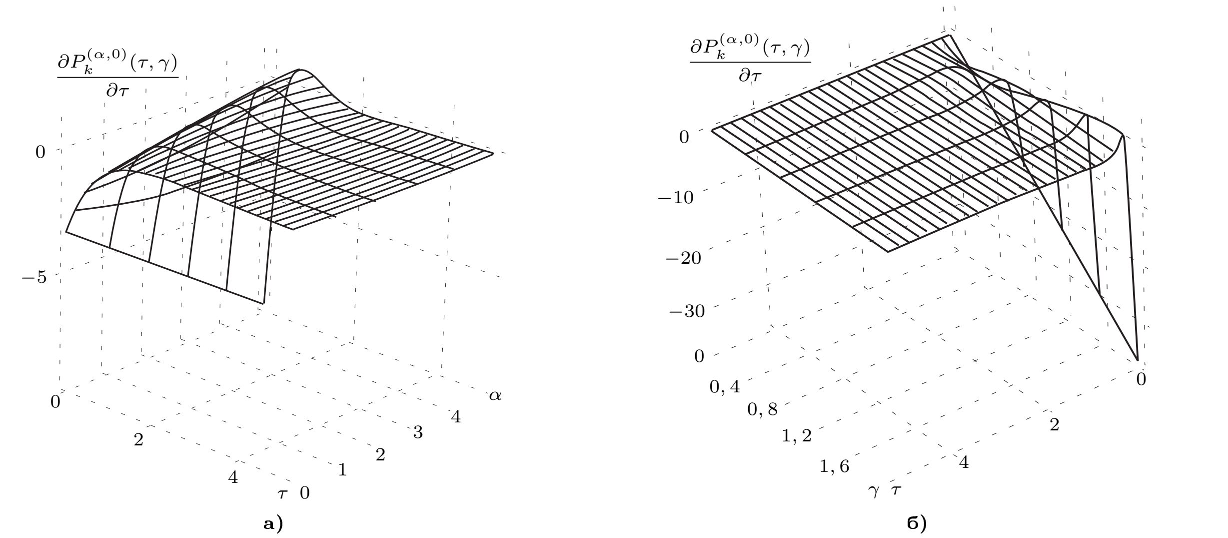


Рис. 1.50. Вид 1-ой производной ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,25$, $c = 2$, $\alpha \in [0;5]$, $\beta = 0$; б) $\gamma \in (0;2]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[1.51]
$$\frac{\partial^2 P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{c^2\gamma^2}{4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}\times$$
$$\times\,(-1)^s(2s+\alpha+1)^2\exp\bigl(-(2s+\alpha+1)c\gamma\tau/2\bigr).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 P_0^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{(\alpha+1)^2c^2\gamma^2}{4}\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr);$$

$$\frac{\partial^2 P_1^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{c^2\gamma^2}{4}\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)^3-(\alpha+2)\times$$
$$\times\,(\alpha+3)^2\exp(-c\gamma\tau)\bigr);$$

$$\frac{\partial^2 P_2^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{c^2\gamma^2}{8}\exp\bigl(-(\alpha+1)c\gamma\tau/2\bigr)\bigl((\alpha+1)^3(\alpha+2)-$$

$$- 2(\alpha+2)(\alpha+3)^3\exp(-c\gamma\tau) + (\alpha+3)(\alpha+4)(\alpha+5)^2 \times \exp(-2c\gamma\tau)\big);$$

$$\frac{\partial^2 P_3^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{c^2\gamma^2}{24}\exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^3(\alpha+2) \times (\alpha+3) - 3(\alpha+2)(\alpha+3)^3(\alpha+4)\exp(-c\gamma\tau) + 3(\alpha+3) \times (\alpha+4)(\alpha+5)^3\exp(-2c\gamma\tau) - (\alpha+4)(\alpha+5)(\alpha+6)(\alpha+7)^2 \times \exp(-3c\gamma\tau)\big);$$

$$\frac{\partial^2 P_4^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{c^2\gamma^2}{96}\exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^3(\alpha+2) \times (\alpha+3)(\alpha+4) - 4(\alpha+2)(\alpha+3)^3(\alpha+4)(\alpha+5)\exp(-c\gamma\tau) + 6(\alpha+3)(\alpha+4)(\alpha+5)^3(\alpha+6)\exp(-2c\gamma\tau) - 4(\alpha+4)(\alpha+5) \times (\alpha+6)(\alpha+7)^3\exp(-3c\gamma\tau) + (\alpha+5)(\alpha+6)(\alpha+7)(\alpha+8) \times (\alpha+9)^2\exp(-4c\gamma\tau)\big);$$

$$\frac{\partial^2 P_5^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^2} = \frac{c^2\gamma^2}{480}\exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^3(\alpha+2) \times (\alpha+3)(\alpha+4)(\alpha+5) - 5(\alpha+2)(\alpha+3)^3(\alpha+4)(\alpha+5)(\alpha+6) \times \exp(-c\gamma\tau) + 10(\alpha+3)(\alpha+4)(\alpha+5)^3(\alpha+6)(\alpha+7) \times \exp(-2c\gamma\tau) - 10(\alpha+4)(\alpha+5)(\alpha+6)(\alpha+7)^3(\alpha+8) \times \exp(-3c\gamma\tau) + 5(\alpha+5)(\alpha+6)(\alpha+7)(\alpha+8)(\alpha+9)^3 \times \exp(-4c\gamma\tau) - (\alpha+6)(\alpha+7)(\alpha+8)(\alpha+9)(\alpha+10)(\alpha+11)^2 \times \exp(-5c\gamma\tau)\big).$$

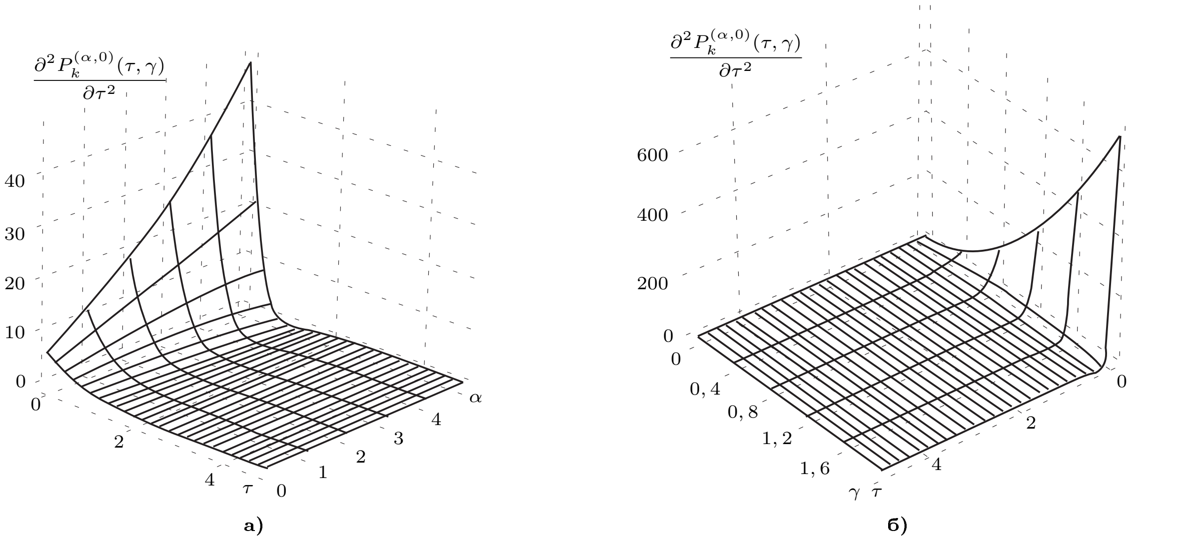


Рис. 1.51. Вид 2-ой производной ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,25$, $c = 2$, $\alpha \in [0;5]$, $\beta = 0$; б) $\gamma \in (0;2]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[1.52]
$$\frac{\partial^3 P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{8}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha} \times (-1)^s(2s+\alpha+1)^3\exp\big(-(2s+\alpha+1)c\gamma\tau/2\big).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 P_0^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{(\alpha+1)^3c^3\gamma^3}{8}\exp\big(-(\alpha+1)c\gamma\tau/2\big);$$

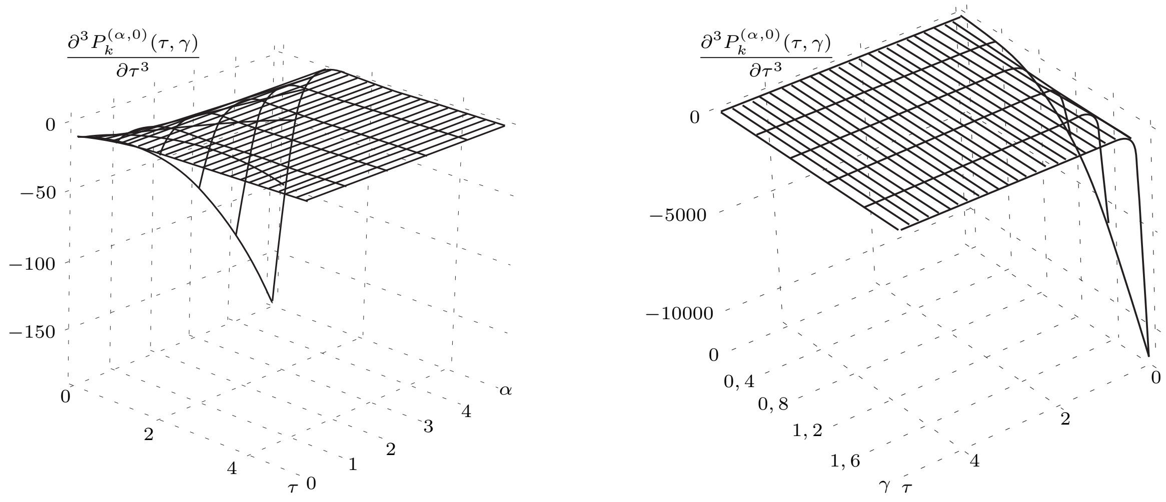
$$\frac{\partial^3 P_1^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{8}\exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^4 - (\alpha+2)(\alpha+3)^3\exp(-c\gamma\tau)\big);$$

$$\frac{\partial^3 P_2^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{16}\exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^4(\alpha+2) - 2(\alpha+2)(\alpha+3)^4\exp(-c\gamma\tau) + (\alpha+3)(\alpha+4)(\alpha+5)^3 \times \exp(-2c\gamma\tau)\big);$$

$$\frac{\partial^3 P_3^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{48}\exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^4(\alpha+2) \times (\alpha+3) - 3(\alpha+2)(\alpha+3)^4(\alpha+4)\exp(-c\gamma\tau) + 3(\alpha+3)(\alpha+4) \times (\alpha+5)^4\exp(-2c\gamma\tau) - (\alpha+4)(\alpha+5)(\alpha+6)(\alpha+7)^3 \times \exp(-3c\gamma\tau)\big);$$

$$\frac{\partial^3 P_4^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{192}\exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^4(\alpha+2) \times (\alpha+3)(\alpha+4) - 4(\alpha+2)(\alpha+3)^4(\alpha+4)(\alpha+5)\exp(-c\gamma\tau) + 6(\alpha+3)(\alpha+4)(\alpha+5)^4(\alpha+6)\exp(-2c\gamma\tau) - 4(\alpha+4)(\alpha+5) \times (\alpha+6)(\alpha+7)^4\exp(-3c\gamma\tau) + (\alpha+5)(\alpha+6)(\alpha+7)(\alpha+8) \times (\alpha+9)^3\exp(-4c\gamma\tau)\big);$$

$$\frac{\partial^3 P_5^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{960}\exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^4(\alpha+2) \times (\alpha+3)(\alpha+4)(\alpha+5) - 5(\alpha+2)(\alpha+3)^4(\alpha+4)(\alpha+5)(\alpha+6) \times \exp(-c\gamma\tau) + 10(\alpha+3)(\alpha+4)(\alpha+5)^4(\alpha+6)(\alpha+7) \times \exp(-2c\gamma\tau) - 10(\alpha+4)(\alpha+5)(\alpha+6)(\alpha+7)^4(\alpha+8) \times \exp(-3c\gamma\tau) + 5(\alpha+5)(\alpha+6)(\alpha+7)(\alpha+8)(\alpha+9)^4 \times \exp(-4c\gamma\tau) - (\alpha+6)(\alpha+7)(\alpha+8)(\alpha+9)(\alpha+10)(\alpha+11)^3 \times \exp(-5c\gamma\tau)\big).$$



а) б)

Рис. 1.52. Вид 3-ой производной ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,25$, $c = 2$, $\alpha \in [0;5]$, $\beta = 0$; б) $\gamma \in (0;2]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[1.53]
$$\frac{\partial^n P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^n} = \Big(-\frac{c\gamma}{2}\Big)^n \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+\alpha}{s+\alpha}\times$$
$$\times(-1)^s(2s+\alpha+1)^n \exp\big(-(2s+\alpha+1)c\gamma\tau/2\big).$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n P_0^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^n} = \Big(-\frac{c\gamma}{2}\Big)^n \exp\big(-(\alpha+1)c\gamma\tau/2\big);$$

$$\frac{\partial^n P_1^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^n} = \Big(-\frac{c\gamma}{2}\Big)^n \exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^{n+1} - (\alpha+2)(\alpha+3)^n\exp(-c\gamma\tau)\big);$$

$$\frac{\partial^n P_2^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^n} = \Big(-\frac{c\gamma}{2}\Big)^n \exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^{n+1}\times(\alpha+2)-2(\alpha+2)(\alpha+3)^{n+1}\exp(-c\gamma\tau)+(\alpha+3)(\alpha+4)(\alpha+5)^n\times\exp(-2c\gamma\tau)\big);$$

$$\frac{\partial^n P_3^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^n} = \Big(-\frac{c\gamma}{2}\Big)^n \exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^{n+1}\times(\alpha+2)(\alpha+3)-3(\alpha+2)(\alpha+3)^{n+1}(\alpha+4)\exp(-c\gamma\tau)+3\times(\alpha+3)(\alpha+4)(\alpha+5)^{n+1}\exp(-2c\gamma\tau)-(\alpha+4)(\alpha+5)(\alpha+6)\times(\alpha+7)^n\exp(-3c\gamma\tau)\big);$$

$$\frac{\partial^n P_4^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^n} = \Big(-\frac{c\gamma}{2}\Big)^n \exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^{n+1}\times(\alpha+2)(\alpha+3)(\alpha+4)-4(\alpha+2)(\alpha+3)^{n+1}(\alpha+4)(\alpha+5)\times\exp(-c\gamma\tau)+6(\alpha+3)(\alpha+4)(\alpha+5)^{n+1}(\alpha+6)\exp(-2c\gamma\tau)-4(\alpha+4)(\alpha+5)(\alpha+6)(\alpha+7)^{n+1}\exp(-3c\gamma\tau)+(\alpha+5)(\alpha+6)\times(\alpha+7)(\alpha+8)(\alpha+9)^n\exp(-4c\gamma\tau)\big);$$

$$\frac{\partial^n P_5^{(\alpha,0)}(\tau,\gamma)}{\partial\tau^n} = \Big(-\frac{c\gamma}{2}\Big)^n \exp\big(-(\alpha+1)c\gamma\tau/2\big)\big((\alpha+1)^{n+1}\times(\alpha+2)(\alpha+3)(\alpha+4)(\alpha+5)-5(\alpha+2)(\alpha+3)^{n+1}(\alpha+4)\times(\alpha+5)(\alpha+6)\exp(-c\gamma\tau)+10(\alpha+3)(\alpha+4)(\alpha+5)^{n+1}(\alpha+6)\times(\alpha+7)\exp(-2c\gamma\tau)-10(\alpha+4)(\alpha+5)(\alpha+6)(\alpha+7)^{n+1}\times(\alpha+8)\exp(-3c\gamma\tau)+5(\alpha+5)(\alpha+6)(\alpha+7)(\alpha+8)(\alpha+9)^{n+1}\times\exp(-4c\gamma\tau)-(\alpha+6)(\alpha+7)(\alpha+8)(\alpha+9)(\alpha+10)(\alpha+11)^n\times\exp(-5c\gamma\tau)\big).$$

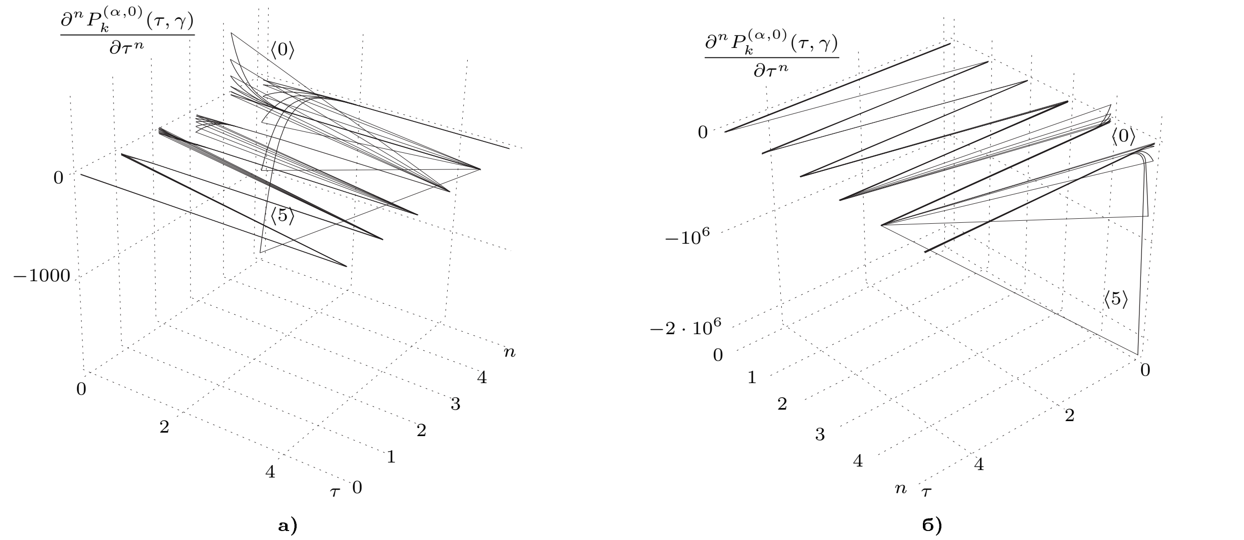


Рис. 1.53. Вид n-ой производной ортогональных функций Якоби 2-ого порядка: а) $n = 0..5$, $\gamma = 0,25$, $c = 2$, $\alpha \in [0;5]$, $\beta = 0$; б) $n = 0..5$, $\gamma \in (0;2]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[1.54]
$$\frac{\partial P_k^{(0,1)}(\tau,\gamma)}{\partial \tau} = -\gamma \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+1}{s} (-1)^s \times \times (2s+1)\exp\bigl(-(2s+1)\gamma\tau\bigr).$$

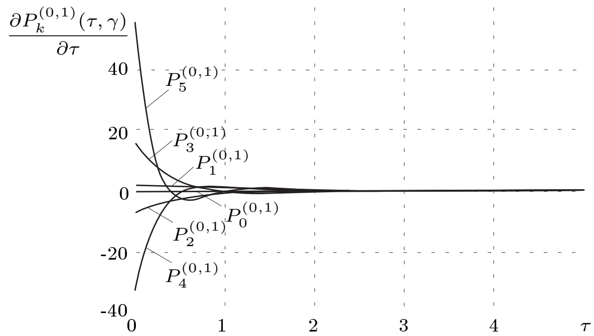


Рис. 1.54. Вид 1-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 1$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial P_0^{(0,1)}(\tau,\gamma)}{\partial \tau} = -\gamma\exp(-\gamma\tau);$$

$$\frac{\partial P_1^{(0,1)}(\tau,\gamma)}{\partial \tau} = \gamma\exp(-\gamma\tau)\bigl(9\exp(-2\gamma\tau) - 1\bigr);$$

$$\frac{\partial P_2^{(0,1)}(\tau,\gamma)}{\partial \tau} = -\gamma\exp(-\gamma\tau)\bigl(50\exp(-4\gamma\tau) - 24\exp(-2\gamma\tau) + + 1\bigr);$$

$$\frac{\partial P_3^{(0,1)}(\tau,\gamma)}{\partial \tau} = \gamma\exp(-\gamma\tau)\bigl(245\exp(-6\gamma\tau) - 225\exp(-4\gamma\tau) + + 45\exp(-2\gamma\tau) - 1\bigr);$$

$$\frac{\partial P_4^{(0,1)}(\tau,\gamma)}{\partial \tau} = -\gamma\exp(-\gamma\tau)\bigl(1134\exp(-8\gamma\tau) - 1568 \times \times \exp(-6\gamma\tau) + 630\exp(-4\gamma\tau) - 72\exp(-2\gamma\tau) + 1\bigr);$$

$$\frac{\partial P_5^{(0,1)}(\tau,\gamma)}{\partial \tau} = \gamma\exp(-\gamma\tau)\bigl(5082\exp(-10\gamma\tau) - 9450 \times \times \exp(-8\gamma\tau) + 5880\exp(-6\gamma\tau) - 1400\exp(-4\gamma\tau) + 105 \times \times \exp(-2\gamma\tau) - 1\bigr).$$

[1.55]
$$\frac{\partial^2 P_k^{(0,1)}(\tau,\gamma)}{\partial \tau^2} = \gamma^2 \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+1}{s} (-1)^s \times \times (2s+1)^2\exp\bigl(-(2s+1)\gamma\tau\bigr).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 P_0^{(0,1)}(\tau,\gamma)}{\partial \tau^2} = \gamma^2\exp(-\gamma\tau);$$

$$\frac{\partial^2 P_1^{(0,1)}(\tau,\gamma)}{\partial \tau^2} = -\gamma^2\exp(-\gamma\tau)\bigl(27\exp(-2\gamma\tau) - 1\bigr);$$

$$\frac{\partial^2 P_2^{(0,1)}(\tau,\gamma)}{\partial \tau^2} = \gamma^2\exp(-\gamma\tau)\bigl(250\exp(-4\gamma\tau) - 72 \times \times \exp(-2\gamma\tau) + 1\bigr);$$

$$\frac{\partial^2 P_3^{(0,1)}(\tau,\gamma)}{\partial \tau^2} = -\gamma^2\exp(-\gamma\tau)\bigl(1715\exp(-6\gamma\tau) - 1125 \times \times \exp(-4\gamma\tau) + 135\exp(-2\gamma\tau) - 1\bigr);$$

$$\frac{\partial^2 P_4^{(0,1)}(\tau,\gamma)}{\partial \tau^2} = \gamma^2\exp(-\gamma\tau)\bigl(10206\exp(-8\gamma\tau) - 10976 \times$$

$\times \exp(-6\gamma\tau) + 3150\exp(-4\gamma\tau) - 216\exp(-2\gamma\tau) + 1\big);$

$$\frac{\partial^2 P_5^{(0,1)}(\tau,\gamma)}{\partial\tau^2} = -\gamma^2\exp(-\gamma\tau)\big(55902\exp(-10\gamma\tau) - 85050\times$$
$$\times\exp(-8\gamma\tau) + 41160\exp(-6\gamma\tau) - 7000\exp(-4\gamma\tau) + 315\times$$
$$\times\exp(-2\gamma\tau) - 1\big).$$

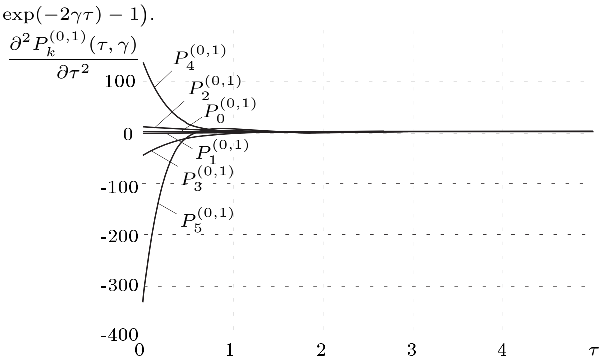


Рис. 1.55. Вид 2-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.56]
$$\frac{\partial^3 P_k^{(0,1)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s\times$$
$$\times(2s+1)^3\exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 P_0^{(0,1)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\exp(-\gamma\tau);$$

$$\frac{\partial^3 P_1^{(0,1)}(\tau,\gamma)}{\partial\tau^3} = \gamma^3\exp(-\gamma\tau)\big(81\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^3 P_2^{(0,1)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\exp(-\gamma\tau)\big(1250\exp(-4\gamma\tau) - 216\times$$
$$\times\exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial^3 P_3^{(0,1)}(\tau,\gamma)}{\partial\tau^3} = \gamma^3\exp(-\gamma\tau)\big(12005\exp(-6\gamma\tau) - 5625\times$$
$$\times\exp(-4\gamma\tau) + 405\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^3 P_4^{(0,1)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\exp(-\gamma\tau)\big(91854\exp(-8\gamma\tau) - 76832\times$$
$$\times\exp(-6\gamma\tau) + 15750\exp(-4\gamma\tau) - 648\exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial^3 P_5^{(0,1)}(\tau,\gamma)}{\partial\tau^3} = \gamma^3\exp(-\gamma\tau)\big(614922\exp(-10\gamma\tau) -$$
$$- 765450\exp(-8\gamma\tau) + 288120\exp(-6\gamma\tau) - 35000\exp(-4\gamma\tau) +$$
$$+ 945\exp(-2\gamma\tau) - 1\big).$$

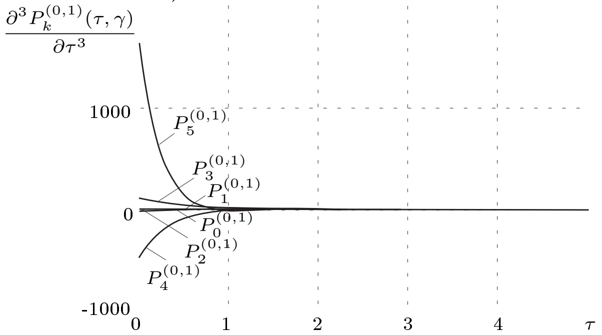


Рис. 1.56. Вид 3-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.57]
$$\frac{\partial^n P_k^{(0,1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}\times$$
$$\times(-1)^s(2s+1)^n\exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n P_0^{(0,1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp(-\gamma\tau);$$

$$\frac{\partial^n P_1^{(0,1)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n\exp(-\gamma\tau)\big(3\cdot 3^n\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^n P_2^{(0,1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp(-\gamma\tau)\big(10\cdot 5^n\exp(-4\gamma\tau) - 8\cdot 3^n\times$$
$$\times\exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial^n P_3^{(0,1)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n\exp(-\gamma\tau)\big(35\cdot 7^n\exp(-6\gamma\tau) -$$
$$- 45\cdot 5^n\exp(-4\gamma\tau) + 15\cdot 3^n\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^n P_4^{(0,1)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n\exp(-\gamma\tau)\big(126\cdot 9^n\exp(-8\gamma\tau) -$$
$$- 224\cdot 7^n\exp(-6\gamma\tau) + 126\cdot 5^n\exp(-4\gamma\tau) - 24\cdot 3^n\exp(-2\gamma\tau) +$$
$$+ 1\big);$$

$$\frac{\partial^n P_5^{(0,1)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n\exp(-\gamma\tau)\big(462\cdot 11^n\exp(-10\gamma\tau) -$$
$$- 1050\cdot 9^n\exp(-8\gamma\tau) + 840\cdot 7^n\exp(-6\gamma\tau) - 280\cdot 5^n\exp(-4\gamma\tau) +$$
$$+ 35\cdot 3^n\exp(-2\gamma\tau) - 1\big).$$

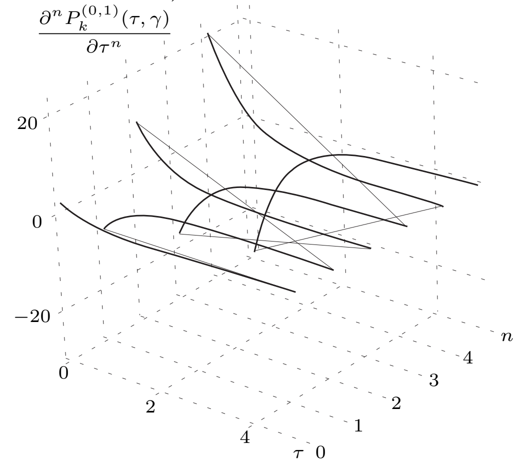


Рис. 1.57. Вид n-ой производной ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.58]
$$\frac{\partial P_k^{(0,2)}(\tau,\gamma)}{\partial\tau} = -\gamma\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s\times$$
$$\times(2s+1)\exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial P_0^{(0,2)}(\tau,\gamma)}{\partial\tau} = -\gamma\exp(-\gamma\tau);$$

$$\frac{\partial P_1^{(0,2)}(\tau,\gamma)}{\partial\tau} = \gamma\exp(-\gamma\tau)\big(12\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial P_2^{(0,2)}(\tau,\gamma)}{\partial\tau} = -\gamma\exp(-\gamma\tau)\big(75\exp(-4\gamma\tau) - 30\exp(-2\gamma\tau) +$$

$+1)$;

$$\frac{\partial P_3^{(0,2)}(\tau,\gamma)}{\partial\tau} = \gamma\exp(-\gamma\tau)\big(392\exp(-6\gamma\tau) - 315\exp(-4\gamma\tau) + 54\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial P_4^{(0,2)}(\tau,\gamma)}{\partial\tau} = -\gamma\exp(-\gamma\tau)\big(1890\exp(-8\gamma\tau) - 2352\times \exp(-6\gamma\tau) + 840\exp(-4\gamma\tau) - 84\exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial P_5^{(0,2)}(\tau,\gamma)}{\partial\tau} = \gamma\exp(-\gamma\tau)\big(8712\exp(-10\gamma\tau) - 14850\times \exp(-8\gamma\tau) + 8400\exp(-6\gamma\tau) - 1800\exp(-4\gamma\tau) + 120\times \exp(-2\gamma\tau) - 1\big).$$

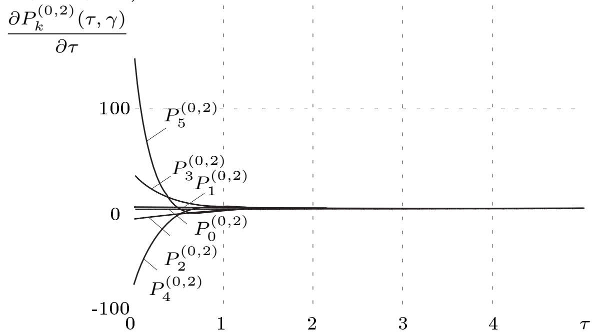


Рис. 1.58. Вид 1-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.59]
$$\frac{\partial^2 P_k^{(0,2)}(\tau,\gamma)}{\partial\tau^2} = \gamma^2\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s\times (2s+1)^2\exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 P_0^{(0,2)}(\tau,\gamma)}{\partial\tau^2} = \gamma^2\exp(-\gamma\tau);$$

$$\frac{\partial^2 P_1^{(0,2)}(\tau,\gamma)}{\partial\tau^2} = -\gamma^2\exp(-\gamma\tau)\big(36\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^2 P_2^{(0,2)}(\tau,\gamma)}{\partial\tau^2} = \gamma^2\exp(-\gamma\tau)\big(375\exp(-4\gamma\tau) - 90\times \exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial^2 P_3^{(0,2)}(\tau,\gamma)}{\partial\tau^2} = -\gamma^2\exp(-\gamma\tau)\big(2744\exp(-6\gamma\tau) - 1575\times \exp(-4\gamma\tau) + 162\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^2 P_4^{(0,2)}(\tau,\gamma)}{\partial\tau^2} = \gamma^2\exp(-\gamma\tau)\big(17010\exp(-8\gamma\tau) - 16464\times \exp(-6\gamma\tau) + 4200\exp(-4\gamma\tau) - 252\exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial^2 P_5^{(0,2)}(\tau,\gamma)}{\partial\tau^2} = -\gamma^2\exp(-\gamma\tau)\big(95832\exp(-10\gamma\tau) - 133650\times \exp(-8\gamma\tau) + 58800\exp(-6\gamma\tau) - 9000\exp(-4\gamma\tau) + 360\times \exp(-2\gamma\tau) - 1\big).$$

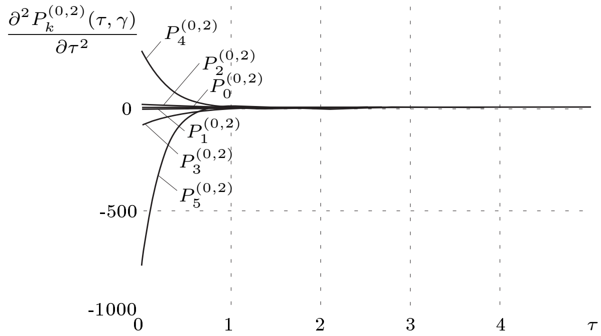


Рис. 1.59. Вид 2-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.60]
$$\frac{\partial^3 P_k^{(0,2)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s\times (2s+1)^3\exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 P_0^{(0,2)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\exp(-\gamma\tau);$$

$$\frac{\partial^3 P_1^{(0,2)}(\tau,\gamma)}{\partial\tau^3} = \gamma^3\exp(-\gamma\tau)\big(108\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^3 P_2^{(0,2)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\exp(-\gamma\tau)\big(1875\exp(-4\gamma\tau) - 270\times \exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial^3 P_3^{(0,2)}(\tau,\gamma)}{\partial\tau^3} = \gamma^3\exp(-\gamma\tau)\big(19208\exp(-6\gamma\tau) - 7875\times \exp(-4\gamma\tau) + 486\exp(-2\gamma\tau) - 1\big);$$

$$\frac{\partial^3 P_4^{(0,2)}(\tau,\gamma)}{\partial\tau^3} = -\gamma^3\exp(-\gamma\tau)\big(153090\exp(-8\gamma\tau) - 115248\times \exp(-6\gamma\tau) + 21000\exp(-4\gamma\tau) - 756\exp(-2\gamma\tau) + 1\big);$$

$$\frac{\partial^3 P_5^{(0,2)}(\tau,\gamma)}{\partial\tau^3} = \gamma^3\exp(-\gamma\tau)\big(1054152\exp(-10\gamma\tau) - 1202850\exp(-8\gamma\tau) + 411600\exp(-6\gamma\tau) - 45000\exp(-4\gamma\tau) + 1080\exp(-2\gamma\tau) - 1\big).$$

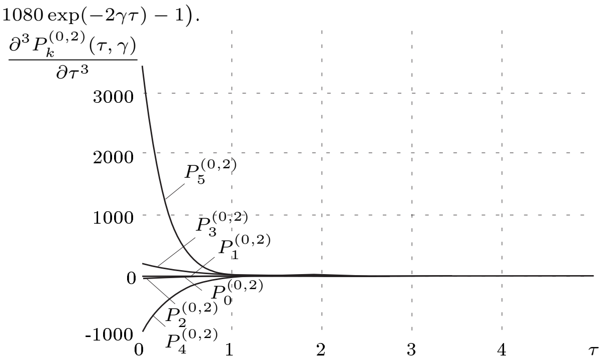


Рис. 1.60. Вид 3-ой производной ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.61] $$\frac{\partial^n P_k^{(0,2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+2}{s} \times (-1)^s (2s+1)^n \exp\bigl(-(2s+1)\gamma\tau\bigr).$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n P_0^{(0,2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp(-\gamma\tau);$$

$$\frac{\partial^n P_1^{(0,2)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n \exp(-\gamma\tau)\bigl(4\cdot 3^n \exp(-2\gamma\tau) - 1\bigr);$$

$$\frac{\partial^n P_2^{(0,2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp(-\gamma\tau)\bigl(15\cdot 5^n \exp(-4\gamma\tau) - 10\cdot 3^n \times \exp(-2\gamma\tau) + 1\bigr);$$

$$\frac{\partial^n P_3^{(0,2)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n \exp(-\gamma\tau)\bigl(56\cdot 7^n \exp(-6\gamma\tau) - 63\cdot 5^n \exp(-4\gamma\tau) + 18\cdot 3^n \exp(-2\gamma\tau) - 1\bigr);$$

$$\frac{\partial^n P_4^{(0,2)}(\tau,\gamma)}{\partial\tau^n} = (-\gamma)^n \exp(-\gamma\tau)\bigl(210\cdot 9^n \exp(-8\gamma\tau) - 336\cdot 7^n \exp(-6\gamma\tau) + 168\cdot 5^n \exp(-4\gamma\tau) - 28\cdot 3^n \exp(-2\gamma\tau) + 1\bigr);$$

$$\frac{\partial^n P_5^{(0,2)}(\tau,\gamma)}{\partial\tau^n} = -(-\gamma)^n \exp(-\gamma\tau)\bigl(792\cdot 11^n \exp(-10\gamma\tau) - 1650\cdot 9^n \exp(-8\gamma\tau) + 1200\cdot 7^n \exp(-6\gamma\tau) - 360\cdot 5^n \exp(-4\gamma\tau) + 40\cdot 3^n \exp(-2\gamma\tau) - 1\bigr).$$

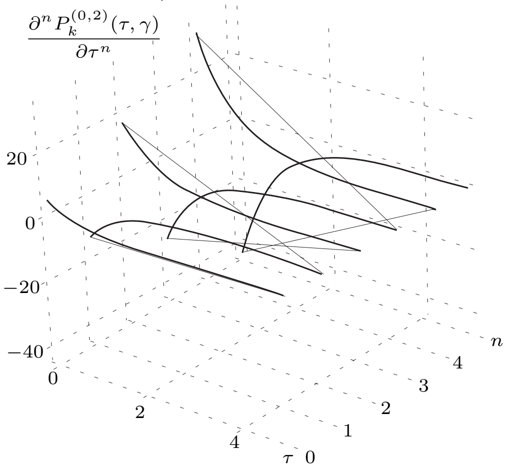


Рис. 1.61. Вид n-ой производной ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.62] $$\frac{\partial P_k^{(0,\beta)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{2} \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+\beta}{s}(-1)^s \times (2s+1)\exp\bigl(-(2s+1)c\gamma\tau/2\bigr).$$

Частные случаи для 1-ой производной функций 0-5 порядков:

$$\frac{\partial P_0^{(0,\beta)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{2}\exp\bigl(-c\gamma\tau/2\bigr);$$

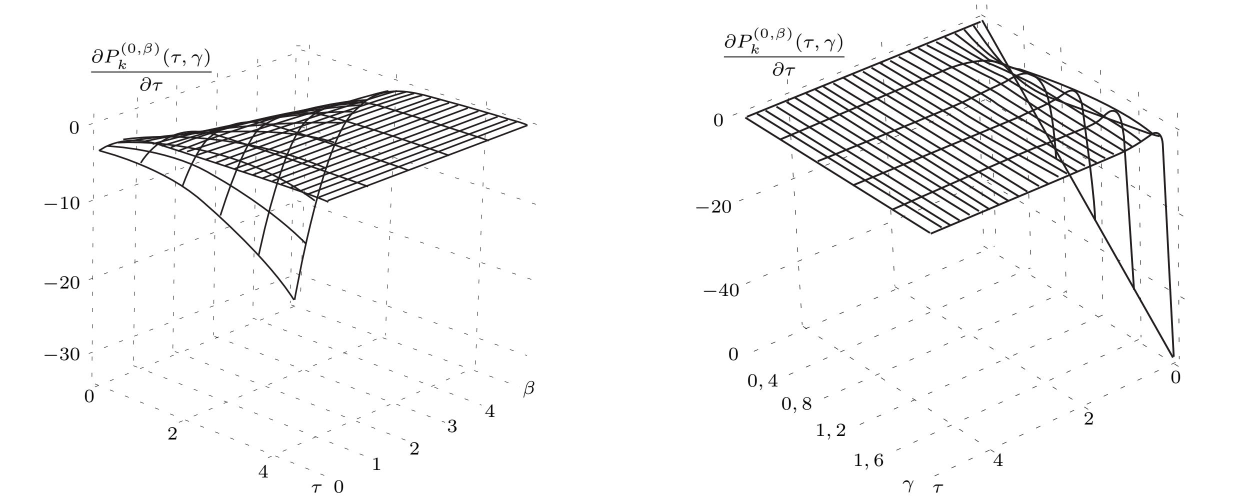
$$\frac{\partial P_1^{(0,\beta)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{2}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1 - 3(\beta+2)\exp(-c\gamma\tau)\bigr);$$

$$\frac{\partial P_2^{(0,\beta)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{2}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1 - 6(\beta+3)\exp(-c\gamma\tau) + 5(\beta+3)(\beta+4)\exp(-2c\gamma\tau)/2\bigr);$$

$$\frac{\partial P_3^{(0,\beta)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{2}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1 - 9(\beta+4)\exp(-c\gamma\tau) + 15(\beta+4)(\beta+5)\exp(-2c\gamma\tau)/2 - 7(\beta+4)(\beta+5)(\beta+6) \times \exp(-3c\gamma\tau)/6\bigr);$$

$$\frac{\partial P_4^{(0,\beta)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{2}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1 - 12(\beta+5)\exp(-c\gamma\tau) + 15(\beta+5)(\beta+6)\exp(-2c\gamma\tau) - 14(\beta+5)(\beta+6)(\beta+7) \times \exp(-3c\gamma\tau)/3 + 3(\beta+5)(\beta+6)(\beta+7)(\beta+8)\exp(-4c\gamma\tau)/8\bigr);$$

$$\frac{\partial P_5^{(0,\beta)}(\tau,\gamma)}{\partial\tau} = -\frac{c\gamma}{2}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1 - 15(\beta+6)\exp(-c\gamma\tau) + 25(\beta+6)(\beta+7)\exp(-2c\gamma\tau) - 35(\beta+6)(\beta+7)(\beta+8) \times \exp(-3c\gamma\tau)/3 + 15(\beta+6)(\beta+7)(\beta+8)(\beta+9)\exp(-4c\gamma\tau)/8 - 11(\beta+6)(\beta+7)(\beta+8)(\beta+9)(\beta+10)\exp(-5c\gamma\tau)/120\bigr).$$



а) б)

Рис. 1.62. Вид 1-ой производной ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta \in [0;5]$; б) $\gamma \in (0;2]$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.63]
$$\frac{\partial^2 P_k^{(0,\beta)}(\tau,\gamma)}{\partial \tau^2} = \frac{c^2\gamma^2}{4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}\times$$
$$\times\,(-1)^s(2s+1)^2\exp\bigl(-(2s+1)c\gamma\tau/2\bigr).$$

Частные случаи для 2-ой производной функций 0-5 порядков:

$$\frac{\partial^2 P_0^{(0,\beta)}(\tau,\gamma)}{\partial \tau^2} = \frac{c^2\gamma^2}{4}\exp\bigl(-c\gamma\tau/2\bigr);$$

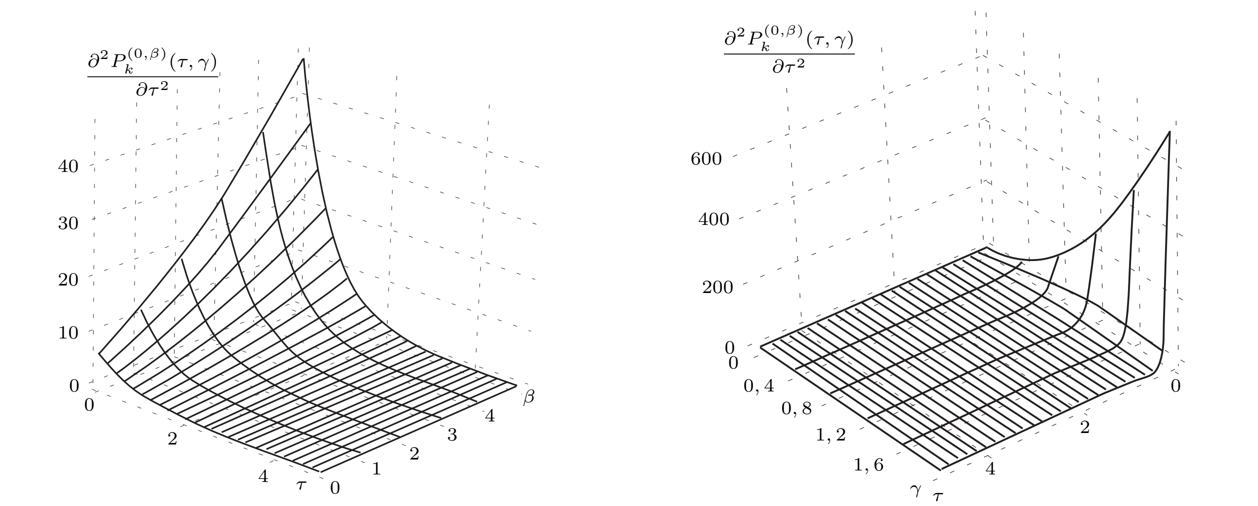
$$\frac{\partial^2 P_1^{(0,\beta)}(\tau,\gamma)}{\partial \tau^2} = \frac{c^2\gamma^2}{4}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-9(\beta+2)\exp(-c\gamma\tau)\bigr);$$

$$\frac{\partial^2 P_2^{(0,\beta)}(\tau,\gamma)}{\partial \tau^2} = \frac{c^2\gamma^2}{4}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-18(\beta+3)\times$$
$$\times\exp(-c\gamma\tau)+25(\beta+3)(\beta+4)\exp(-2c\gamma\tau)/2\bigr);$$

$$\frac{\partial^2 P_3^{(0,\beta)}(\tau,\gamma)}{\partial \tau^2} = \frac{c^2\gamma^2}{4}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-27(\beta+4)\times$$
$$\times\exp(-c\gamma\tau)+75(\beta+4)(\beta+5)\exp(-2c\gamma\tau)/2-49(\beta+4)(\beta+5)\times$$
$$\times\,(\beta+6)\exp(-3c\gamma\tau)/6\bigr);$$

$$\frac{\partial^2 P_4^{(0,\beta)}(\tau,\gamma)}{\partial \tau^2} = \frac{c^2\gamma^2}{4}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-36(\beta+5)\times$$
$$\times\exp(-c\gamma\tau)+75(\beta+5)(\beta+6)\exp(-2c\gamma\tau)-98(\beta+5)(\beta+6)\times$$
$$\times\,(\beta+7)\exp(-3c\gamma\tau)/3+27(\beta+5)(\beta+6)(\beta+7)(\beta+8)\times$$
$$\times\exp(-4c\gamma\tau)/8\bigr);$$

$$\frac{\partial^2 P_5^{(0,\beta)}(\tau,\gamma)}{\partial \tau^2} = \frac{c^2\gamma^2}{4}exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-45(\beta+6)\exp(-c\gamma\tau)+$$
$$+\,125(\beta+6)(\beta+7)\exp(-2c\gamma\tau)-245(\beta+6)(\beta+7)(\beta+8)\times$$
$$\times\exp(-3c\gamma\tau/3+135(\beta+6)(\beta+7)(\beta+8)(\beta+9)\exp(-4c\gamma\tau)/8-$$
$$-\,121(\beta+6)(\beta+7)(\beta+8)(\beta+9)(\beta+10)\exp(-5c\gamma\tau)/120\bigr).$$



а) б)

Рис. 1.63. Вид 2-ой производной ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta \in [0;5]$; б) $\gamma \in (0;2]$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.64]
$$\frac{\partial^3 P_k^{(0,\beta)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{8}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}\times$$
$$\times\,(-1)^s(2s+1)^3\exp\bigl(-(2s+1)c\gamma\tau/2\bigr).$$

Частные случаи для 3-ой производной функций 0-5 порядков:

$$\frac{\partial^3 P_0^{(0,\beta)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{8}\exp\bigl(-c\gamma\tau/2\bigr);$$

$$\frac{\partial^3 P_1^{(0,\beta)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{8}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-27(\beta+2)\times$$
$$\times\exp(-c\gamma\tau)\bigr);$$

$$\frac{\partial^3 P_2^{(0,\beta)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{8}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-54(\beta+3)\times$$
$$\times\exp(-c\gamma\tau)+125(\beta+3)(\beta+4)\exp(-2c\gamma\tau)/2\bigr);$$

$$\frac{\partial^3 P_3^{(0,\beta)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{8}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-81(\beta+4)\times$$
$$\times\exp(-c\gamma\tau)+375(\beta+4)(\beta+5)\exp(-2c\gamma\tau)/2-343(\beta+4)\times$$
$$\times(\beta+5)(\beta+6)\exp(-3c\gamma\tau)/6\bigr);$$

$$\frac{\partial^3 P_4^{(0,\beta)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{8}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-108(\beta+5)\times$$
$$\times\exp(-c\gamma\tau)+375(\beta+5)(\beta+6)\exp(-2c\gamma\tau)-686(\beta+5)\times$$
$$\times(\beta+6)(\beta+7)\exp(-3c\gamma\tau)/3+243(\beta+5)(\beta+6)(\beta+7)(\beta+8)\times$$
$$\times\exp(-4c\gamma\tau)/8\bigr);$$

$$\frac{\partial^3 P_5^{(0,\beta)}(\tau,\gamma)}{\partial\tau^3} = -\frac{c^3\gamma^3}{8}\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-135(\beta+6)\times$$
$$\times\exp(-c\gamma\tau)+625(\beta+6)(\beta+7)\exp(-2c\gamma\tau)-1715(\beta+6)\times$$
$$\times(\beta+7)(\beta+8)\exp(-3c\gamma\tau)/3+1215(\beta+6)(\beta+7)(\beta+8)(\beta+9)\times$$
$$\times\exp(-4c\gamma\tau)/8-1331(\beta+6)(\beta+7)(\beta+8)(\beta+9)(\beta+10)\times$$
$$\times\exp(-5c\gamma\tau)/120\bigr).$$

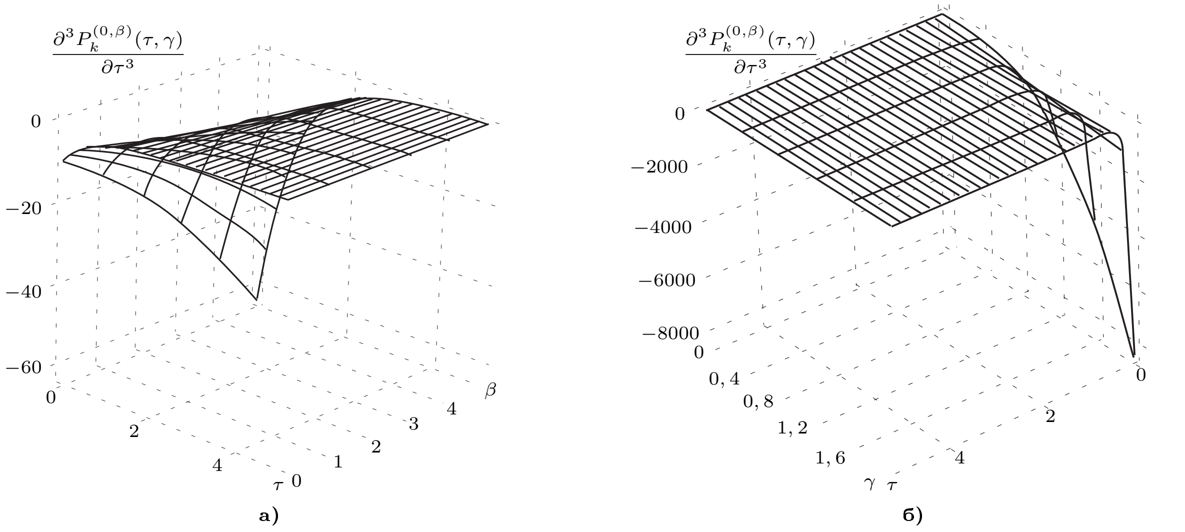


Рис. 1.64. Вид 3-ой производной ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta \in [0;5]$; б) $\gamma \in (0;2]$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.65]
$$\frac{\partial^n P_k^{(0,\beta)}(\tau,\gamma)}{\partial\tau^n} = \Bigl(-\frac{c\gamma}{2}\Bigr)^n\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}\times$$
$$(-1)^s(2s+1)^n\exp\bigl(-(2s+1)c\gamma\tau/2\bigr).$$

Частные случаи для n-ой производной функций 0-5 порядков:

$$\frac{\partial^n P_0^{(0,\beta)}(\tau,\gamma)}{\partial\tau^n} = \Bigl(-\frac{c\gamma}{2}\Bigr)^n\exp\bigl(-c\gamma\tau/2\bigr);$$

$$\frac{\partial^n P_1^{(0,\beta)}(\tau,\gamma)}{\partial\tau^n} = \Bigl(-\frac{c\gamma}{2}\Bigr)^n\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-3^n(\beta+2)\times$$
$$\times\exp(-c\gamma\tau)\bigr);$$

$$\frac{\partial^n P_2^{(0,\beta)}(\tau,\gamma)}{\partial\tau^n} = \Bigl(-\frac{c\gamma}{2}\Bigr)^n\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-2\cdot 3^n(\beta+3)\times$$
$$\times\exp(-c\gamma\tau)+5^n(\beta+3)(\beta+4)\exp(-2c\gamma\tau)/2\bigr);$$

$$\frac{\partial^n P_3^{(0,\beta)}(\tau,\gamma)}{\partial\tau^n} = \Bigl(-\frac{c\gamma}{2}\Bigr)^n\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-3\cdot 3^n(\beta+4)\times$$
$$\times\exp(-c\gamma\tau)+3\cdot 5^n(\beta+4)\times$$
$$\times(\beta+5)\exp(-2c\gamma\tau)/2-7^n(\beta+4)(\beta+5)(\beta+6)\exp(-3c\gamma\tau)/6\bigr);$$

$$\frac{\partial^n P_4^{(0,\beta)}(\tau,\gamma)}{\partial\tau^n} = \Bigl(-\frac{c\gamma}{2}\Bigr)^n\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-4\cdot 3^n(\beta+5)\times$$
$$\times\exp(-c\gamma\tau)+3\cdot 5^n(\beta+5)(\beta+6)\exp(-2c\gamma\tau)-2\cdot 7^n(\beta+5)\times$$
$$\times(\beta+6)(\beta+7)\exp(-3c\gamma\tau)/3+9^n(\beta+5)(\beta+6)(\beta+7)(\beta+8)\times$$
$$\times\exp(-4c\gamma\tau)/24\bigr);$$

$$\frac{\partial^n P_5^{(0,\beta)}(\tau,\gamma)}{\partial\tau^n} = \Bigl(-\frac{c\gamma}{2}\Bigr)^n\exp\bigl(-c\gamma\tau/2\bigr)\bigl(1-5\cdot 3^n(\beta+6)\times$$
$$\times\exp(-c\gamma\tau)+5\cdot 5^n(\beta+6)(\beta+7)\exp(-2c\gamma\tau)-5\cdot 7^n(\beta+6)\times$$
$$\times(\beta+7)(\beta+8)\exp(-3c\gamma\tau)+5\cdot 9^n(\beta+6)(\beta+7)(\beta+8)(\beta+9)\times$$
$$\times\exp(-4c\gamma\tau)-11^n(\beta+6)(\beta+7)(\beta+8)(\beta+9)(\beta+10)\times$$
$$\times\exp(-5c\gamma\tau)/120\bigr).$$

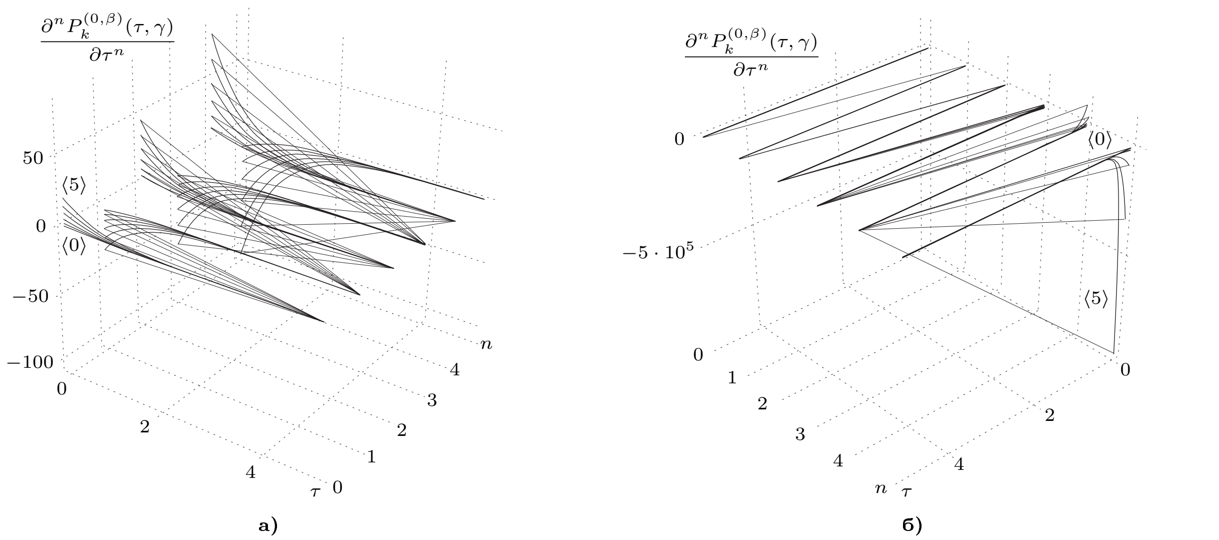


Рис. 1.65. Вид n-ой производной ортогональных функций Якоби 2-ого порядка: а) $n = 0..5$, $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta \in [0;5]$; б) $n = 0..5$, $\gamma \in (0;2]$, $c = 2$, $\alpha = 0$, $\beta = 1$

1.3 Аналитические соотношения для неопределенных интегралов от ортогональных функций

[1.66]
$$\int L_k(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k}{s}(-\gamma)^s\times \times\sum_{j=0}^{s}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s-j}}{(s-j)!}.$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$\int L_0(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)$;

$\int L_1(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau+1)$;

$\int L_2(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2+2)$;

$\int L_3(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3-3\gamma^2\tau^2+6\gamma\tau+6)$;

$\int L_4(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4-8\gamma^3\tau^3+24\gamma^2\tau^2+ +24)$;

$\int L_5(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5-15\gamma^4\tau^4+80\gamma^3\tau^3- -120\gamma^2\tau^2+120\gamma\tau+120)$.

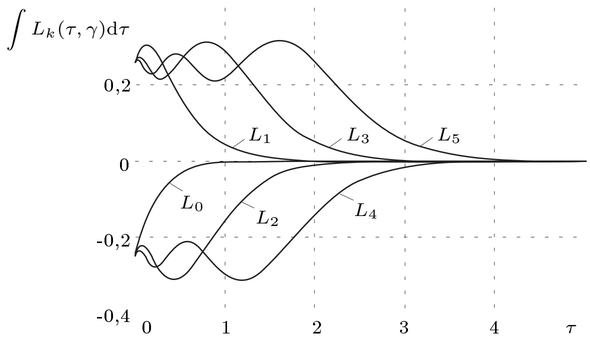


Рис. 1.66. Вид неопределенного интеграла от ортогональных функций Лагерра 0-5 порядков; $\gamma = 8$

[1.67]
$$\int \tau L_k(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k}{s}(-\gamma)^s\times \times(s+1)\sum_{j=0}^{s+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+1-j}}{(s+1-j)!}.$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$\int \tau L_0(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau+2)$;

$\int \tau L_1(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2+3\gamma\tau+6)$;

$\int \tau L_2(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3+2\gamma^2\tau^2+10\gamma\tau+ +20)$;

$\int \tau L_3(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4-\gamma^3\tau^3+12\gamma^2\tau^2+ +42\gamma\tau+84)$;

$\int \tau L_4(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5-6\gamma^4\tau^4+24\gamma^3\tau^3+$

$+ 48\gamma^2\tau^2 + 216\gamma\tau + 432);$

$$\int \tau L_5(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^6\tau^6 - 13\gamma^5\tau^5 + 70\gamma^4\tau^4 -$$
$$- 40\gamma^3\tau^3 + 360\gamma^2\tau^2 + 1320\gamma\tau + 2640).$$

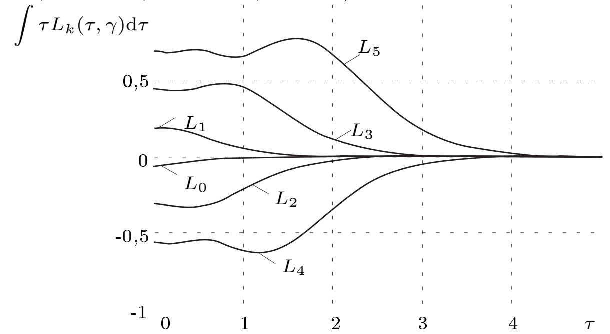


Рис. 1.67. Вид неопределенного интеграла 1-ого рода от ортогональных функций Лагерра 0-5 порядков; $\gamma = 8$

[1.68]
$$\int \tau^2 L_k(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k}{s}(-\gamma)^s\times$$
$$\times (s+1)(s+2)\sum_{j=0}^{s+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+2-j}}{(s+2-j)!}.$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$$\int \tau^2 L_0(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 + 4\gamma\tau + 8);$$

$$\int \tau^2 L_1(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 + 5\gamma^2\tau^2 + 20\gamma\tau + 40);$$

$$\int \tau^2 L_2(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 + 4\gamma^3\tau^3 + 26\gamma^2\tau^2 +$$
$$+ 104\gamma\tau + 208);$$

$$\int \tau^2 L_3(\tau,\gamma)\mathrm{d}\tau = \frac{1}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 + \gamma^4\tau^4 + 26\gamma^3\tau^3 +$$
$$+ 150\gamma^2\tau^2 + 600\gamma\tau + 1200);$$

$$\int \tau^2 L_4(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^6\tau^6 - 4\gamma^5\tau^5 + 32\times$$
$$\times \gamma^4\tau^4 + 160\gamma^3\tau^3 + 984\gamma^2\tau^2 + 3936\gamma\tau + 7872);$$

$$\int \tau^2 L_5(\tau,\gamma)\mathrm{d}\tau = \frac{\gamma^3}{960}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^7\tau^7 - 11\gamma^6\tau^6 - 68\gamma^5\tau^5 +$$
$$+ 80\gamma^4\tau^4 + 1240\gamma^3\tau^3 + 7320\gamma^2\tau^2 + 29280\gamma\tau + 58560).$$

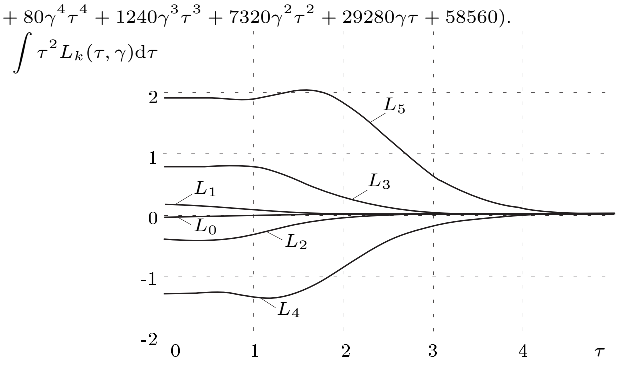


Рис. 1.68. Вид неопределенного интеграла 2-ого рода от ортогональных функций Лагерра 0-5 порядков; $\gamma = 8$

[1.69]
$$\int \tau^3 L_k(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k}{s}(-\gamma)^s\times$$
$$\times (s+1)(s+2)(s+3)\sum_{j=0}^{s+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+3-j}}{(s+3-j)!}.$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$$\int \tau^3 L_0(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 + 6\gamma^2\tau^2 + 24\gamma\tau + 48);$$

$$\int \tau^3 L_1(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 + 7\gamma^3\tau^3 + 42\gamma^2\tau^2 +$$
$$+ 168\gamma\tau + 336);$$

$$\int \tau^3 L_2(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 + 6\gamma^4\tau^4 + 50\gamma^3\tau^3 +$$
$$+ 300\gamma^2\tau^2 + 1200\gamma\tau + 2400);$$

$$\int \tau^3 L_3(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^6\tau^6 + 3\gamma^5\tau^5 + 48\gamma^4\tau^4 +$$
$$+ 378\gamma^3\tau^3 + 2268\gamma^2\tau^2 + 9072\gamma\tau + 18144);$$

$$\int \tau^3 L_4(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^7\tau^7 - 2\gamma^6\tau^6 + 48\times$$
$$\times \gamma^5\tau^5 + 384\gamma^4\tau^4 + 3096\gamma^3\tau^3 + 18576\gamma^2\tau^2 + 74304\gamma\tau + 148608);$$

$$\int \tau^3 L_5(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^8\tau^8 - 9\gamma^7\tau^7 + 74\gamma^6\tau^6 +$$
$$+ 288\gamma^5\tau^5 + 3480\gamma^4\tau^4 + 27720\gamma^3\tau^3 + 166320\gamma^2\tau^2 + 665280\gamma\tau +$$
$$+ 1330560).$$

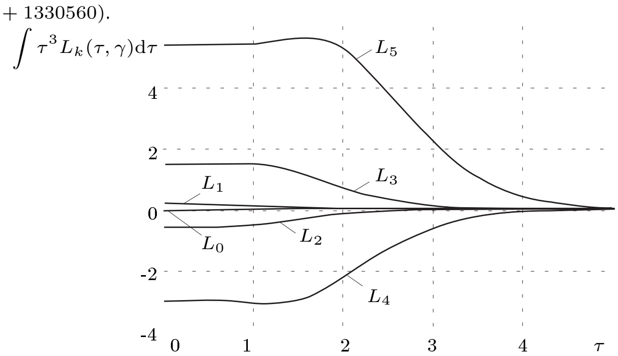


Рис. 1.69. Вид неопределенного интеграла 3-ого рода от ортогональных функций Лагерра 0-5 порядков; $\gamma = 8$

[1.70]
$$\int \tau^n L_k(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k}{s}(-\gamma)^s\times$$
$$\times \frac{(s+n)!}{s!}\sum_{j=0}^{s+n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+n-j}}{(s+n-j)!}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n L_0(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n L_1(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$
$$- \gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!}\Bigg);$$

$$\int \tau^n L_2(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} - 2\gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!} + \frac{1}{2}\gamma^2(n+2)!\times\sum_{j=0}^{n+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{2+n-j}}{(2+n-j)!}\Bigg);$$

$$\int \tau^n L_3(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} - 3\gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!} + \frac{3}{2}\gamma^2(n+2)!\times\sum_{j=0}^{n+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{2+n-j}}{(2+n-j)!} - \frac{1}{6}\gamma^3(n+3)!\sum_{j=0}^{n+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{3+n-j}}{(3+n-j)!}\Bigg);$$

$$\int \tau^n L_4(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} - 4\gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!} + 3\gamma^2(n+2)!\times\sum_{j=0}^{n+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{2+n-j}}{(2+n-j)!} - \frac{2}{3}\gamma^3(n+3)!\sum_{j=0}^{n+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{3+n-j}}{(3+n-j)!} + \frac{1}{24}\gamma^4(n+4)!\sum_{j=0}^{n+4}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{4+n-j}}{(4+n-j)!}\Bigg);$$

$$\int \tau^n L_5(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} - 5\gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!} + 5\gamma^2(n+2)!\times\sum_{j=0}^{n+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{2+n-j}}{(2+n-j)!} - \frac{5}{3}\gamma^3(n+3)!\sum_{j=0}^{n+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{3+n-j}}{(3+n-j)!} + \frac{5}{24}\gamma^4(n+4)!\sum_{j=0}^{n+4}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{4+n-j}}{(4+n-j)!} - \frac{1}{120}\gamma^5(n+5)!\times\sum_{j=0}^{n+5}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{5+n-j}}{(5+n-j)!}\Bigg).$$

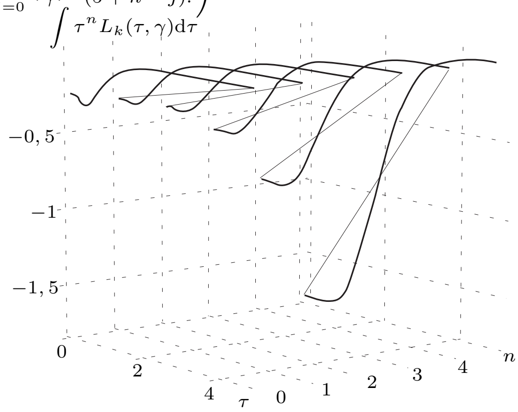


Рис. 1.70. Вид неопределенного интеграла n-ого рода от ортогональных функций Лагерра 2-ого порядка; $n = 0..5$, $\gamma = 8$

[1.71]
$$\int L_k^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+1}{k-s}\times(-\gamma)^s\sum_{j=0}^{s}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s-j}}{(s-j)!}.$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int L_0^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$

$$\int L_1^{(1)}(\tau,\gamma)\mathrm{d}\tau = 2\exp\Big(-\frac{\gamma\tau}{2}\Big)\tau;$$

$$\int L_2^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 - 2\gamma\tau + 2);$$

$$\int L_3^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3}\exp\Big(-\frac{\gamma\tau}{2}\Big)\tau(\gamma^2\tau^2 - 6\gamma\tau + 12);$$

$$\int L_4^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 - 12\gamma^3\tau^3 + 48\gamma^2\tau^2 - 48\gamma\tau + 24);$$

$$\int L_5^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60}\exp\Big(-\frac{\gamma\tau}{2}\Big)\tau(\gamma^4\tau^4 - 20\gamma^3\tau^3 + 140\gamma^2\tau^2 - 360\gamma\tau + 360).$$

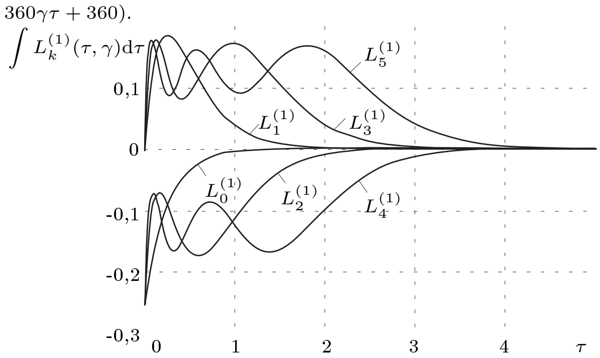


Рис. 1.71. Вид неопределенного интеграла от ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 8$, $\alpha = 1$

[1.72]
$$\int \tau L_k^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+1}{k-s}\times(-\gamma)^s(s+1)\sum_{j=0}^{s+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+1-j}}{(s+1-j)!}.$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int \tau L_0^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau + 2);$$

$$\int \tau L_1^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 + 2\gamma\tau + 4);$$

$$\int \tau L_2^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 + 6\gamma\tau + 12);$$

$$\int \tau L_3^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 - 4\gamma^3\tau^3 + 12\gamma^2\tau^2 + 24\gamma\tau + 48);$$

$$\int \tau L_4^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 - 10\gamma^4\tau^4 + 40\times\gamma^3\tau^3 + 120\gamma\tau + 240);$$

$$\int \tau L_5^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^6\tau^6 - 18\gamma^5\tau^5 + 120\times\gamma^4\tau^4 - 240\gamma^3\tau^3 + 360\gamma^2\tau^2 + 720\gamma\tau + 1440).$$

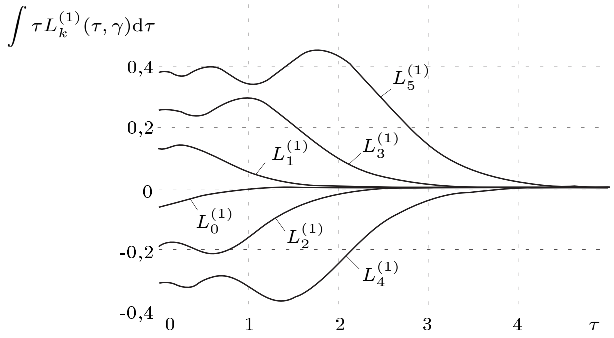


Рис. 1.72. Вид неопределенного интеграла 1-ого рода от ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 8,\ \alpha = 1$

[1.73]
$$\int \tau^2 L_k^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+1}{k-s}\times$$
$$\times(-\gamma)^s(s+1)(s+2)\sum_{j=0}^{s+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+2-j}}{(s+2-j)!}.$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$$\int \tau^2 L_0^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 + 4\gamma\tau + 8);$$

$$\int \tau^2 L_1^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 + 4\gamma^2\tau^2 + 16\gamma\tau + 32);$$

$$\int \tau^2 L_2^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 + 2\gamma^3\tau^3 + 18\gamma^2\tau^2 + $$
$$+ 72\gamma\tau + 144);$$

$$\int \tau^2 L_3^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 - 2\gamma^4\tau^4 + 20\gamma^3\tau^3 + $$
$$+ 96\gamma^2\tau^2 + 384\gamma\tau + 768);$$

$$\int \tau^2 L_4^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^6\tau^6 - 8\gamma^5\tau^5 + 40\times$$
$$\times\gamma^4\tau^4 + 80\gamma^3\tau^3 + 600\gamma^2\tau^2 + 2400\gamma\tau + 4800);$$

$$\int \tau^2 L_5^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{\gamma^3}{960}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^7\tau^7 - 16\gamma^6\tau^6 + 108\times$$
$$\times\gamma^5\tau^5 - 120\gamma^4\tau^4 + 840\gamma^3\tau^3 + 4320\gamma^2\tau^2 + 17280\gamma\tau + 34560).$$

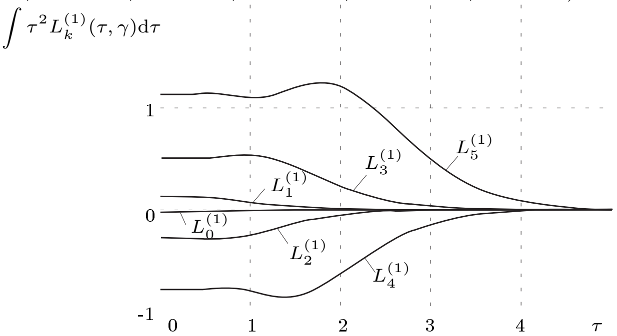


Рис. 1.73. Вид неопределенного интеграла 2-ого рода от ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 8,\ \alpha = 1$

[1.74]
$$\int \tau^3 L_k^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+1}{k-s}\times$$
$$\times(-\gamma)^s(s+1)(s+2)(s+3)\sum_{j=0}^{s+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+3-j}}{(s+3-j)!}.$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$$\int \tau^3 L_0^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 + 6\gamma^2\tau^2 + 24\gamma\tau + $$
$$+ 48);$$

$$\int \tau^3 L_1^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 + 6\gamma^3\tau^3 + 36\gamma^2\tau^2 + $$
$$+ 144\gamma\tau + 288);$$

$$\int \tau^3 L_2^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 + 4\gamma^4\tau^4 + 38\gamma^3\tau^3 + $$
$$+ 228\gamma^2\tau^2 + 912\gamma\tau + 1824);$$

$$\int \tau^3 L_3^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^6\tau^6 + 36\gamma^4\tau^4 + 264\times$$
$$\times\gamma^3\tau^3 + 1584\gamma^2\tau^2 + 6336\gamma\tau + 12672);$$

$$\int \tau^3 L_4^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^7\tau^7 - 6\gamma^6\tau^6 + 48\times$$
$$\times\gamma^5\tau^5 + 240\gamma^4\tau^4 + 2040\gamma^3\tau^3 + 12240\gamma^2\tau^2 + 48960\gamma\tau + 97920);$$

$$\int \tau^3 L_5^{(1)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^8\tau^8 - 14\gamma^7\tau^7 + 104\times$$
$$\times\gamma^6\tau^6 + 48\gamma^5\tau^5 + 2280\gamma^4\tau^4 + 17520\gamma^3\tau^3 + 105120\gamma^2\tau^2 + $$
$$+ 420480\gamma\tau + 840960).$$

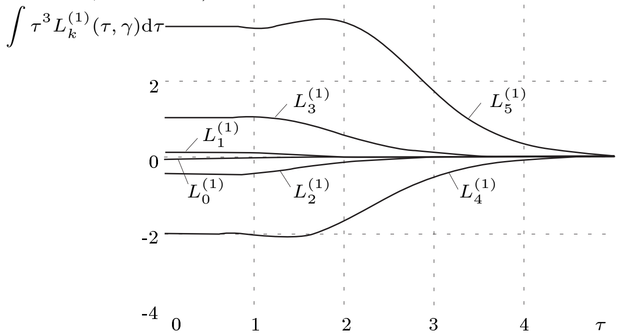


Рис. 1.74. Вид неопределенного интеграла 3-ого рода от ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 8,\ \alpha = 1$

[1.75]
$$\int \tau^n L_k^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+1}{k-s}\times$$
$$\times(-\gamma)^s\frac{(s+n)!}{s!}\sum_{j=0}^{s+n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+n-j}}{(s+n-j)!}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n L_0^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n L_1^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} - $$

$$- 2\gamma(n+1)! \sum_{j=0}^{n+1} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{1+n-j}}{(1+n-j)!}\Bigg);$$

$$\int \tau^n L_2^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$

$$- 3\gamma(n+1)! \sum_{j=0}^{n+1} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{1+n-j}}{(1+n-j)!} + \frac{3}{2}\gamma^2(n+2)! \times$$

$$\times \sum_{j=0}^{n+2} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{2+n-j}}{(2+n-j)!}\Bigg);$$

$$\int \tau^n L_3^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$

$$- 4\gamma(n+1)! \sum_{j=0}^{n+1} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{1+n-j}}{(1+n-j)!} + 3\gamma^2(n+2)! \times$$

$$\times \sum_{j=0}^{n+2} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{2+n-j}}{(2+n-j)!} - \frac{2}{3}\gamma^3(n+3)!\sum_{j=0}^{n+3} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{3+n-j}}{(3+n-j)!}\Bigg);$$

$$\int \tau^n L_4^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$

$$- 5\gamma(n+1)! \sum_{j=0}^{n+1} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{1+n-j}}{(1+n-j)!} + 5\gamma^2(n+2)! \times$$

$$\times \sum_{j=0}^{n+2} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{2+n-j}}{(2+n-j)!} - \frac{5}{3}\gamma^3(n+3)!\sum_{j=0}^{n+3} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{3+n-j}}{(3+n-j)!} +$$

$$+ \frac{5}{24}\gamma^4(n+4)!\sum_{j=0}^{n+4} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{4+n-j}}{(4+n-j)!}\Bigg);$$

$$\int \tau^n L_5^{(1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$

$$- 6\gamma(n+1)! \sum_{j=0}^{n+1} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{1+n-j}}{(1+n-j)!} + \frac{15}{2}\gamma^2(n+2)! \times$$

$$\times \sum_{j=0}^{n+2} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{2+n-j}}{(2+n-j)!} - \frac{10}{3}\gamma^3(n+3)!\sum_{j=0}^{n+3} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{3+n-j}}{(3+n-j)!} +$$

$$+ \frac{5}{8}\gamma^4(n+4)!\sum_{j=0}^{n+4} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{4+n-j}}{(4+n-j)!} - \frac{1}{20}\gamma^5(n+5)! \times$$

$$\times \sum_{j=0}^{n+5} \Big(\frac{2}{\gamma}\Big)^j \frac{\tau^{5+n-j}}{(5+n-j)!}\Bigg).$$

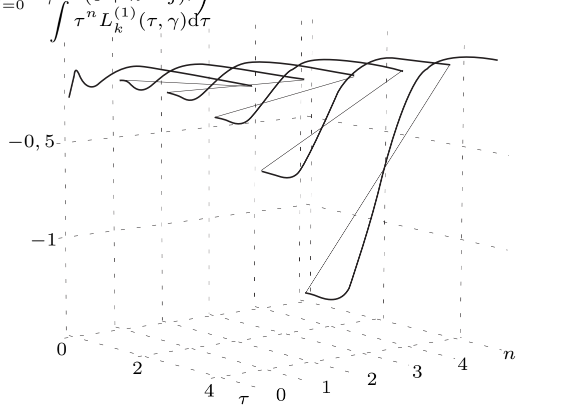


Рис. 1.75. Вид неопределенного интеграла n-ого рода от ортогональных функций Сонина-Лагерра 2-ого порядка; $n = 0..5$, $\gamma = 8$, $\alpha = 1$

[1.76] $$\int L_k^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+2}{k-s}\times$$

$$\times(-\gamma)^s\sum_{j=0}^{s}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s-j}}{(s-j)!}.$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int L_0^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$

$$\int L_1^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau-1);$$

$$\int L_2^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2-4\gamma\tau+4);$$

$$\int L_3^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3-9\gamma^2\tau^2+24\gamma\tau-12);$$

$$\int L_4^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4-16\gamma^3\tau^3+84\gamma^2\tau^2 -$$

$$- 144\gamma\tau+72);$$

$$\int L_5^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5-25\gamma^4\tau^4+220\gamma^3\tau^3 -$$

$$- 780\gamma^2\tau^2+1080\gamma\tau-360).$$

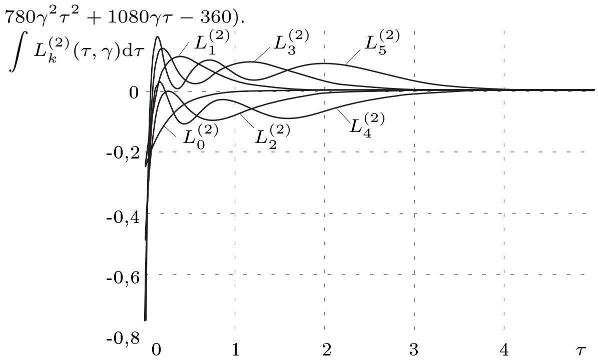


Рис. 1.76. Вид неопределенного интеграла от ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 8$, $\alpha = 2$

[1.77] $$\int \tau L_k^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+2}{k-s}\times$$

$$\times(-\gamma)^s(s+1)\sum_{j=0}^{s+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+1-j}}{(s+1-j)!}.$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int \tau L_0^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau+2);$$

$$\int \tau L_1^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2+\gamma\tau+2);$$

$$\int \tau L_2^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3-2\gamma^2\tau^2+4\gamma\tau+8);$$

$$\int \tau L_3^{(2)}3(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4-7\gamma^3\tau^3+18\gamma^2\tau^2 +$$

$$+ 12\gamma\tau+24);$$

$$\int \tau L_4^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5-14\gamma^4\tau^4+68 \times$$

$$\times \gamma^3\tau^3-72\gamma^2\tau^2+72\gamma\tau+144);$$

$$\int \tau L_5^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^6\tau^6 - 23\gamma^5\tau^5 + 190 \times$$
$$\times\, \gamma^4\tau^4 - 580\gamma^3\tau^3 + 720\gamma^2\tau^2 + 360\gamma\tau + 720).$$

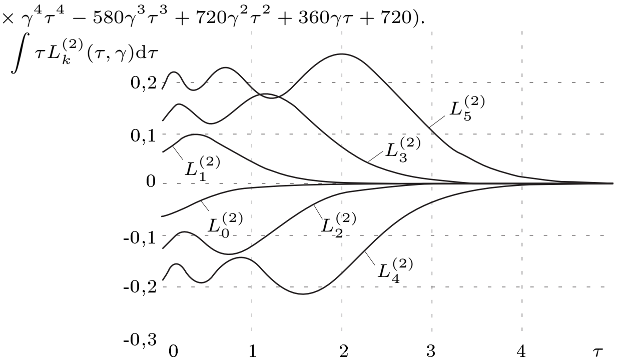


Рис. 1.77. Вид неопределенного интеграла 1-ого рода от ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 8,\ \alpha = 2$

[1.78]
$$\int \tau^2 L_k^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+2}{k-s}\times$$
$$\times\, (-\gamma)^s(s+1)(s+2)\sum_{j=0}^{s+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+2-j}}{(s+2-j)!}.$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$$\int \tau^2 L_0^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 + 4\gamma\tau + 8);$$

$$\int \tau^2 L_1^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 + 3\gamma^2\tau^2 + 12\gamma\tau + 24);$$

$$\int \tau^2 L_2^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 + 12\gamma^2\tau^2 + 48\gamma\tau +$$
$$+\, 96);$$

$$\int \tau^2 L_3^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 - 5\gamma^4\tau^4 + 20\gamma^3\tau^3 +$$
$$+\, 60\gamma^2\tau^2 + 240\gamma\tau + 480);$$

$$\int \tau^2 L_4^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^6\tau^6 - 12\gamma^5\tau^5 + 60 \times$$
$$\times\, \gamma^4\tau^4 + 360\gamma^2\tau^2 + 1440\gamma\tau + 2880);$$

$$\int \tau^2 L_5^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{\gamma^3}{960}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^7\tau^7 - 21\gamma^6\tau^6 + 168 \times$$
$$\times\, \gamma^5\tau^5 - 420\gamma^4\tau^4 + 840\gamma^3\tau^3 + 2520\gamma^2\tau^2 + 10080\gamma\tau + 20160).$$

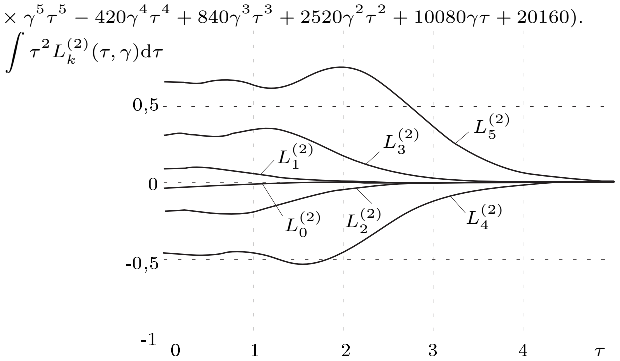


Рис. 1.78. Вид неопределенного интеграла 2-ого рода от ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 8,\ \alpha = 2$

[1.79]
$$\int \tau^3 L_k^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+2}{k-s}\times$$
$$\times\, (-\gamma)^s(s+1)(s+2)(s+3)\sum_{j=0}^{s+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+3-j}}{(s+3-j)!}.$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$$\int \tau^3 L_0^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 + 6\gamma^2\tau^2 + 24\gamma\tau +$$
$$+\, 48);$$

$$\int \tau^3 L_1^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^4\tau^4 + 5\gamma^3\tau^3 + 30\gamma^2\tau^2 +$$
$$+\, 120\gamma\tau + 240);$$

$$\int \tau^3 L_2^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^5\tau^5 + 2\gamma^4\tau^4 + 28\gamma^3\tau^3 +$$
$$+\, 168\gamma^2\tau^2 + 672\gamma\tau + 1344);$$

$$\int \tau^3 L_3^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^6\tau^6 - 3\gamma^5\tau^5 + 30\gamma^4\tau^4 +$$
$$+\, 180\gamma^3\tau^3 + 1080\gamma^2\tau^2 + 4320\gamma\tau + 8640);$$

$$\int \tau^3 L_4^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^7\tau^7 - 10\gamma^6\tau^6 + 60 \times$$
$$\times\, \gamma^5\tau^5 + 120\gamma^4\tau^4 + 1320\gamma^3\tau^3 + 7920\gamma^2\tau^2 + 31680\gamma\tau + 63360);$$

$$\int \tau^3 L_5^{(2)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^8\tau^8 - 19\gamma^7\tau^7 + 154 \times$$
$$\times\, \gamma^6\tau^6 - 252\gamma^5\tau^5 + 1680\gamma^4\tau^4 + 10920\gamma^3\tau^3 + 65520\gamma^2\tau^2 +$$
$$+\, 262080\gamma\tau + 524160).$$

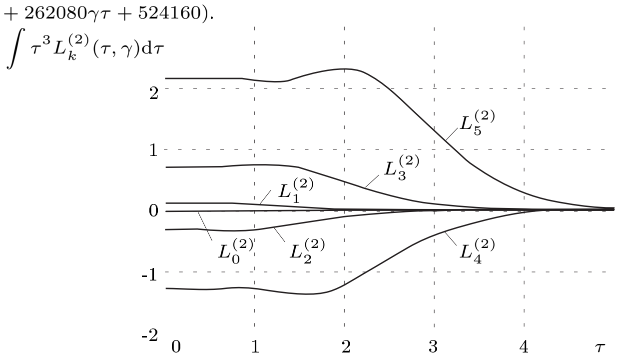


Рис. 1.79. Вид неопределенного интеграла 3-ого рода от ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 8,\ \alpha = 2$

[1.80]
$$\int \tau^n L_k^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+2}{k-s}\times$$
$$\times\, (-\gamma)^s\frac{(s+n)!}{s!}\sum_{j=0}^{s+n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+n-j}}{(s+n-j)!}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n L_0^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n L_1^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$

$$- 3\gamma(n+1)! \sum_{j=0}^{n+1} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{1+n-j}}{(1+n-j)!}\Bigg);$$

$$\int \tau^n L_2^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\Biggl(n!\sum_{j=0}^{n}\left(\frac{2}{\gamma}\right)^j \frac{\tau^{n-j}}{(n-j)!} -$$
$$- 4\gamma(n+1)! \sum_{j=0}^{n+1} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{1+n-j}}{(1+n-j)!} + 3\gamma^2(n+2)! \times$$
$$\times \sum_{j=0}^{2+n} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{2+n-j}}{(2+n-j)!}\Biggr);$$

$$\int \tau^n L_3^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\Biggl(n!\sum_{j=0}^{n}\left(\frac{2}{\gamma}\right)^j \frac{\tau^{n-j}}{(n-j)!} -$$
$$- 5\gamma(n+1)! \sum_{j=0}^{n+1} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{1+n-j}}{(1+n-j)!} + 5\gamma^2(n+2)! \times$$
$$\times \sum_{j=0}^{n+2} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{2+n-j}}{(2+n-j)!} - \frac{5}{3}\gamma^3(n+3)! \sum_{j=0}^{n+3} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{3+n-j}}{(3+n-j)!}\Biggr);$$

$$\int \tau^n L_4^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\Biggl(n!\sum_{j=0}^{n}\left(\frac{2}{\gamma}\right)^j \frac{\tau^{n-j}}{(n-j)!} -$$
$$- 6\gamma(n+1)! \sum_{j=0}^{n+1} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{1+n-j}}{(1+n-j)!} + \frac{15}{2}\gamma^2(n+2)! \times$$
$$\times \sum_{j=0}^{n+2} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{2+n-j}}{(2+n-j)!} - \frac{10}{3}\gamma^3(n+3)! \sum_{j=0}^{n+3} \left(\frac{2}{\gamma}\right)^j \times$$
$$\times \frac{\tau^{3+n-j}}{(3+n-j)!} + \frac{5}{8}\gamma^4(n+4)! \sum_{j=0}^{n+4} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{4+n-j}}{(4+n-j)!}\Biggr);$$

$$\int \tau^n L_5^{(2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\Biggl(n!\sum_{j=0}^{n}\left(\frac{2}{\gamma}\right)^j \frac{\tau^{n-j}}{(n-j)!} -$$
$$- 7\gamma(n+1)! \sum_{j=0}^{1+n} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{1+n-j}}{(1+n-j)!} + \frac{21}{2}\gamma^2(n+2)! \times$$
$$\times \sum_{j=0}^{n+2} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{2+n-j}}{(2+n-j)!} - \frac{35}{6}\gamma^3(n+3)! \sum_{j=0}^{n+3} \left(\frac{2}{\gamma}\right)^j \times$$
$$\times \frac{\tau^{3+n-j}}{(3+n-j)!} + \frac{35}{24}\gamma^4(n+4)! \sum_{j=0}^{n+4} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{4+n-j}}{(4+n-j)!} -$$
$$- \frac{7}{40}\gamma^5(n+5)! \sum_{j=0}^{n+5} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{5+n-j}}{(5+n-j)!}\Biggr).$$

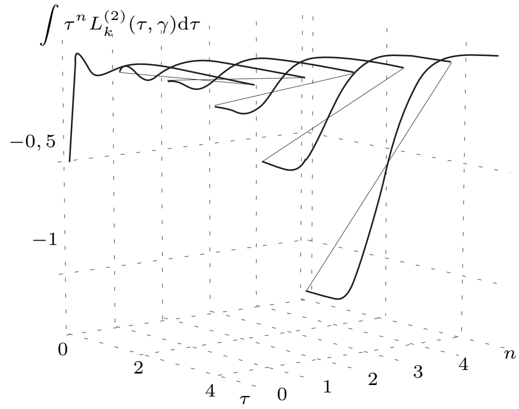
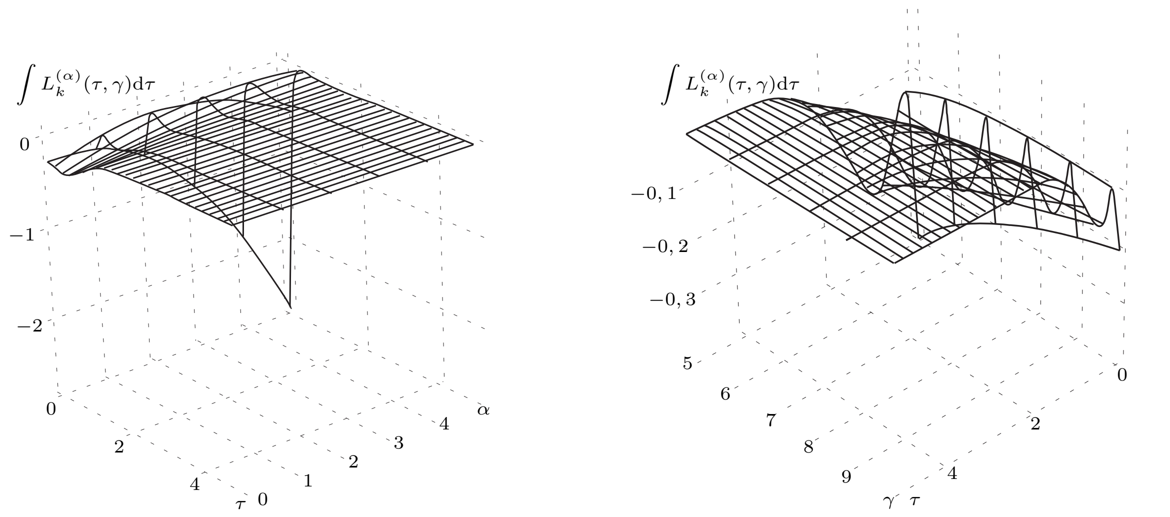


Рис. 1.80. Вид неопределенного интеграла n-ого рода от ортогональных функций Сонина-Лагерра 2-ого порядка; $n = 0..5$, $\gamma = 8$, $\alpha = 2$

[1.81]
$$\int L_k^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\times$$
$$\times(-\gamma)^s\sum_{j=0}^{s}\left(\frac{2}{\gamma}\right)^j\frac{\tau^{s-j}}{(s-j)!}.$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int L_0^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr);$$
$$\int L_1^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma\tau-\alpha+1);$$
$$\int L_2^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\bigl(\gamma^2\tau^2-2\alpha\gamma\tau+\alpha^2-\alpha+2\bigr);$$
$$\int L_3^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\bigl(\gamma^3\tau^3-3(\alpha+1)\gamma^2\tau^2+3\times$$
$$\times(\alpha^2+\alpha+2)\gamma\tau-\alpha^3-5\alpha+6\bigr);$$
$$\int L_4^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\bigl(\gamma^4\tau^4-4(\alpha+2)\gamma^3\tau^3+6\times$$
$$\times(\alpha^2+3\alpha+4)\gamma^2\tau^2-4(\alpha^3+3\alpha^2+8\alpha)\gamma\tau+\alpha^4+2\alpha^3+11\alpha^2-$$
$$-14\alpha+24\bigr);$$
$$\int L_5^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\bigl(\gamma^5\tau^5-5(\alpha+3)\gamma^4\tau^4+10\times$$
$$\times(\alpha^2+5\alpha+8)\gamma^3\tau^3-10(\alpha^3+6\alpha^2+17\alpha+12)\gamma^2\tau^2+5(\alpha^4+6\alpha^3+$$
$$+23\alpha^2+18\alpha+24)\gamma\tau-\alpha^5-5\alpha^4-25\alpha^3+5\alpha^2-94\alpha+120\bigr).$$



а) **б)**

Рис. 1.81. Вид неопределенного интеграла от ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 8$, $\alpha \in [0;5]$; б) $\gamma \in [5;10]$, $\alpha = 1$

[1.82]
$$\int \tau L_k^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\times$$
$$\times(-\gamma)^s(s+1)\sum_{j=0}^{s+1}\Bigl(\frac{2}{\gamma}\Bigr)^j\frac{\tau^{s+1-j}}{(s+1-j)!}.$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int \tau L_0^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^2}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)(\gamma\tau+2);$$

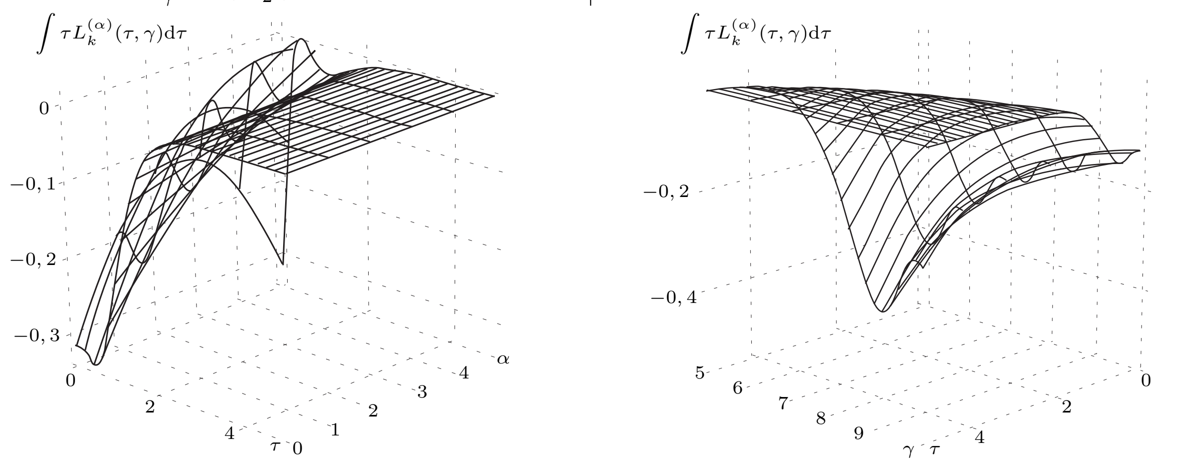
$$\int \tau L_1^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^2}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\bigl(\gamma^2\tau^2-(\alpha-3)\gamma\tau-2\alpha+6\bigr);$$

$$\int \tau L_2^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^2}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\bigl(\gamma^3\tau^3-2(\alpha-1)\gamma^2\tau^2+$$
$$+(\alpha^2-5\alpha+10)\gamma\tau+2\alpha^2-10\alpha+20\bigr);$$

$$\int \tau L_3^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma^2}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\bigl(\gamma^4\tau^4-(3\alpha+1)\gamma^3\tau^3+$$
$$+3(\alpha^2-\alpha+4)\gamma^2\tau^2-(\alpha^3-6\alpha^2+23\alpha-42)\gamma\tau-2(\alpha^3+6\alpha^2-$$
$$-23\alpha+42)\bigr);$$

$$\int \tau L_4^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^2}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\bigl(\gamma^5\tau^5-2(2\alpha+3)\gamma^4\tau^4+$$
$$+2(3\alpha^2+5\alpha+12)\gamma^3\tau^3-4(\alpha^3+11\alpha-12)\gamma^2\tau^2+(\alpha^4+6\alpha^3+$$
$$+35\alpha^2-126\alpha+216)\gamma\tau+2(\alpha^4-6\alpha^3+35\alpha^2-126\alpha+216)\bigr);$$

$$\int \tau L_5^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma^2}\exp\Bigl(-\frac{\gamma\tau}{2}\Bigr)\bigl(\gamma^6\tau^6-(5\alpha+13)\gamma^5\tau^5+$$
$$+10(\alpha^2+4\alpha+7)\gamma^4\tau^4-10(\alpha^3+4\alpha^2+15\alpha+4)\gamma^3\tau^3+5(\alpha^4+2\alpha^3+$$
$$+23\alpha^2-26\alpha+72)\gamma^2\tau^2-(\alpha^5-5\alpha^4+45\alpha^3-235\alpha^2+794\alpha-$$
$$-1320)\gamma\tau-2(\alpha^5-5\alpha^4+45\alpha^3-235\alpha^2+794\alpha-1320)\bigr).$$



а) **б)**

Рис. 1.82. Вид неопределенного интеграла 1-ого рода от ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 8$, $\alpha \in [0;5]$; б) $\gamma \in [5;10]$, $\alpha = 1$

[1.83] $$\int \tau^2 L_k^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\times$$
$$\times(-\gamma)^s(s+1)(s+2)\sum_{j=0}^{s+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+2-j}}{(s+2-j)!}.$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$$\int \tau^2 L_0^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^2\tau^2+4\gamma\tau+8);$$

$$\int \tau^2 L_1^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^3\tau^3-(\alpha-5)\gamma^2\tau^2-4\times$$
$$\times(\alpha-5)\gamma\tau-8(\alpha-5)\big);$$

$$\int \tau^2 L_2^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^4\tau^4-2(\alpha-2)\gamma^3\tau^3+$$
$$+(\alpha^2-9\alpha+26)\gamma^2\tau^2+4(\alpha^2-9\alpha+26)\gamma\tau+8(\alpha^2-9\alpha+26)\big);$$

$$\int \tau^2 L_3^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^5\tau^5-(3\alpha-1)\gamma^4\tau^4+$$
$$+(3\alpha^2-9\alpha+26)\gamma^3\tau^3-(\alpha^3-12\alpha^2+65\alpha-150)\gamma^2\tau^2-4\times$$
$$\times(\alpha^3-12\alpha^2+65\alpha-150)\gamma\tau-8(\alpha^3+12\alpha^2-65\alpha+150)\big);$$

$$\int \tau^2 L_4^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^6\tau^6-4(\alpha+1)\gamma^5\tau^5+$$
$$+2(3\alpha^2+\alpha+16)\gamma^4\tau^4-4(\alpha^3-3\alpha^2+22\alpha-40)\gamma^3\tau^3+$$
$$+(\alpha^4-14\alpha^3+107\alpha^2-478\alpha+984)\gamma^2\tau^2+4(\alpha^4-14\alpha^3+107\times$$
$$\times\alpha^2-478\alpha+984)\gamma\tau+8(\alpha^4-14\alpha^3+107\alpha^2-478\alpha+984)\big);$$

$$\int \tau^2 L_5^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma^3}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^7\tau^7-(5\alpha+11)\gamma^6\tau^6+$$
$$+2(5\alpha^2+15\alpha+34)\gamma^5\tau^5-10(\alpha^3+2\alpha^2+17\alpha-8)\gamma^4\tau^4+5\times$$
$$\times(\alpha^4-2\alpha^3+39\alpha^2-118\alpha+248)\gamma^3\tau^3-(\alpha^5-15\alpha^4+145\alpha^3-945\times$$
$$\times\alpha^2+3814\alpha-7320)\gamma^2\tau^2-4(\alpha^5-15\alpha^4+145\alpha^3-945\alpha^2+3814\times$$
$$\times\alpha-7320)\gamma\tau-8(\alpha^5-15\alpha^4+145\alpha^3-945\alpha^2+3814\alpha-7320)\big).$$

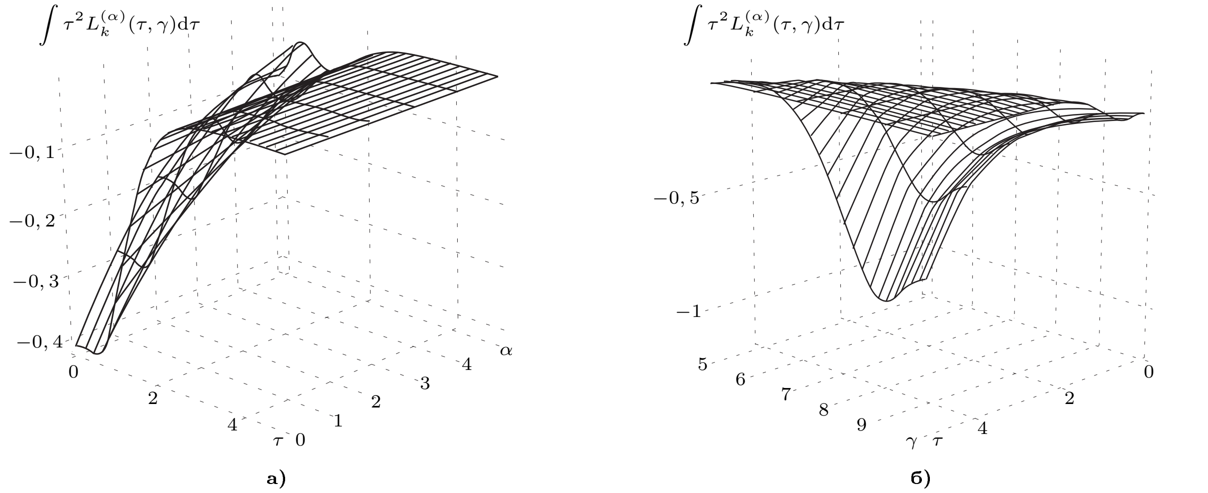


Рис. 1.83. Вид неопределенного интеграла 2-ого рода от ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 8$, $\alpha \in [0;5]$; б) $\gamma \in [5;10]$, $\alpha = 1$

[1.84] $$\int \tau^3 L_k^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\times$$
$$\times(-\gamma)^s(s+1)(s+2)(s+3)\sum_{j=0}^{s+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+3-j}}{(s+3-j)!}.$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$$\int \tau^3 L_0^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma^3\tau^3+6\gamma^2\tau^2+24\gamma\tau+$$
$$+48);$$

$$\int \tau^3 L_1^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{2}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^4\tau^4-(\alpha-7)\gamma^3\tau^3-6\times$$
$$\times(\alpha-7)\gamma^2\tau^2-24(\alpha-7)\gamma\tau-48(\alpha-7)\big);$$

$$\int \tau^3 L_2^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^5\tau^5-2(\alpha-3)\gamma^4\tau^4+$$
$$+(\alpha^2-13\alpha+50)\gamma^3\tau^3+6(\alpha^2-13\alpha+50)\gamma^2\tau^2+24(\alpha^2-13\alpha+$$
$$+50)\gamma\tau+48(\alpha^2-13\alpha+50)\big);$$

$$\int \tau^3 L_3^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{3\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^6\tau^6-3(\alpha-1)\gamma^5\tau^5+3\times$$
$$\times(\alpha^2-5\alpha+16)\gamma^4\tau^4-(\alpha^3-18\alpha^2+131\alpha-378)\gamma^3\tau^3-6(\alpha^3-$$
$$-18\alpha^2+131\alpha-378)\gamma^2\tau^2-24(\alpha^3-18\alpha^2+131\alpha-378)\gamma\tau-$$
$$-48(\alpha^3-18\alpha^2+131\alpha-378)\big);$$

$$\int \tau^3 L_4^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^7\tau^7-2(2\alpha+1)\times$$
$$\times\gamma^6\tau^6+6(\alpha^2-\alpha+8)\gamma^5\tau^5-4(\alpha^3-6\alpha^2+41\alpha-96)\gamma^4\tau^4+$$
$$+4(\alpha^4-22\alpha^3+227\alpha^2-1262\alpha+3096)\gamma^3\tau^3+6(\alpha^4-22\alpha^3+227\times$$
$$\times\alpha^2-1262\alpha+3096)\gamma^2\tau^2+24(\alpha^4-22\alpha^3+227\alpha^2-1262\alpha+$$
$$+3096)\gamma\tau+48(\alpha^4-22\alpha^3+227\alpha^2-1262\alpha+3096)\big);$$

$$\int \tau^3 L_5^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{60\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(\gamma^8\tau^8-(5\alpha+9)\gamma^7\tau^7+$$
$$+2(5\alpha^2+10\alpha+37)\gamma^6\tau^6-2(5\alpha^3-115\alpha+144)\gamma^5\tau^5+5(\alpha^4-$$
$$-6\alpha^3+71\alpha^2-306\alpha+696)\gamma^4\tau^4-(\alpha^5-25\alpha^4+325\alpha^3-2615\alpha^2+$$
$$+12514\alpha-27720)\gamma^3\tau^3-6(\alpha^5-25\alpha^4+325\alpha^3-2615\alpha^2+$$
$$+12514\alpha-27720)\gamma^2\tau^2-24(\alpha^5-25\alpha^4+325\alpha^3-2615\alpha^2+$$
$$+12514\alpha-27720)\gamma\tau-48(\alpha^5-25\alpha^4+325\alpha^3-2615\alpha^2+12514\times$$
$$\times\alpha-27720)\big).$$

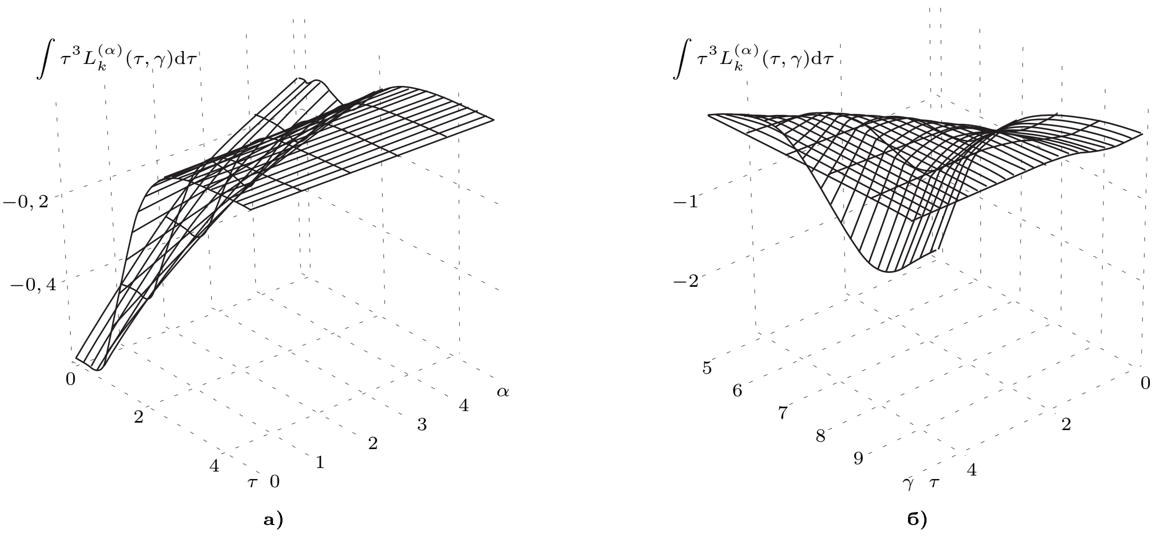


Рис. 1.84. Вид неопределенного интеграла 3-ого рода от ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 8$, $\alpha \in [0; 5]$; б) $\gamma \in [5; 10]$, $\alpha = 1$

[1.85]
$$\int \tau^n L_k^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\times$$
$$\times(-\gamma)^s\frac{(s+n)!}{s!}\sum_{j=0}^{s+n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{s+n-j}}{(s+n-j)!}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n L_0^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n L_1^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$
$$-(\alpha+1)\gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!}\Bigg);$$

$$\int \tau^n L_2^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$
$$-(\alpha+2)\gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!} +$$
$$+(\alpha+1)(\alpha+2)\gamma^2(n+2)!\frac{1}{4}\sum_{j=0}^{n+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{2+n-j}}{(2+n-j)!}\Bigg);$$

$$\int \tau^n L_3^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$
$$-(\alpha+3)\gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!} +$$
$$+(\alpha+2)(\alpha+3)\gamma^2(n+2)!\frac{1}{4}\sum_{j=0}^{n+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{2+n-j}}{(2+n-j)!} -$$
$$-(\alpha+1)(\alpha+2)(\alpha+3)\gamma^3(n+3)!\frac{1}{36}\sum_{j=0}^{n+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{3+n-j}}{(3+n-j)!}\Bigg);$$

$$\int \tau^n L_4^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$
$$-(\alpha+4)\gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!} +$$
$$+(\alpha+3)(\alpha+4)\gamma^2(n+2)!\frac{1}{4}\sum_{j=0}^{n+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{2+n-j}}{(2+n-j)!} -$$
$$-(\alpha+2)(\alpha+3)(\alpha+4)\gamma^3(n+3)!\frac{1}{36}\sum_{j=0}^{n+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{3+n-j}}{(3+n-j)!} +$$
$$+(\alpha+4)!\gamma^4(n+4)!\frac{1}{24\alpha!}\sum_{j=0}^{n+4}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{4+n-j}}{(4+n-j)!}\Bigg);$$

$$\int \tau^n L_5^{(\alpha)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\Bigg(n!\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!} -$$
$$-(\alpha+5)\gamma(n+1)!\sum_{j=0}^{n+1}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{1+n-j}}{(1+n-j)!} +$$
$$+(\alpha+4)(\alpha+5)\gamma^2(n+2)!\frac{1}{4}\sum_{j=0}^{n+2}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{2+n-j}}{(2+n-j)!} -$$
$$-(\alpha+3)(\alpha+4)(\alpha+5)\gamma^3(n+3)!\frac{1}{36}\sum_{j=0}^{n+3}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{3+n-j}}{(3+n-j)!} +$$
$$+(\alpha+4)!\gamma^4(n+4)!\frac{1}{576(\alpha+1)!}\sum_{j=0}^{n+4}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{4+n-j}}{(4+n-j)!} -$$
$$-(\alpha+5)!\gamma^5(n+5)!\frac{1}{14400\alpha!}\sum_{j=0}^{n+5}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{5+n-j}}{(5+n-j)!}\Bigg).$$

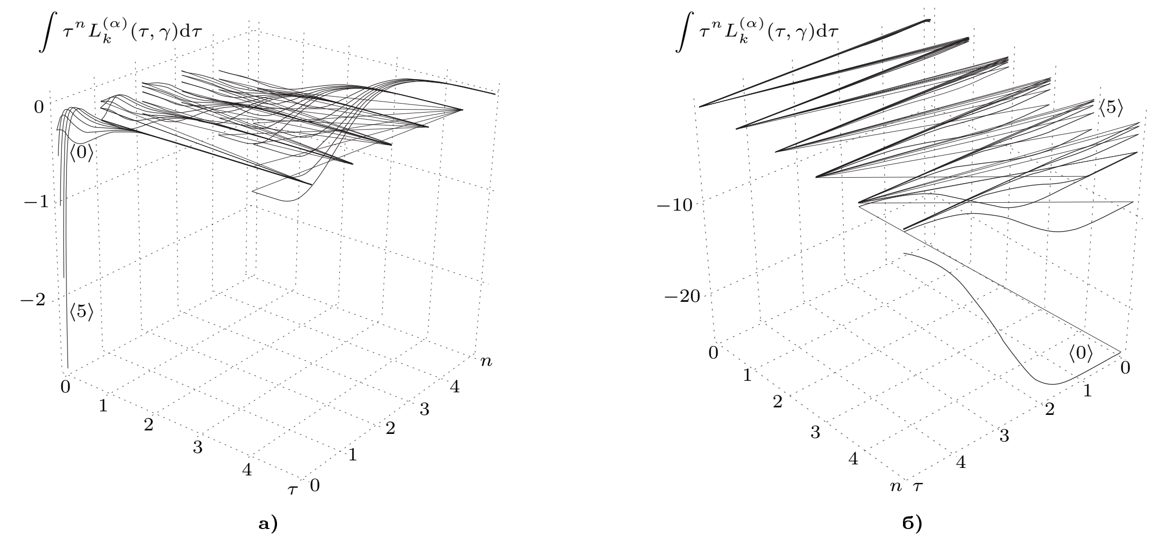


Рис. 1.85. Вид неопределенного интеграла n-ого рода от ортогональных функций Сонина-Лагерра 2-ого порядка: а) $n = 0..5$, $\gamma = 8$, $\alpha \in [0;5]$; б) $n = 0..5$, $\gamma \in [5;10]$, $\alpha = 1$

$$[\mathbf{1.86}] \quad \int P_k^{(-1/2,0)}(\tau,\gamma) = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}\times \times \frac{2(-1)^s}{\gamma(4s+1)}\exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big).$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int P_0^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big);$$

$$\int P_1^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{5\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(5-3\exp(-2\gamma\tau));$$

$$\int P_2^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{36\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(27-54\exp(-2\gamma\tau)+ + 35\exp(-4\gamma\tau));$$

$$\int P_3^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{104\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(65-273\times \times\exp(-2\gamma\tau)+455\exp(-4\gamma\tau)-231\exp(-6\gamma\tau));$$

$$\int P_4^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1088\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(595-4284\times \times\exp(-2\gamma\tau)+13090\exp(-4\gamma\tau)-15708\exp(-6\gamma\tau)+ + 6435\exp(-8\gamma\tau));$$

$$\int P_5^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2688\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)(1323-14553\times \times\exp(-2\gamma\tau)+70070\exp(-4\gamma\tau)-145530\exp(-6\gamma\tau)+ + 135135\exp(-8\gamma\tau)-46189\exp(-10\gamma\tau)).$$

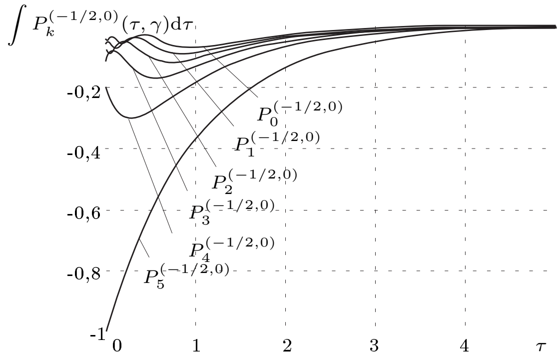


Рис. 1.86. Вид неопределенного интеграла от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

$$[\mathbf{1.87}] \quad \int \tau P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}\times \times(-1)^s\exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big)\Big(\frac{2\tau}{\gamma(4s+1)}+\frac{4}{\gamma^2(4s+1)^2}\Big).$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int \tau P_0^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)(\gamma\tau+2);$$

$$\int \tau P_1^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{25\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(50-6\times \times\exp(-2\gamma\tau)+\gamma\tau(25-15\exp(-2\gamma\tau))\big);$$

$$\int \tau P_2^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1620\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(2430-972\times \times\exp(-2\gamma\tau)+350\exp(-4\gamma\tau)+\gamma\tau(1215-2430\exp(-2\gamma\tau)+ + 1575\exp(-4\gamma\tau))\big);$$

$$\int \tau P_3^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{60840\gamma^2}\exp\Big(-\frac{\gamma\tau}{2}\Big)\big(76059- - 63882\exp(-2\gamma\tau)+59150\exp(-4\gamma\tau)-20790\exp(-6\gamma\tau)+$$

$+\gamma\tau(38025 - 159705\exp(-2\gamma\tau) + 266175\exp(-4\gamma\tau) -$
$- 135135\exp(-6\gamma\tau)))$;

$\int \tau P_4^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{1}{10820160\gamma^2}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(11834550 -$
$- 17041752\exp(-2\gamma\tau) + 28928900\exp(-4\gamma\tau) - 24033240\,\times$
$\times\exp(-6\gamma\tau) + 7528950\exp(-8\gamma\tau) + \gamma\tau(5917275 -$
$- 42604380\exp(-2\gamma\tau) + 130180050\exp(-4\gamma\tau) - 156216060\,\times$
$\times\exp(-6\gamma\tau) + 63996075\exp(-8\gamma\tau)))$;

$\int \tau P_5^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{1}{187125120\gamma^2}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\,\times$
$\times\big(184201290 - 405242838\exp(-2\gamma\tau) + 1083982900\,\times$
$\times\exp(-4\gamma\tau) - 1558626300\exp(-6\gamma\tau) + 1106755650\,\times$
$\times\exp(-8\gamma\tau) - 306233070\exp(-10\gamma\tau) + \gamma\tau(92100645 -$
$- 1013107095\exp(-2\gamma\tau) + 4877923050\exp(-4\gamma\tau) -$
$- 10131070950\exp(-6\gamma\tau) + 9407423025\exp(-8\gamma\tau) -$
$- 3215447235\exp(-10\gamma\tau))\big)$.

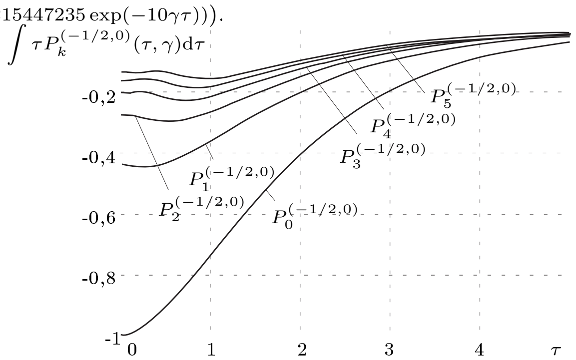


Рис. 1.87. Вид неопределенного интеграла 1-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

[1.88] $\displaystyle\int \tau^2 P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau =$

$$= -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big)\times$$
$$\times\Big(\frac{2\tau^2}{\gamma(4s+1)} + \frac{8\tau}{\gamma^2(4s+1)^2} + \frac{16}{\gamma^3(4s+1)^3}\Big).$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$\int \tau^2 P_0^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{2}{\gamma^3}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)(\gamma^2\tau^2 + 4\gamma\tau + 8)$;

$\int \tau^2 P_1^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{1}{125\gamma^3}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(1000 - 24\,\times$
$\times\exp(-2\gamma\tau) + \gamma\tau(500 - 60\exp(-2\gamma\tau)) + \gamma^2\tau^2(125 - 75\,\times$
$\times\exp(-2\gamma\tau))\big)$;

$\int \tau^2 P_2^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{1}{72900\gamma^3}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(437400 -$
$- 34992\exp(-2\gamma\tau) + 7000\exp(-4\gamma\tau) + \gamma\tau(218700 - 87480\,\times$
$\times\exp(-2\gamma\tau) + 31500\exp(-4\gamma\tau)) + \gamma^2\tau^2(54675 - 109350\,\times$
$\times\exp(-2\gamma\tau) + 70875\exp(-4\gamma\tau))\big)$;

$\int \tau^2 P_3^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{1}{35591400\gamma^3}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\,\times$
$\times\big(177957000 - 29896776\exp(-2\gamma\tau) + 15379000\exp(-4\gamma\tau) -$
$- 3742200\exp(-6\gamma\tau) + \gamma\tau(88978500 - 74741940\exp(-2\gamma\tau) +$
$+ 69205500\exp(-4\gamma\tau) - 24324300\exp(-6\gamma\tau)) + \gamma^2\tau^2\,\times$
$\times(22244625 - 93427425\exp(-2\gamma\tau) + 155712375\exp(-4\gamma\tau) -$
$- 79053975\exp(-6\gamma\tau))\big)$;

$\int \tau^2 P_4^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{1}{107606491200\gamma^3}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\,\times$
$\times\big(470778399000 - 135584178912\exp(-2\gamma\tau) + 127865738000\,\times$
$\times\exp(-4\gamma\tau) - 73541714400\exp(-6\gamma\tau) + 17617743000\,\times$
$\times\exp(-8\gamma\tau) + \gamma\tau(235389199500 - 338960447280\exp(-2\gamma\tau) +$
$+ 575395821000\exp(-4\gamma\tau) - 478021143600\exp(-6\gamma\tau) +$
$+ 149750815500\exp(-8\gamma\tau)) + \gamma^2\tau^2(58847299875 -$
$- 423700559100\exp(-2\gamma\tau) + 1294640597250\exp(-4\gamma\tau) -$
$- 1553568716700\exp(-6\gamma\tau) + 636440965875\exp(-8\gamma\tau))\big)$;

$\int \tau^2 P_5^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{1}{13026715228800\gamma^3}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\,\times$
$\times\big(51292691213400 - 22568784133896\exp(-2\gamma\tau) +$
$+ 33538430926000\exp(-4\gamma\tau) - 33385775346000\exp(-6\gamma\tau) +$
$+ 18128657547000\exp(-8\gamma\tau) - 4060650508200\exp(-10\gamma\tau) +$
$+ \gamma\tau(25646345606700 - 56421960334740\exp(-2\gamma\tau) +$
$+ 150922939167000\exp(-4\gamma\tau) - 217007539749000\,\times$
$\times\exp(-6\gamma\tau) + 154093589149500\exp(-8\gamma\tau) -$
$- 42636830336100\exp(-10\gamma\tau)) + \gamma^2\tau^2(6411586401675 -$
$- 70527450418425\exp(-2\gamma\tau) + 339576613125750\exp(-4\gamma\tau) -$
$- 705274504184250\exp(-6\gamma\tau) + 654897753885375\,\times$
$\times\exp(-8\gamma\tau) - 223843359264525\exp(-10\gamma\tau))\big)$.

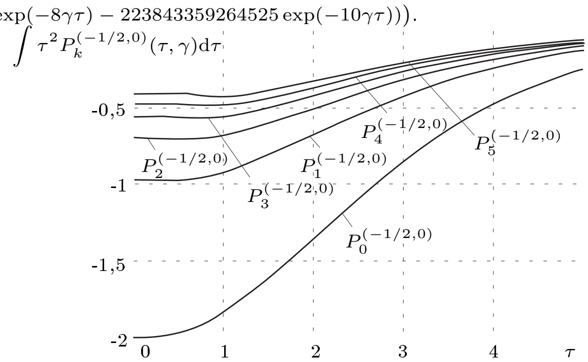


Рис. 1.88. Вид неопределенного интеграла 2-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

[1.89] $\displaystyle\int \tau^3 P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau =$

$$= -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big)\times$$
$$\times\Big(\frac{2\tau^3}{\gamma(4s+1)} + \frac{12\tau^2}{\gamma^2(4s+1)^2} +$$
$$+ \frac{48\tau}{\gamma^3(4s+1)^3} + \frac{96}{\gamma^4(4s+1)^4}\Big).$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$\int \tau^3 P_0^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{2}{\gamma^4}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)(\gamma^3\tau^3 + 6\gamma^2\tau^2 +$
$+ 24\gamma\tau + 48)$;

$\int \tau^3 P_1^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{1}{625\gamma^4}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(30000 -$
$- 144\exp(-2\gamma\tau) + \gamma\tau(15000 - 360\exp(-2\gamma\tau)) + \gamma^2\tau^2(3750 -$
$- 450\exp(-2\gamma\tau)) + \gamma^3\tau^3(625 - 375\exp(-2\gamma\tau))\big)$;

$\int \tau^3 P_2^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\dfrac{1}{1093500\gamma^4}\exp\Big(-\dfrac{\gamma\tau}{2}\Big)\big(39366000 -$
$- 629856\exp(-2\gamma\tau) + 70000\exp(-4\gamma\tau) + \gamma\tau(19683000 -$

$- 1574640\exp(-2\gamma\tau) + 315000\exp(-4\gamma\tau)) + \gamma^2\tau^2(4920750 -$
$- 1968300\exp(-2\gamma\tau) + 708750\exp(-4\gamma\tau)) + \gamma^3\tau^3(820125 -$
$- 1640250\exp(-2\gamma\tau) + 1063125\exp(-4\gamma\tau)));$

$$\int \tau^3 P_3^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{6940323000\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big) \times$$
$\times \big(208209690000 - 6995845584\exp(-2\gamma\tau) + 1999270000 \times$
$\times \exp(-4\gamma\tau) - 336798000\exp(-6\gamma\tau) + \gamma\tau(104104845000 -$
$- 17489613960\exp(-2\gamma\tau) + 8996715000\exp(-4\gamma\tau) -$
$- 2189187000\exp(-6\gamma\tau)) + \gamma^2\tau^2(26026211250 - 21862017450 \times$
$\times \exp(-2\gamma\tau) + 20242608750\exp(-4\gamma\tau) - 7114857750 \times$
$\times \exp(-6\gamma\tau)) + \gamma^3\tau^3(4337701875 - 18218347875\exp(-2\gamma\tau) +$
$+ 30363913125\exp(-4\gamma\tau) - 15415525125\exp(-6\gamma\tau)))\big);$

$$\int \tau^3 P_4^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{356715518328000\gamma^4}\exp\Big(-\frac{\gamma\tau}{2}\Big) \times$$
$\times \big(9363782356110000 - 539353863711936\exp(-2\gamma\tau) +$
$+ 282583280980000\exp(-4\gamma\tau) - 112518823032000\exp(-6\gamma\tau) +$
$+ 20612759310000\exp(-8\gamma\tau) + \gamma\tau(4681891178055000 -$
$- 1348384659279840\exp(-2\gamma\tau) + 1271624764410000 \times$
$\times \exp(-4\gamma\tau) - 731372349708000\exp(-6\gamma\tau) + 175208454135000 \times$
$\times \exp(-8\gamma\tau)) + \gamma^2\tau^2(1170472794513750 - 1685480824099800 \times$
$\times \exp(-2\gamma\tau) + 2861155719922500\exp(-4\gamma\tau) -$
$- 2376960136551000\exp(-6\gamma\tau) + 744635930073750 \times$
$\times \exp(-8\gamma\tau)) + \gamma^3\tau^3(195078799085625 - 1404567353416500 \times$
$\times \exp(-2\gamma\tau) + 4291733579883750\exp(-4\gamma\tau) -$
$- 5150080295860500\exp(-6\gamma\tau) + 2109801801875625 \times$
$\times \exp(-8\gamma\tau)))\big);$

$$\int \tau^3 P_5^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{302284926884304000\gamma^4} \times$$
$\times \exp\Big(-\frac{\gamma\tau}{2}\Big)\big(7141481397641682000 - 628450362992468016 \times$
$\times \exp(-2\gamma\tau) + 518839526425220000\exp(-4\gamma\tau) -$
$- 357561653955660000\exp(-6\gamma\tau) + 148473705309930000 \times$
$\times \exp(-8\gamma\tau) - 26922112869366000\exp(-10\gamma\tau) +$
$+ \gamma\tau(3570740698820841000 - 1571125907481170040\exp(-2\gamma\tau) +$
$+ 2334777868913490000\exp(-4\gamma\tau) - 2324150750711790000 \times$
$\times \exp(-6\gamma\tau) + 1262026495134405000\exp(-8\gamma\tau) -$
$- 282682185128343000\exp(-10\gamma\tau)) + \gamma^2\tau^2 \times$
$\times (892685174705210250 - 1963907384351462550\exp(-2\gamma\tau) +$
$+ 5253250205055352500\exp(-4\gamma\tau) - 7553489939813317500 \times$
$\times \exp(-6\gamma\tau) + 5363612604321221250\exp(-8\gamma\tau) -$
$- 1484081471923800750\exp(-10\gamma\tau)) + \gamma^3\tau^3 \times$
$\times (148780862450868375 - 1636589486959552125\exp(-2\gamma\tau) +$
$+ 7879875307583028750\exp(-4\gamma\tau) - 16365894869595521250 \times$
$\times \exp(-6\gamma\tau) + 15196902378910126875\exp(-8\gamma\tau) -$
$- 5194285151733302625\exp(-10\gamma\tau)))\big).$

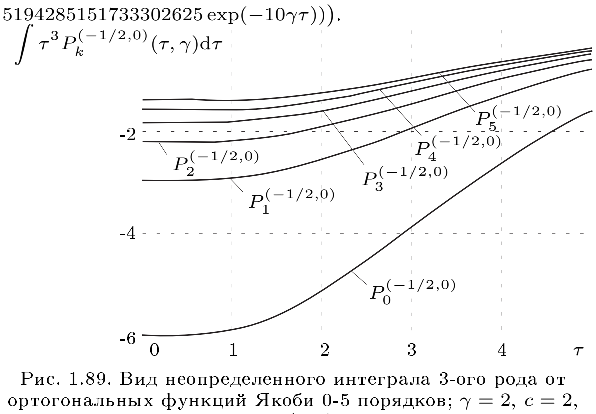


Рис. 1.89. Вид неопределенного интеграла 3-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

[1.90]
$$\int \tau^n P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau =$$
$$= -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\exp\Big(-\frac{(4s+1)}{2}\gamma\tau\Big)\times$$
$$\times\sum_{j=0}^{n}\frac{n!\tau^{n-j}}{(n-j)!\Big(\dfrac{\gamma(4s+1)}{2}\Big)^{j+1}}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n P_0^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp\Big(-\frac{\gamma\tau}{2}\Big)\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n P_1^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp\Big(-\frac{\gamma\tau}{2}\Big)\times$$
$$\times\left(\frac{n!}{2}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{\gamma}{2}\Big)^{j+1}} - \right.$$
$$\left. - \frac{3n!}{2}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{5\gamma}{2}\Big)^{j+1}}\right);$$

$$\int \tau^n P_2^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp\Big(-\frac{\gamma\tau}{2}\Big)\times$$
$$\times\left(\frac{3n!}{8}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{\gamma}{2}\Big)^{j+1}} - \right.$$
$$- \frac{15n!}{4}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{5\gamma}{2}\Big)^{j+1}} +$$
$$\left. + \frac{35n!}{8}\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{9\gamma}{2}\Big)^{j+1}}\right);$$

$$\int \tau^n P_3^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp\Big(-\frac{\gamma\tau}{2}\Big)\times$$
$$\times\left(\frac{5n!}{16}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{\gamma}{2}\Big)^{j+1}} - \right.$$
$$- \frac{105n!}{16}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{5\gamma}{2}\Big)^{j+1}} +$$
$$+ \frac{315n!}{16}\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{9\gamma}{2}\Big)^{j+1}} -$$
$$\left. - \frac{231n!}{16}\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{13\gamma}{2}\Big)^{j+1}}\right);$$

$$\int \tau^n P_4^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp\Big(-\frac{\gamma\tau}{2}\Big)\times$$
$$\times\left(\frac{35n!}{128}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{\gamma}{2}\Big)^{j+1}} - \right.$$

$$-\frac{315n!}{32}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{5\gamma}{2}\right)^{j+1}}+$$

$$+\frac{3465n!}{64}\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{9\gamma}{2}\right)^{j+1}}-$$

$$-\frac{3003n!}{32}\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{13\gamma}{2}\right)^{j+1}}+$$

$$\left.+\frac{6435n!}{128}\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{17\gamma}{2}\right)^{j+1}}\right);$$

$$\int\tau^n P_5^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau=-\exp\left(-\frac{\gamma\tau}{2}\right)\times$$

$$\times\left(\frac{63n!}{256}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{\gamma}{2}\right)^{j+1}}-\right.$$

$$-\frac{3465n!}{256}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{5\gamma}{2}\right)^{j+1}}+$$

$$+\frac{15015n!}{128}\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{9\gamma}{2}\right)^{j+1}}-$$

$$-\frac{45045n!}{128}\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{13\gamma}{2}\right)^{j+1}}+$$

$$+\frac{109395n!}{256}\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{17\gamma}{2}\right)^{j+1}}-$$

$$\left.-\frac{46189n!}{256}\exp(-10\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{21\gamma}{2}\right)^{j+1}}\right).$$

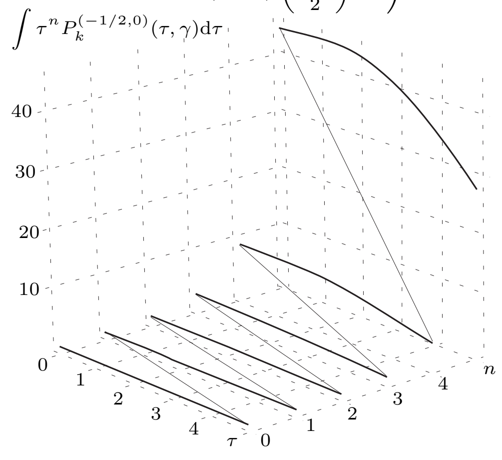


Рис. 1.90. Вид неопределенного интеграла n-ого рода от ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 2$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

[1.91]
$$\int Leg_k(\tau,\gamma)\mathrm{d}\tau=-\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}\times$$
$$\times\frac{(-1)^s}{\gamma(2s+1)}\exp\bigl(-(2s+1)\gamma\tau\bigr).$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int Leg_0(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{\gamma}\exp(-\gamma\tau);$$

$$\int Leg_1(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{3\gamma}\exp(-\gamma\tau)(3-2\exp(-2\gamma\tau));$$

$$\int Leg_2(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{5\gamma}\exp(-\gamma\tau)(5-10\exp(-2\gamma\tau)+6\times$$
$$\times\exp(-4\gamma\tau));$$

$$\int Leg_3(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{7\gamma}\exp(-\gamma\tau)(7-28\exp(-2\gamma\tau)+42\times$$
$$\times\exp(-4\gamma\tau)-20\exp(-6\gamma\tau));$$

$$\int Leg_4(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{9\gamma}\exp(-\gamma\tau)(9-60\exp(-2\gamma\tau)+162\times$$
$$\times\exp(-4\gamma\tau)-180\exp(-6\gamma\tau)+70\exp(-8\gamma\tau));$$

$$\int Leg_5(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{11\gamma}\exp(-\gamma\tau)(11-110\exp(-2\gamma\tau)+462\times$$
$$\times\exp(-4\gamma\tau)-880\exp(-6\gamma\tau)+770\exp(-8\gamma\tau)-252\times$$
$$\times\exp(-10\gamma\tau)).$$

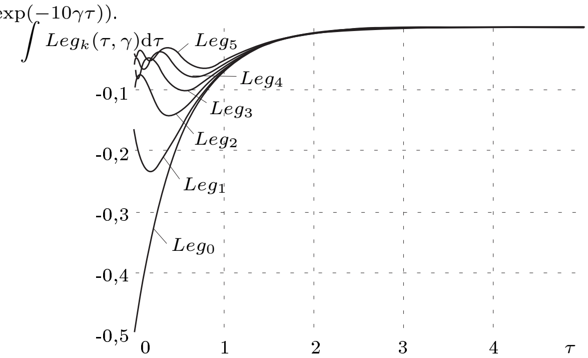


Рис. 1.91. Вид неопределенного интеграла от ортогональных функций Лежандра 0-5 порядков; $\gamma = 2$, $c = 2$

[1.92]
$$\int\tau Leg_k(\tau,\gamma)\mathrm{d}\tau=-\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\times$$
$$\times\exp\bigl(-(2s+1)\gamma\tau\bigr)\left(\frac{\tau}{\gamma(2s+1)}+\frac{1}{\gamma^2(2s+1)^2}\right).$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int\tau Leg_0(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{\gamma^2}\exp(-\gamma\tau)(\gamma\tau+1);$$

$$\int\tau Leg_1(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{9\gamma^2}\exp(-\gamma\tau)\bigl(9-2\exp(-2\gamma\tau)+\gamma\tau(9-$$
$$-6\exp(-2\gamma\tau))\bigr);$$

$$\int\tau Leg_2(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{75\gamma^2}\exp(-\gamma\tau)\bigl(75-50\exp(-2\gamma\tau)+$$
$$+18\exp(-4\gamma\tau)+\gamma\tau(75-150\exp(-2\gamma\tau)+90\exp(-4\gamma\tau))\bigr);$$

$$\int\tau Leg_3(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{735\gamma^2}\exp(-\gamma\tau)\bigl(735-980\exp(-2\gamma\tau)+$$
$$+882\exp(-4\gamma\tau)-300\exp(-6\gamma\tau)+\gamma\tau(735-2940\exp(-2\gamma\tau)+$$
$$+4410\exp(-4\gamma\tau)-2100\exp(-6\gamma\tau))\bigr);$$

$\int \tau Leg_4(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2835\gamma^2}\exp(-\gamma\tau)\big(12835 - 6300 \times$
$\times \exp(-2\gamma\tau) + 10206\exp(-4\gamma\tau) - 8100\exp(-6\gamma\tau) + 2450 \times$
$\times \exp(-8\gamma\tau) + \gamma\tau(2835 - 18900\exp(-2\gamma\tau) + 51030\exp(-4\gamma\tau) -$
$- 56700\exp(-6\gamma\tau) + 22050\exp(-8\gamma\tau))\big);$

$\int \tau Leg_5(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{38115\gamma^2}\exp(-\gamma\tau)\big(38115 -$
$- 127050\exp(-2\gamma\tau) + 320166\exp(-4\gamma\tau) - 435600\exp(-6\gamma\tau) +$
$+ 296450\exp(-8\gamma\tau) - 79380\exp(-10\gamma\tau) + \gamma\tau(38115 -$
$- 381150\exp(-2\gamma\tau) + 1600830\exp(-4\gamma\tau) - 3049200 \times$
$\times \exp(-6\gamma\tau) + 2668050\exp(-8\gamma\tau) - 873180\exp(-10\gamma\tau))\big).$

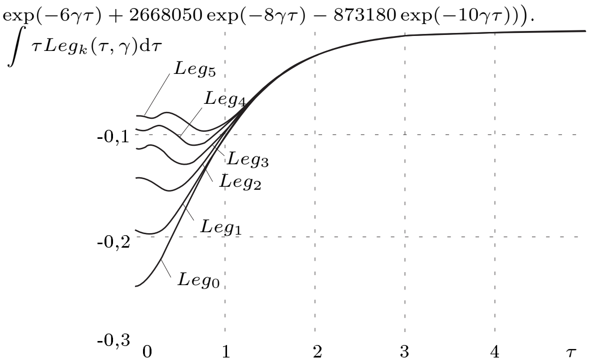


Рис. 1.92. Вид неопределенного интеграла 1-ого рода от ортогональных функций Лежандра 0-5 порядков; $\gamma = 2$, $c = 2$

[1.93]
$$\int \tau^2 Leg_k(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s \times \exp\big(-(2s+1)\gamma\tau\big)\left(\frac{\tau^2}{\gamma(2s+1)} + \frac{2\tau}{\gamma^2(2s+1)^2} + \frac{2}{\gamma^3(2s+1)^3}\right).$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$\int \tau^2 Leg_0(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^3}\exp(-\gamma\tau)(\gamma^2\tau^2 + 2\gamma\tau + 2);$

$\int \tau^2 Leg_1(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{27\gamma^3}\exp(-\gamma\tau)\big(54 - 4\exp(-2\gamma\tau) +$
$+ \gamma\tau(54 - 12\exp(-2\gamma\tau) + \gamma^2\tau^2(27 - 18\exp(-2\gamma\tau)))\big);$

$\int \tau^2 Leg_2(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1125\gamma^3}\exp(-\gamma\tau)\big(2250 - 500 \times$
$\times \exp(-2\gamma\tau) + 108\exp(-4\gamma\tau) + \gamma\tau(2250 - 1500\exp(-2\gamma\tau) +$
$+ 540\exp(-4\gamma\tau)) + \gamma^2\tau^2(1125 - 2250\exp(-2\gamma\tau) + 1350 \times$
$\times \exp(-4\gamma\tau))\big);$

$\int \tau^2 Leg_3(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{77175\gamma^3}\exp(-\gamma\tau)\big(154350 -$
$- 68600\exp(-2\gamma\tau) + 37044\exp(-4\gamma\tau) - 9000\exp(-6\gamma\tau) +$
$+ \gamma\tau(154350 - 205800\exp(-2\gamma\tau) + 185220\exp(-4\gamma\tau) -$
$- 63000\exp(-6\gamma\tau)) + \gamma^2\tau^2(77175 - 308700\exp(-2\gamma\tau) +$
$+ 463050\exp(-4\gamma\tau) - 220500\exp(-6\gamma\tau))\big);$

$\int \tau^2 Leg_4(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{893025\gamma^3}\exp(-\gamma\tau)\big(1786050 -$
$- 1323000\exp(-2\gamma\tau) + 1285956\exp(-4\gamma\tau) - 729000 \times$
$\times \exp(-6\gamma\tau) + 171500\exp(-8\gamma\tau) + \gamma\tau(1786050 - 3969000 \times$
$\times \exp(-2\gamma\tau) + 6429780\exp(-4\gamma\tau) - 5103000\exp(-6\gamma\tau) +$
$+ 1543500\exp(-8\gamma\tau)) + \gamma^2\tau^2(893025 - 5953500\exp(-2\gamma\tau) +$
$+ 16074450\exp(-4\gamma\tau) - 17860500\exp(-6\gamma\tau) + 6945750 \times$
$\times \exp(-8\gamma\tau))\big);$

$\int \tau^2 Leg_5(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{132068475\gamma^3}\exp(-\gamma\tau)\big(264136950 -$
$- 293485500\exp(-2\gamma\tau) + 443750076\exp(-4\gamma\tau) - 431244000 \times$
$\times \exp(-6\gamma\tau) + 228266500\exp(-8\gamma\tau) - 50009400\exp(-10\gamma\tau) +$
$+ \gamma\tau(264136950 - 880456500\exp(-2\gamma\tau) + 2218750380 \times$
$\times \exp(-4\gamma\tau) - 3018708000\exp(-6\gamma\tau) + 2054398500 \times$
$\times \exp(-8\gamma\tau) - 550103400\exp(-10\gamma\tau)) + \gamma^2\tau^2(132068475 -$
$- 1320684750\exp(-2\gamma\tau) + 5546875950\exp(-4\gamma\tau) -$
$- 10565478000\exp(-6\gamma\tau) + 9244793250\exp(-8\gamma\tau) -$
$- 3025568700\exp(-10\gamma\tau))\big).$

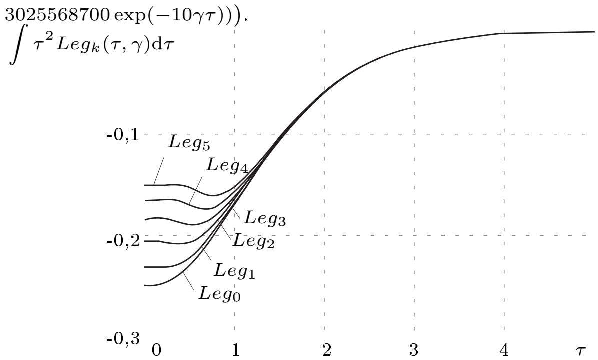


Рис. 1.93. Вид неопределенного интеграла 2-ого рода от ортогональных функций Лежандра 0-5 порядков; $\gamma = 2$, $c = 2$

[1.94]
$$\int \tau^3 Leg_k(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s \times \exp\big(-(2s+1)\gamma\tau\big)\left(\frac{\tau^3}{\gamma(2s+1)} + \frac{3\tau^2}{\gamma^2(2s+1)^2} + \frac{6\tau}{\gamma^3(2s+1)^3} + \frac{6}{\gamma^4(2s+1)^4}\right).$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$\int \tau^3 Leg_0(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^4}\exp(-\gamma\tau)(\gamma^3\tau^3 + 3\gamma^2\tau^2 + 6\gamma\tau + 6);$

$\int \tau^3 Leg_1(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{27\gamma^4}\exp(-\gamma\tau)\big(162 - 4\exp(-2\gamma\tau) +$
$+ \gamma\tau(162 - 12\exp(-2\gamma\tau) + \gamma^2\tau^2(81 - 18\exp(-2\gamma\tau)) + \gamma^3\tau^3 \times$
$\times (27 - 18\exp(-2\gamma\tau))\big);$

$\int \tau^3 Leg_2(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{5625\gamma^4}\exp(-\gamma\tau)\big(33750 - 2500 \times$
$\times \exp(-2\gamma\tau) + 324\exp(-4\gamma\tau) + \gamma\tau(33750 - 7500\exp(-2\gamma\tau) +$
$+ 1620\exp(-4\gamma\tau)) + \gamma^2\tau^2(16875 - 11250\exp(-2\gamma\tau) + 4050 \times$
$\times \exp(-4\gamma\tau)) + \gamma^3\tau^3(5625 - 11250\exp(-2\gamma\tau) + 6750 \times$
$\times \exp(-4\gamma\tau))\big);$

$\int \tau^3 Leg_3(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2701125\gamma^4}\exp(-\gamma\tau)\big(16206750 -$
$- 2401000\exp(-2\gamma\tau) + 777924\exp(-4\gamma\tau) - 135000\exp(-6\gamma\tau) +$
$+ \gamma\tau(16206750 - 7203000\exp(-2\gamma\tau) + 3889620\exp(-4\gamma\tau) -$
$- 945000\exp(-6\gamma\tau)) + \gamma^2\tau^2(8103375 - 10804500\exp(-2\gamma\tau) +$
$+ 9724050\exp(-4\gamma\tau) - 3307500\exp(-6\gamma\tau)) + \gamma^3\tau^3(2701125 -$
$- 10804500\exp(-2\gamma\tau) + 16206750\exp(-4\gamma\tau) - 7717500 \times$
$\times \exp(-6\gamma\tau))\big);$

$\int \tau^3 Leg_4(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{93767625\gamma^4}\exp(-\gamma\tau)\big(562605750 -$

$$- 138915000\exp(-2\gamma\tau) + 81015228\exp(-4\gamma\tau) - 32805000 \times$$
$$\times \exp(-6\gamma\tau) + 6002500\exp(-8\gamma\tau) + \gamma\tau(562605750 -$$
$$- 416745000\exp(-2\gamma\tau) + 405076140\exp(-4\gamma\tau) - 229635000 \times$$
$$\times \exp(-6\gamma\tau) + 54022500\exp(-8\gamma\tau)) + \gamma^2\tau^2(281302875 -$$
$$- 625117500\exp(-2\gamma\tau) + 1012690350\exp(-4\gamma\tau) - 803722500 \times$$
$$\times \exp(-6\gamma\tau) + 243101250\exp(-8\gamma\tau)) + \gamma^3\tau^3(93767625 -$$
$$- 625117500\exp(-2\gamma\tau) + 1687817250\exp(-4\gamma\tau) -$$
$$- 1875352500\exp(-6\gamma\tau) + 729303750\exp(-8\gamma\tau))\big);$$

$$\int \tau^3 Leg_5(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{152539088625\gamma^4}\exp(-\gamma\tau) \times$$
$$\times \big(915234531750 - 338975752500\exp(-2\gamma\tau) +$$
$$+ 307518802668\exp(-4\gamma\tau) - 213465780000\exp(-6\gamma\tau) +$$
$$+ 87882602500\exp(-8\gamma\tau) - 15752961000\exp(-10\gamma\tau) +$$
$$+ \gamma\tau(915234531750 - 1016927257500\exp(-2\gamma\tau) +$$
$$+ 1537594013340\exp(-4\gamma\tau) - 1494260460000\exp(-6\gamma\tau) +$$
$$+ 790943422500\exp(-8\gamma\tau) - 173282571000\exp(-10\gamma\tau)) +$$
$$+ \gamma^2\tau^2(457617265875 - 1525390886250\exp(-2\gamma\tau) +$$
$$+ 3843985033350\exp(-4\gamma\tau) - 5229911610000\exp(-6\gamma\tau) +$$
$$+ 3559245401250\exp(-8\gamma\tau) - 953054140500\exp(-10\gamma\tau)) +$$
$$+ \gamma^3\tau^3(152539088625 - 1525390886250\exp(-2\gamma\tau) +$$
$$+ 6406641722250\exp(-4\gamma\tau) - 12203127090000\exp(-6\gamma\tau) +$$
$$+ 10677736203750\exp(-8\gamma\tau) - 3494531848500\exp(-10\gamma\tau))\big).$$

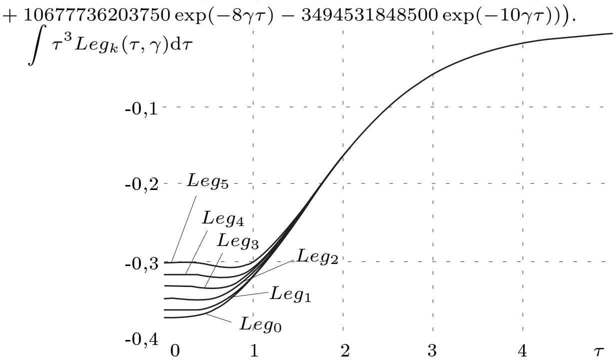


Рис. 1.94. Вид неопределенного интеграла 3-ого рода от ортогональных функций Лежандра 0-5 порядков; $\gamma = 2$, $c = 2$

[1.95]
$$\int \tau^n Leg_k(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s \times$$
$$\times \exp\big(-(2s+1)\gamma\tau\big)\sum_{j=0}^{n}\frac{n!\tau^{n-j}}{(n-j)!\Big(\gamma(2s+1)\Big)^{j+1}}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n Leg_0(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp(-\gamma\tau)\sum_{j=0}^{n}\left(\frac{2}{\gamma}\right)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n Leg_1(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\gamma\Big)^{j+1}} -\right.$$
$$\left.- n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n Leg_2(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\gamma\Big)^{j+1}} -\right.$$
$$- 6n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} +$$
$$\left.+ 6n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n Leg_3(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\gamma\Big)^{j+1}} -\right.$$
$$- 12n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} +$$
$$+ 30n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} -$$
$$\left.- 20n!\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n Leg_4(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\gamma\Big)^{j+1}} -\right.$$
$$- 20n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} +$$
$$+ 90n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} -$$
$$- 140n!\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}} +$$
$$\left.+ 70n!\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(9\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n Leg_5(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\gamma\Big)^{j+1}} -\right.$$
$$- 30n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} +$$
$$+ 210n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} -$$
$$- 560n!\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}} +$$
$$+ 630n!\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(9\gamma\Big)^{j+1}} -$$
$$\left.- 252n!\exp(-10\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(11\gamma\Big)^{j+1}}\right).$$

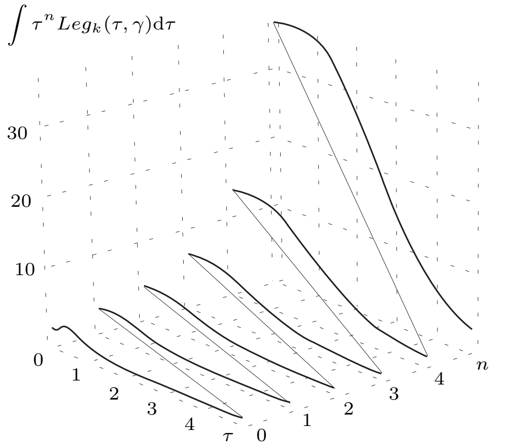


Рис. 1.95. Вид неопределенного интеграла n-ого рода от ортогональных функций Лежандра 2-ого порядка; $n = 0..5$, $\gamma = 2$, $c = 2$

[1.96]
$$\int P_k^{(1/2,0)}(\tau,\gamma) = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}\times \times\frac{2(-1)^s}{\gamma(4s+3)}\exp\Big(-\frac{(4s+3)}{2}\gamma\tau\Big).$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int P_0^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{3\gamma}\exp\Big(-\frac{3\gamma\tau}{2}\Big);$$

$$\int P_1^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{7\gamma}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(7-5\exp(-2\gamma\tau));$$

$$\int P_2^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{44\gamma}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(55-110\times \times\exp(-2\gamma\tau)+ +63\exp(-4\gamma\tau));$$

$$\int P_3^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{120\gamma}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(175-675\times \times\exp(-2\gamma\tau)+945\exp(-4\gamma\tau)-429\exp(-6\gamma\tau));$$

$$\int P_4^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1216\gamma}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(1995-12540\times \times\exp(-2\gamma\tau)+31122\exp(-4\gamma\tau)-32604\exp(-6\gamma\tau)+12155\times \times\exp(-8\gamma\tau));$$

$$\int P_5^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2944\gamma}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(5313-49335\times \times\exp(-2\gamma\tau)+188370\exp(-4\gamma\tau)-335478\exp(-6\gamma\tau)+ +279565\exp(-8\gamma\tau)-88179\exp(-10\gamma\tau)).$$

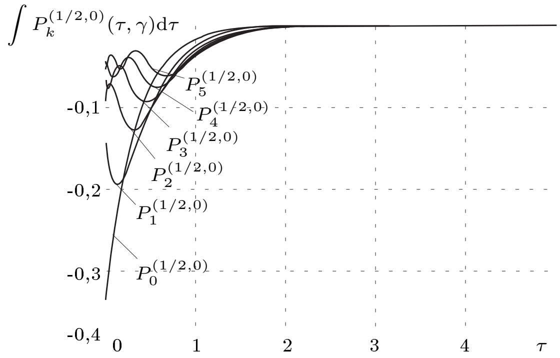


Рис. 1.96. Вид неопределенного интеграла от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[1.97]
$$\int \tau P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}\times \times(-1)^s\exp\Big(-\frac{(4s+3)}{2}\gamma\tau\Big)\Big(\frac{2\tau}{\gamma(4s+3)}+\frac{4}{\gamma^2(4s+3)^2}\Big).$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int \tau P_0^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{9\gamma^2}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(3\gamma\tau+2);$$

$$\int \tau P_1^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{147\gamma^2}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(98-30\times \times\exp(-2\gamma\tau)+\gamma\tau(147-105\exp(-2\gamma\tau)));$$

$$\int \tau P_2^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{10164\gamma^2}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(8470-7260\times \times\exp(-2\gamma\tau)+2646\exp(-4\gamma\tau)+\gamma\tau(12705-25410\exp(-2\gamma\tau)+ +14553\exp(-4\gamma\tau)));$$

$$\int \tau P_3^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{138600\gamma^2}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(134750- -222750\exp(-2\gamma\tau)+198450\exp(-4\gamma\tau)-66066\exp(-6\gamma\tau)+ +\gamma\tau(202125-779625\exp(-2\gamma\tau)+1091475\exp(-4\gamma\tau)- -495495\exp(-6\gamma\tau)));$$

$$\int \tau P_4^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{8895040\gamma^2}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(9728950- -26208600\exp(-2\gamma\tau)+41392260\exp(-4\gamma\tau)-31799768\times \times\exp(-6\gamma\tau)+9359350\exp(-8\gamma\tau)+\gamma\tau(14593425-91730100\times \times\exp(-2\gamma\tau)+227657430\exp(-4\gamma\tau)-238498260\exp(-6\gamma\tau)+ +88913825\exp(-8\gamma\tau)));$$

$$\int \tau P_5^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{495313280\gamma^2}\times \times\exp\Big(-\frac{3\gamma\tau}{2}\Big)(595923790-2371533450\exp(-2\gamma\tau)+ +5762238300\exp(-4\gamma\tau)-7525666148\exp(-6\gamma\tau)+ +4951096150\exp(-8\gamma\tau)-1290058770\exp(-10\gamma\tau)+ +\gamma\tau(893885685-8300367075\exp(-2\gamma\tau)+31692310650\times \times\exp(-4\gamma\tau)-56442496110\exp(-6\gamma\tau)+47035413425\times \times\exp(-8\gamma\tau)-14835675855\exp(-10\gamma\tau))).$$

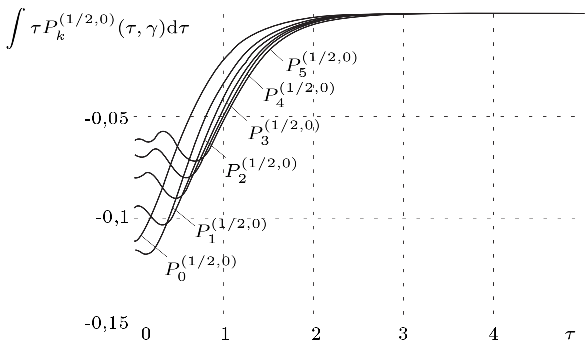


Рис. 1.97. Вид неопределенного интеграла 1-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[1.98] $$\int \tau^2 P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = \\ = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s \exp\Big(-\frac{(4s+3)}{2}\gamma\tau\Big)\times \\ \times\Big(\frac{2\tau^2}{\gamma(4s+3)} + \frac{8\tau}{\gamma^2(4s+3)^2} + \frac{16}{\gamma^3(4s+3)^3}\Big).$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$\int \tau^2 P_0^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{27\gamma^3}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(9\gamma^2\tau^2 + 12\gamma\tau + 8)$;

$\int \tau^2 P_1^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3087\gamma^3}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(2744 - 360 \times \exp(-2\gamma\tau) + \gamma\tau(4116 - 1260\exp(-2\gamma\tau) + \gamma^2\tau^2(3087 - 2205 \times \exp(-2\gamma\tau)))\big)$;

$\int \tau^2 P_2^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2347884\gamma^3}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(2608760 - 958320\exp(-2\gamma\tau) + 222264\exp(-4\gamma\tau) + \gamma\tau(3913140 - 3354120\exp(-2\gamma\tau) + 1222452\exp(-4\gamma\tau)) + \gamma^2\tau^2(2934855 - 5869710\exp(-2\gamma\tau) + 3361743\exp(-4\gamma\tau))\big)$;

$\int \tau^2 P_3^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{160083000\gamma^3}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times \big(207515000 - 147015000\exp(-2\gamma\tau) + 83349000\exp(-4\gamma\tau) - 20348328\exp(-6\gamma\tau) + \gamma\tau(311272500 - 514552500 \times \exp(-2\gamma\tau) + +458419500\exp(-4\gamma\tau) - 152612460 \times \exp(-6\gamma\tau)) + \gamma^2\tau^2(233454375 - -900466875 \times \exp(-2\gamma\tau) + 1260653625\exp(-4\gamma\tau) - 572296725 \times \exp(-6\gamma\tau))\big)$;

$\int \tau^2 P_4^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{195201652800\gamma^3}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times \big(284669077000 - 328655844000\exp(-2\gamma\tau) + 330310234800 \times \exp(-4\gamma\tau) - 186092242336\exp(-6\gamma\tau) + 43240197000 \times \exp(-8\gamma\tau) + \gamma\tau(427003615500 - 1150295454000\exp(-2\gamma\tau) + 1816706291400\exp(-4\gamma\tau) - 1395691817520\exp(-6\gamma\tau) + 410781871500\exp(-8\gamma\tau)) + \gamma^2\tau^2(320252711625 - 2013017044500\exp(-2\gamma\tau) + 4995942301350\exp(-4\gamma\tau) - 5233844315700\exp(-6\gamma\tau) + 1951213889625\exp(-8\gamma\tau))\big)$;

$\int \tau^2 P_5^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{250001948380800\gamma^3}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times \big(401044792194200 - 683997677649000\exp(-2\gamma\tau) + 1057601217582000\exp(-4\gamma\tau) - 1012924560856208 \times \exp(-6\gamma\tau) + 526103476899000\exp(-8\gamma\tau) - 113241358830600\exp(-10\gamma\tau) + \gamma\tau(601567188291300 - 2393991871771500\exp(-2\gamma\tau) + 5816806696701000 \times \exp(-4\gamma\tau) - 7596934206421560\exp(-6\gamma\tau) + 4997983030540500\exp(-8\gamma\tau) - 1302275626551900 \times \exp(-10\gamma\tau)) + \gamma^2\tau^2(451175391218475 - 4189485775600125 \times \exp(-2\gamma\tau) + 15996218415927750\exp(-4\gamma\tau) - 28488503274080850\exp(-6\gamma\tau) + 4997983030540500 \times \exp(-8\gamma\tau) - 1302275626551900\exp(-10\gamma\tau))\big)$.

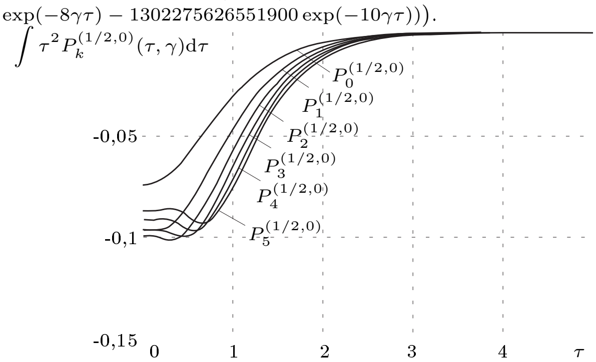


Рис. 1.98. Вид неопределенного интеграла 2-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[1.99] $$\int \tau^3 P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau \\ = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s \exp\Big(-\frac{(4s+3)}{2}\gamma\tau\Big)\times \\ \times\Big(\frac{2\tau^3}{\gamma(4s+3)} + \frac{12\tau^2}{\gamma^2(4s+31)^2} + \\ + \frac{48\tau}{\gamma^3(4s+3)^3} + \frac{96}{\gamma^4(4s+3)^4}\Big).$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$\int \tau^3 P_0^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{27\gamma^4}\exp\Big(-\frac{3\gamma\tau}{2}\Big)(9\gamma^3\tau^3 + 18\gamma^2\tau^2 + 24\gamma\tau + 16)$;

$\int \tau^3 P_1^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{21609\gamma^4}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(38416 - 2160 \times \exp(-2\gamma\tau) + \gamma\tau(57624 - 7560\exp(-2\gamma\tau)) + \gamma^2\tau^2 \times (43218 - 13230\exp(-2\gamma\tau)) + \gamma^3\tau^3(21609 - 15435 \times \exp(-2\gamma\tau))\big)$;

$\int \tau^3 P_2^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{180787068\gamma^4}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times \big(401749040 - 63249120\exp(-2\gamma\tau) + 9335088\exp(-4\gamma\tau) + \gamma\tau(602623560 - 221371920\exp(-2\gamma\tau) + 51342984 \times \exp(-4\gamma\tau)) + \gamma^2\tau^2(451967670 - 387400860\exp(-2\gamma\tau) + 141193206\exp(-4\gamma\tau)) + \gamma^3\tau^3(225983835 - 451967670 \times \exp(-2\gamma\tau) + 258854211\exp(-4\gamma\tau))\big)$;

$\int \tau^3 P_3^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{61631955000\gamma^4}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times \big(159786550000 - 48514950000\exp(-2\gamma\tau) + 17503290000\exp(-4\gamma\tau) - 3133642512\exp(-6\gamma\tau) + \gamma\tau(239679825000 - 169802325000\exp(-2\gamma\tau) + 96268095000\exp(-4\gamma\tau) - 23502318840\exp(-6\gamma\tau)) + \gamma^2\tau^2(179759868750 - 297154068750\exp(-2\gamma\tau) + 264737261250\exp(-4\gamma\tau) - 88133695650\exp(-6\gamma\tau)) +$

$+\gamma^3\tau^3(89879934375 - 346679746875\exp(-2\gamma\tau) +$
$+ 485351645625\exp(-4\gamma\tau) - 220334239125\exp(-6\gamma\tau)))$;

$$\int \tau^3 P_4^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1427900090232000\gamma^4}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times$$
$\times\big(4164708596510000 - 2060672141880000\exp(-2\gamma\tau) +$
$+ 1317937836852000\exp(-4\gamma\tau) - 544505901075136\times$
$\times\exp(-6\gamma\tau) + 99884855070000\exp(-8\gamma\tau) + \gamma\tau\times$
$\times(6247062894765000 - 7212352496580000\exp(-2\gamma\tau) +$
$+ 7248658102686000\exp(-4\gamma\tau) - 4083794258063520\times$
$\times\exp(-6\gamma\tau) + 948906123165000\exp(-8\gamma\tau)) + \gamma^2\tau^2\times$
$\times(4685297171073750 - 12621616869015000\exp(-2\gamma\tau) +$
$+ 19933809782386500\exp(-4\gamma\tau) - 15314228467738200\times$
$\times\exp(-6\gamma\tau) + 4507304085033750\exp(-8\gamma\tau)) + \gamma^3\tau^3\times$
$\times(2342648585536875 - 14725219680517500\exp(-2\gamma\tau) +$
$+ 36545317934375250\exp(-4\gamma\tau) - 38285571169345500\times$
$\times\exp(-6\gamma\tau) + 14273129602606875\exp(-8\gamma\tau)))$;

$$\int \tau^3 P_5^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{420615778053276960000\gamma^4}\times$$
$\times\exp\Big(-\frac{3\gamma\tau}{2}\Big)\big(1349475621254263580000 -$
$- 986393050937622900000\exp(-2\gamma\tau) + 970560637375001400000\times$
$\times\exp(-4\gamma\tau) - 681677970965010859840\exp(-6\gamma\tau) +$
$+ 279518777276438700000\exp(-8\gamma\tau) - 49701632390750340000\times$
$\times\exp(-10\gamma\tau) + \gamma\tau(2024213431881395370000 -$
$- 3452375678281680150000\exp(-2\gamma\tau) + 5338083505562507700000\times$
$\times\exp(-4\gamma\tau) - 5112584782237581448800\exp(-6\gamma\tau) +$
$+ 2655428384126167650000\exp(-8\gamma\tau) - 571568772493628910000\times$
$\times\exp(-10\gamma\tau)) + \gamma^2\tau^2(1518160073911046527500 -$
$- 6041657436992940262500\exp(-2\gamma\tau) + 14679729640296896175000\times$
$\times\exp(-4\gamma\tau) - 19172192933390930433000\exp(-6\gamma\tau) +$
$+ 12613284824599296337500\exp(-8\gamma\tau) - 3286520441838366232500\times$
$\times\exp(-10\gamma\tau)) + \gamma^3\tau^3(759080036955523263750 -$
$- 7048600343158430306250\exp(-2\gamma\tau) + 26912837673877642987500\times$
$\times\exp(-4\gamma\tau) - 47930482333477326082500\exp(-6\gamma\tau) +$
$+ 39942068611231105068750\exp(-8\gamma\tau) -$
$- 12598328360380403891250\exp(-10\gamma\tau)))$.

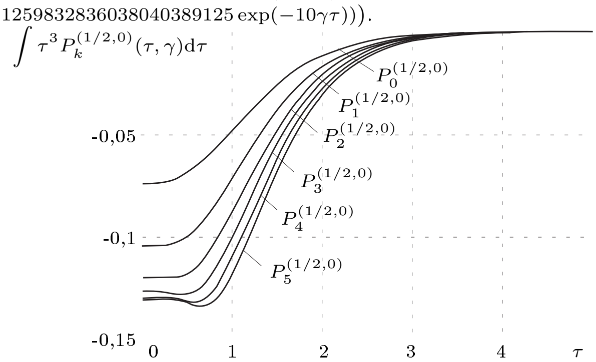


Рис. 1.99. Вид неопределенного интеграла 3-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[1.100]
$$\int \tau^n P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau =$$
$$= -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s\exp\Big(-\frac{(4s+3)}{2}\gamma\tau\Big)\times$$
$$\times\sum_{j=0}^{n}\frac{n!\tau^{n-j}}{(n-j)!\Big(\dfrac{\gamma(4s+3)}{2}\Big)^{j+1}}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n P_0^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{3\gamma}\exp\Big(-\frac{3\gamma\tau}{2}\Big)\sum_{j=0}^{n}\Big(\frac{2}{3\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n P_1^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times$$
$$\times\left(\frac{3n!}{2}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{3\gamma}{2}\Big)^{j+1}} - \right.$$
$$\left. -\frac{5n!}{2}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{7\gamma}{2}\Big)^{j+1}}\right);$$

$$\int \tau^n P_2^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times$$
$$\times\left(\frac{15n!}{8}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{3\gamma}{2}\Big)^{j+1}} - \right.$$
$$-\frac{35n!}{4}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{7\gamma}{2}\Big)^{j+1}} +$$
$$\left. +\frac{63n!}{8}\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{11\gamma}{2}\Big)^{j+1}}\right);$$

$$\int \tau^n P_3^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times$$
$$\times\left(\frac{35n!}{16}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{3\gamma}{2}\Big)^{j+1}} - \right.$$
$$-\frac{315n!}{16}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{7\gamma}{2}\Big)^{j+1}} +$$
$$+\frac{693n!}{16}\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{11\gamma}{2}\Big)^{j+1}} -$$
$$\left. -\frac{429n!}{16}\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{15\gamma}{2}\Big)^{j+1}}\right);$$

$$\int \tau^n P_4^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp\Big(-\frac{3\gamma\tau}{2}\Big)\times$$
$$\times\left(\frac{315n!}{128}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\dfrac{3\gamma}{2}\Big)^{j+1}} - \right.$$

$$-\frac{1155n!}{32}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{7\gamma}{2}\right)^{j+1}}+$$

$$+\frac{9009n!}{64}\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{11\gamma}{2}\right)^{j+1}}-$$

$$-\frac{6435n!}{32}\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{15\gamma}{2}\right)^{j+1}}+$$

$$\left.+\frac{12155n!}{128}\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{19\gamma}{2}\right)^{j+1}}\right);$$

$$\int\tau^n P_5^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau=-\exp\Bigl(-\frac{3\gamma\tau}{2}\Bigr)\times$$

$$\times\left(\frac{693n!}{256}\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{3\gamma}{2}\right)^{j+1}}-\right.$$

$$-\frac{15015n!}{256}\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{7\gamma}{2}\right)^{j+1}}+$$

$$+\frac{45045n!}{128}\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{11\gamma}{2}\right)^{j+1}}-$$

$$-\frac{109395n!}{128}\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{15\gamma}{2}\right)^{j+1}}+$$

$$+\frac{230945n!}{256}\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{19\gamma}{2}\right)^{j+1}}-$$

$$\left.-\frac{88179n!}{256}\exp(-10\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\left(\frac{23\gamma}{2}\right)^{j+1}}\right).$$

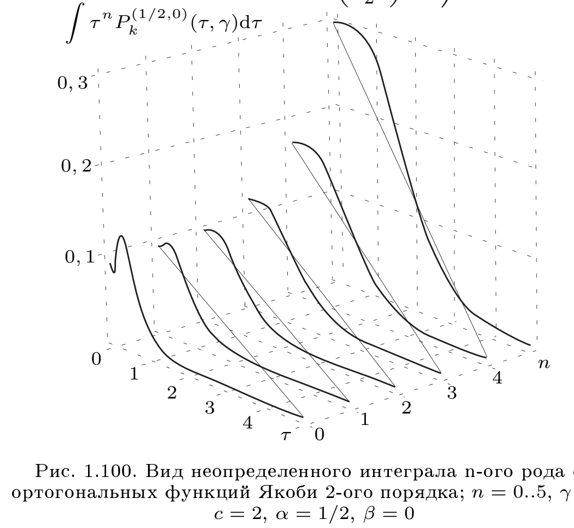


Рис. 1.100. Вид неопределенного интеграла n-ого рода от ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 2$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[1.101]
$$\int P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}\times$$
$$\times\frac{(-1)^s}{\gamma(s+1)}\exp\bigl(-(s+1)\gamma\tau\bigr).$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int P_0^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{\gamma}\exp(-\gamma\tau);$$

$$\int P_1^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{2\gamma}\exp(-\gamma\tau)(4-3\exp(-\gamma\tau));$$

$$\int P_2^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{3\gamma}\exp(-\gamma\tau)(9-18\exp(-\gamma\tau)+10\times$$
$$\times\exp(-2\gamma\tau));$$

$$\int P_3^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{4\gamma}\exp(-\gamma\tau)(16-60\exp(-\gamma\tau)+80\times$$
$$\times\exp(-2\gamma\tau)-35\exp(-3\gamma\tau));$$

$$\int P_4^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{5\gamma}\exp(-\gamma\tau)(25-150\exp(-\gamma\tau)+350\times$$
$$\times\exp(-2\gamma\tau)-350\exp(-3\gamma\tau)+126\exp(-4\gamma\tau));$$

$$\int P_5^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{6\gamma}\exp(-\gamma\tau)(36-315\exp(-\gamma\tau)+1120\times$$
$$\times\exp(-2\gamma\tau)-1890\exp(-3\gamma\tau)+1512\exp(-4\gamma\tau)-462\times$$
$$\times\exp(-5\gamma\tau)).$$

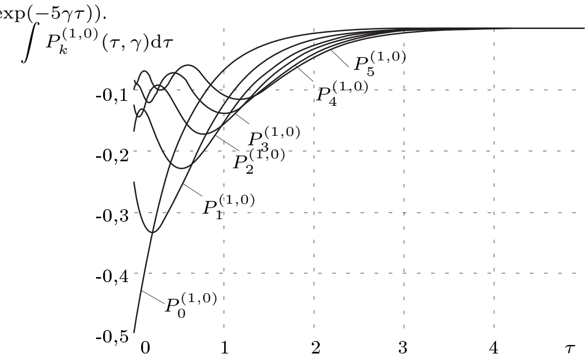


Рис. 1.101. Вид неопределенного интеграла от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 1$, $\alpha = 1$, $\beta = 0$

[1.102]
$$\int\tau P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}\times$$
$$\times(-1)^s\exp\bigl(-(s+1)\gamma\tau\bigr)\left(\frac{\tau}{\gamma(s+1)}+\frac{1}{\gamma^2(s+1)^2}\right).$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int\tau P_0^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{\gamma^2}\exp(-\gamma\tau)(\gamma\tau+1);$$

$$\int\tau P_1^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{4\gamma^2}\exp(-\gamma\tau)\bigl(8-3\exp(-\gamma\tau)+\gamma\tau(8-$$
$$-6\exp(-\gamma\tau))\bigr);$$

$$\int\tau P_2^{(1,0)}(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{9\gamma^2}\exp(-\gamma\tau)\bigl(27-27\exp(-\gamma\tau)+10\times$$
$$\times\exp(-2\gamma\tau)+\gamma\tau(27-54\exp(-\gamma\tau)+30\exp(-2\gamma\tau))\bigr);$$

$$\int\tau P_3(1,0)(\tau,\gamma)\mathrm{d}\tau=-\frac{1}{48\gamma^2}\exp(-\gamma\tau)\bigl(192-360\exp(-\gamma\tau)+$$
$$+320\exp(-2\gamma\tau)-105\exp(-3\gamma\tau)+\gamma\tau(192-720\exp(-\gamma\tau)+$$
$$+960\exp(-2\gamma\tau)-420\exp(-3\gamma\tau))\bigr);$$

$$\int \tau P_4^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{150\gamma^2}\exp(-\gamma\tau)\big(750 - 2250 \times \exp(-\gamma\tau) + 3500\exp(-2\gamma\tau) - 2625\exp(-3\gamma\tau) + 756 \times \exp(-4\gamma\tau) + \gamma\tau(750 - 4500\exp(-\gamma\tau) + 10500\exp(-2\gamma\tau) - 10500\exp(-3\gamma\tau) + 3780\exp(-4\gamma\tau))\big);$$

$$\int \tau P_5^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{180\gamma^2}\exp(-\gamma\tau)\big(1080 - 4725 \times \exp(-\gamma\tau) + 11200\exp(-2\gamma\tau) - 14175\exp(-3\gamma\tau) + 9072\exp(-4\gamma\tau) - 2310\exp(-5\gamma\tau) + \gamma\tau(1080 - 9450 \times \exp(-\gamma\tau) + 33600\exp(-2\gamma\tau) - 56700\exp(-3\gamma\tau) + 45360 \times \exp(-4\gamma\tau) - 13860\exp(-5\gamma\tau))\big).$$

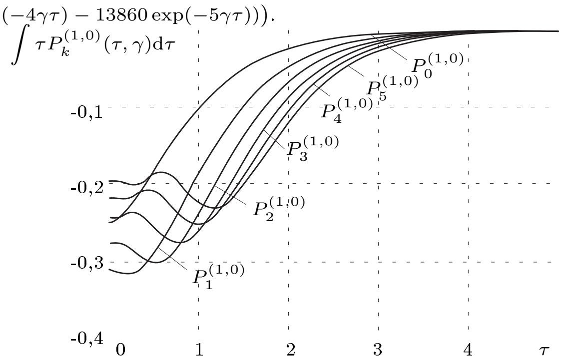


Рис. 1.102. Вид неопределенного интеграла 1-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 1$, $\alpha = 1$, $\beta = 0$

[1.103]
$$\int \tau^2 P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}\times (-1)^s\exp\big(-(s+1)\gamma\tau\big)\left(\frac{\tau^2}{\gamma(s+1)} + \frac{2\tau}{\gamma^2(s+1)^2} + \frac{2}{\gamma^3(s+1)^3}\right).$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$$\int \tau^2 P_0^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^3}\exp(-\gamma\tau)(\gamma^2\tau^2 + 2\gamma\tau + 2);$$

$$\int \tau^2 P_1^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{4\gamma^3}\exp(-\gamma\tau)\big(16 - 3\exp(-\gamma\tau) + \gamma\tau(16 - 6\exp(-\gamma\tau) + \gamma^2\tau^2(8 - 6\exp(-\gamma\tau)))\big);$$

$$\int \tau^2 P_2^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{27\gamma^3}\exp(-\gamma\tau)\big(162 - 81\exp(-\gamma\tau) + 20\exp(-2\gamma\tau) + \gamma\tau(162 - 162\exp(-\gamma\tau) + 60\exp(-2\gamma\tau)) + \gamma^2\tau^2(81 - 162\exp(-\gamma\tau) + 90\exp(-2\gamma\tau))\big);$$

$$\int \tau^2 P_3^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{288\gamma^3}\exp(-\gamma\tau)\big(2304 - 2160 \times \exp(-\gamma\tau) + 1280\exp(-2\gamma\tau) - 315\exp(-3\gamma\tau) + \gamma\tau(2304 - 4320\exp(-\gamma\tau) + 3840\exp(-2\gamma\tau) - 1260\exp(-3\gamma\tau)) + \gamma^2\tau^2(1152 - 4320\exp(-\gamma\tau) + 5760\exp(-2\gamma\tau) - 2520 \times \exp(-3\gamma\tau))\big);$$

$$\int \tau^2 P_4^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{4500\gamma^3}\exp(-\gamma\tau)\big(45000 - 67500 \times \exp(-\gamma\tau) + 70000\exp(-2\gamma\tau) - 39375\exp(-3\gamma\tau) + 9072 \times \exp(-4\gamma\tau) + \gamma\tau(45000 - 135000\exp(-\gamma\tau) + 210000 \times \exp(-2\gamma\tau) - 157500\exp(-3\gamma\tau) + 45360\exp(-4\gamma\tau)) + \gamma^2\tau^2 \times (22500 - 135000\exp(-\gamma\tau) + 315000\exp(-2\gamma\tau) - 315000 \times \exp(-3\gamma\tau) + 113400\exp(-4\gamma\tau))\big);$$

$$\int \tau^2 P_5^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{5400\gamma^3}\exp(-\gamma\tau)\big(64800 - 141750 \times \exp(-\gamma\tau) + 224000\exp(-2\gamma\tau) - 212625\exp(-3\gamma\tau) + 108864 \times \exp(-4\gamma\tau) - 23100\exp(-5\gamma\tau) + \gamma\tau(64800 - 283500 \times \exp(-\gamma\tau) + 672000\exp(-2\gamma\tau) - 850500\exp(-3\gamma\tau) + 544320\exp(-4\gamma\tau) - 138600\exp(-5\gamma\tau)) + \gamma^2\tau^2(32400 - 283500\exp(-\gamma\tau) + 1008000\exp(-2\gamma\tau) - 1701000\exp(-3\gamma\tau) + 1360800\exp(-4\gamma\tau) - 415800\exp(-5\gamma\tau))\big).$$

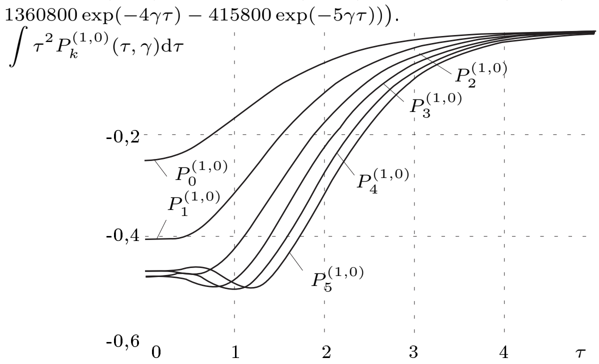


Рис. 1.103. Вид неопределенного интеграла 2-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 1$, $\alpha = 1$, $\beta = 0$

[1.104]
$$\int \tau^3 P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}\times (-1)^s\exp\big(-(s+1)\gamma\tau\big)\left(\frac{\tau^3}{\gamma(s+1)} + \frac{3\tau^2}{\gamma^2(s+1)^2} + \frac{6\tau}{\gamma^3(s+1)^3} + \frac{6}{\gamma^4(s+1)^4}\right).$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$$\int \tau^3 P_0^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^4}\exp(-\gamma\tau)(\gamma^3\tau^3 + 3\gamma^2\tau^2 + 6\gamma\tau + 6);$$

$$\int \tau^3 P_1^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{8\gamma^4}\exp(-\gamma\tau)\big(96 - 9\exp(-\gamma\tau) + \gamma\tau(96 - 18\exp(-\gamma\tau) + \gamma^2\tau^2(48 - 18\exp(-\gamma\tau)) + \gamma^3\tau^3(16 - 12\exp(-\gamma\tau)))\big);$$

$$\int \tau^3 P_2^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{54\gamma^4}\exp(-\gamma\tau)\big(972 - 243\exp(-\gamma\tau) + 40\exp(-2\gamma\tau) + \gamma\tau(972 - 486\exp(-\gamma\tau) + 120\exp(-2\gamma\tau)) + \gamma^2\tau^2(486 - 486\exp(-\gamma\tau) + 180\exp(-2\gamma\tau)) + \gamma^3\tau^3(162 - 324\exp(-\gamma\tau) + 180\exp(-2\gamma\tau))\big);$$

$$\int \tau^3 P_3^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1152\gamma^4}\exp(-\gamma\tau)\big(27648 - 12960 \times \exp(-\gamma\tau) + 5120\exp(-2\gamma\tau) - 945\exp(-3\gamma\tau) + \gamma\tau(27648 - 25920\exp(-\gamma\tau) + 15360\exp(-2\gamma\tau) - 3780\exp(-3\gamma\tau)) + \gamma^2\tau^2(13824 - 25920\exp(-\gamma\tau) + 23040\exp(-2\gamma\tau) - 7560 \times \exp(-3\gamma\tau)) + \gamma^3\tau^3(4608 - 17280\exp(-\gamma\tau) + 23040 \times \exp(-2\gamma\tau) - 10080\exp(-3\gamma\tau))\big);$$

$$\int \tau^3 P_4^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{90000\gamma^4}\exp(-\gamma\tau)\big(2700000 - 2025000\exp(-\gamma\tau) + 1400000\exp(-2\gamma\tau) - 590625 \times \exp(-3\gamma\tau) + 108864\exp(-4\gamma\tau) + \gamma\tau(2700000 - 4050000 \times \exp(-\gamma\tau) + 4200000\exp(-2\gamma\tau) - 2362500\exp(-3\gamma\tau) + 544320\exp(-4\gamma\tau)) + \gamma^2\tau^2(1350000 - 4050000\exp(-\gamma\tau) +$$

$$+\,6300000\exp(-2\gamma\tau) - 4725000\exp(-3\gamma\tau) + 1360800\times$$
$$\times\exp(-4\gamma\tau)\big) + \gamma^3\tau^3(450000 - 2700000\exp(-\gamma\tau) +$$
$$+\,6300000\exp(-2\gamma\tau) - 6300000\exp(-3\gamma\tau) + 2268000\times$$
$$\times\exp(-4\gamma\tau)\big)\big);$$

$$\int \tau^3 P_5^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{108000\gamma^4}\exp(-\gamma\tau)\big(3888000 -$$
$$-\,4252500\exp(-\gamma\tau) + 4480000\exp(-2\gamma\tau) - 3189375\times$$
$$\times\exp(-3\gamma\tau) + 1306368\exp(-4\gamma\tau) - 231000\exp(-5\gamma\tau) +$$
$$+\,\gamma\tau(3888000 - 8505000\exp(-\gamma\tau) + 13440000\exp(-2\gamma\tau) -$$
$$-\,12757500\exp(-3\gamma\tau) + 6531840\exp(-4\gamma\tau) - 1386000\times$$
$$\times\exp(-5\gamma\tau)) + \gamma^2\tau^2(1944000 - 8505000\exp(-\gamma\tau) +$$
$$+\,20160000\exp(-2\gamma\tau) - 25515000\exp(-3\gamma\tau) + 16329600\times$$
$$\times\exp(-4\gamma\tau) - 4158000\exp(-5\gamma\tau)) + \gamma^3\tau^3(648000 -$$
$$-\,5670000\exp(-\gamma\tau) + 20160000\exp(-2\gamma\tau) - 34020000\times$$
$$\times\exp(-3\gamma\tau) + 27216000\exp(-4\gamma\tau) - 8316000\exp(-5\gamma\tau))\big).$$

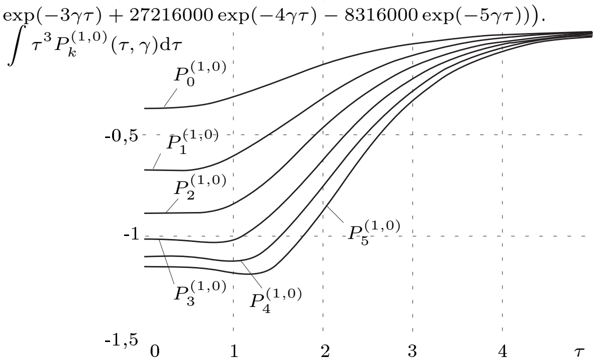


Рис. 1.104. Вид неопределенного интеграла 3-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 1$, $\alpha = 1$, $\beta = 0$

[1.105]
$$\int \tau^n P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}\times$$
$$\times(-1)^s\exp\big(-(s+1)\gamma\tau\big)\sum_{j=0}^{n}\frac{n!\tau^{n-j}}{(n-j)!\big(\gamma(s+1)\big)^{j+1}}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n P_0^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp(-\gamma\tau)\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n P_1^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(2n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(\gamma\big)^{j+1}} -\right.$$
$$\left.-\ n!\exp(-\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(2\gamma\big)^{j+1}}\right);$$

$$\int \tau^n P_2^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(3n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(\gamma\big)^{j+1}} -\right.$$
$$-\ 12n!\exp(-\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(2\gamma\big)^{j+1}} +$$
$$\left.+\ 10n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(3\gamma\big)^{j+1}}\right);$$

$$\int \tau^n P_3^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(4n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(\gamma\big)^{j+1}} -\right.$$
$$-\ 30n!\exp(-\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(2\gamma\big)^{j+1}} +$$
$$+\ 60n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(3\gamma\big)^{j+1}} -$$
$$\left.-\ 35n!\exp(-3\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(4\gamma\big)^{j+1}}\right);$$

$$\int \tau^n P_4^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(5n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(\gamma\big)^{j+1}} -\right.$$
$$-\ 60n!\exp(-\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(2\gamma\big)^{j+1}} +$$
$$+\ 210n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(3\gamma\big)^{j+1}} -$$
$$-\ 280n!\exp(-3\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(4\gamma\big)^{j+1}} +$$
$$\left.+\ 126n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(5\gamma\big)^{j+1}}\right);$$

$$\int \tau^n P_5^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(6n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(\gamma\big)^{j+1}} -\right.$$
$$-\ 105n!\exp(-\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(2\gamma\big)^{j+1}} +$$
$$+\ 560n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(3\gamma\big)^{j+1}} -$$
$$-\ 1260n!\exp(-3\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(4\gamma\big)^{j+1}} +$$
$$+\ 1260n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(5\gamma\big)^{j+1}} -$$
$$\left.-\ 462n!\exp(-5\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(6\gamma\big)^{j+1}}\right).$$

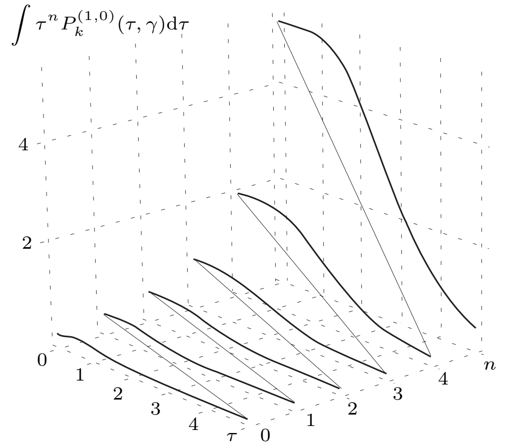


Рис. 1.105. Вид неопределенного интеграла n-ого рода от ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 2$, $c = 1$, $\alpha = 1$, $\beta = 0$

[1.106]
$$\int P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}\times \times \frac{(-1)^s}{\gamma(2s+3)}\exp\bigl(-(2s+3)\gamma\tau\bigr).$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int P_0^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3\gamma}\exp(-3\gamma\tau);$$

$$\int P_1^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{5\gamma}\exp(-3\gamma\tau)(5-4\exp(-2\gamma\tau));$$

$$\int P_2^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{7\gamma}\exp(-3\gamma\tau)(14-28\exp(-2\gamma\tau)+15\times \times\exp(-4\gamma\tau));$$

$$\int P_3^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{9\gamma}\exp(-3\gamma\tau)(30-56\exp(-2\gamma\tau)+108\times \times\exp(-4\gamma\tau)-135\exp(-6\gamma\tau));$$

$$\int P_4^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{11\gamma}\exp(-3\gamma\tau)(55-308\exp(-2\gamma\tau)+ +660\exp(-4\gamma\tau)-616\exp(-6\gamma\tau)+210\exp(-8\gamma\tau));$$

$$\int P_5^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{13\gamma}\exp(-3\gamma\tau)(91-728\exp(-2\gamma\tau)+ +2340\exp(-4\gamma\tau)-3640\exp(-6\gamma\tau)+2730\exp(-8\gamma\tau)-792\times \times\exp(-10\gamma\tau)).$$

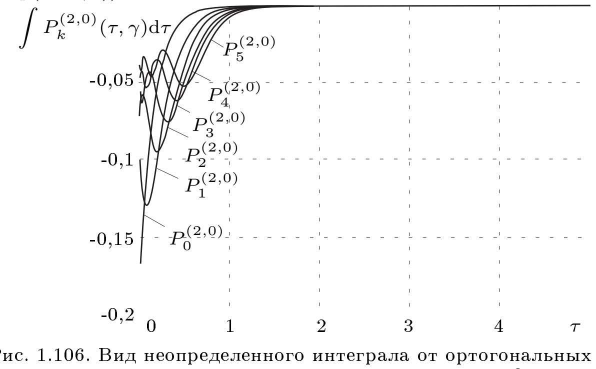


Рис. 1.106. Вид неопределенного интеграла от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 2$, $\beta = 0$

[1.107]
$$\int \tau P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}\times \times(-1)^s\exp\bigl(-(2s+3)\gamma\tau\bigr)\left(\frac{\tau}{\gamma(2s+3)}+\frac{1}{\gamma^2(2s+3)^2}\right).$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int \tau P_0^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{9\gamma^2}\exp(-3\gamma\tau)(3\gamma\tau+1);$$

$$\int \tau P_1^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{75\gamma^2}\exp(-3\gamma\tau)\bigl(25-12\exp(-2\gamma\tau)+ +\gamma\tau(75-60\exp(-2\gamma\tau))\bigr);$$

$$\int \tau P_2^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{735\gamma^2}\exp(-3\gamma\tau)\bigl(490-588\times \times\exp(-2\gamma\tau)+225\exp(-4\gamma\tau)+\gamma\tau(1470-2940\exp(-2\gamma\tau)+ +1575\exp(-4\gamma\tau))\bigr);$$

$$\int \tau P_3(2,0)(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2835\gamma^2}\exp(-3\gamma\tau)\bigl(3150-6804\times \times\exp(-2\gamma\tau)+6075\exp(-4\gamma\tau)-1960\exp(-6\gamma\tau)+\gamma\tau(9450- -34020\exp(-2\gamma\tau)+42535\exp(-4\gamma\tau)-17640\exp(-6\gamma\tau))\bigr);$$

$$\int \tau P_4^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{38115\gamma^2}\exp(-3\gamma\tau)\bigl(63525-213444\times \times\exp(-2\gamma\tau)+326700\exp(-4\gamma\tau)-237160\exp(-6\gamma\tau)+ +66150\exp(-8\gamma\tau)+\gamma\tau(190575-1067220\exp(-2\gamma\tau)+ +2286900\exp(-4\gamma\tau)-2134440\exp(-6\gamma\tau)+727650\exp(-8\gamma\tau))\bigr);$$

$$\int \tau P_5^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{585585\gamma^2}\exp(-3\gamma\tau)\bigl(1366365- -6558552\exp(-2\gamma\tau)+15057900\exp(-4\gamma\tau)-18218200\times \times\exp(-6\gamma\tau)+11179350\exp(-8\gamma\tau)-2744280\exp(-10\gamma\tau)+ +\gamma\tau(4099095-32792760\exp(-2\gamma\tau)+105405300\exp(-4\gamma\tau)- +163963800\exp(-6\gamma\tau)+122972850\exp(-8\gamma\tau)-35675640\times \times\exp(-10\gamma\tau))\bigr).$$

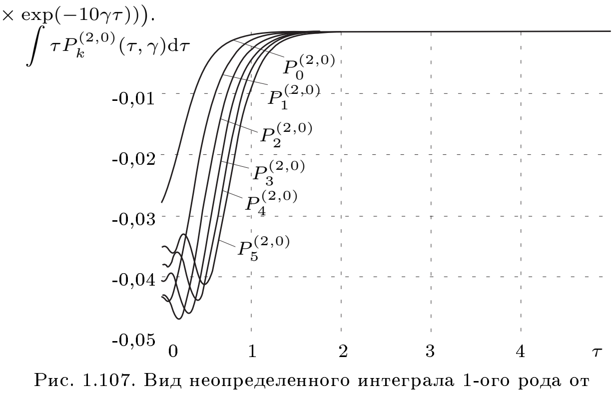


Рис. 1.107. Вид неопределенного интеграла 1-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 2$, $\beta = 0$

[1.108]
$$\int \tau^2 P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}\times \times(-1)^s\exp\bigl(-(2s+3)\gamma\tau\bigr)\left(\frac{\tau^2}{\gamma(2s+3)}+ +\frac{2\tau}{\gamma^2(2s+3)^2}+\frac{2}{\gamma^3(2s+3)^3}\right).$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$$\int \tau^2 P_0^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{27\gamma^3}\exp(-3\gamma\tau)(9\gamma^2\tau^2+6\gamma\tau+2);$$

$$\int \tau^2 P_1^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1125\gamma^3}\exp(-3\gamma\tau)\big(250-72\times \exp(-2\gamma\tau)+\gamma\tau(750-360\exp(-2\gamma\tau)+\gamma^2\tau^2(1125-900\times \exp(-2\gamma\tau)))\big);$$

$$\int \tau^2 P_2^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{77175\gamma^3}\exp(-3\gamma\tau)\big(34300-24696\times \exp(-2\gamma\tau)+6750\exp(-4\gamma\tau)+\gamma\tau(102900-123480\times \exp(-2\gamma\tau)+47250\exp(-4\gamma\tau))+\gamma^2\tau^2(154350-308700\times \exp(-2\gamma\tau)+165375\exp(-4\gamma\tau))\big);$$

$$\int \tau^2 P_3^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{893025\gamma^3}\exp(-3\gamma\tau)\big(661500-857304\exp(-2\gamma\tau)+546750\exp(-4\gamma\tau)-137200\exp(-6\gamma\tau)+\gamma\tau(1984500-4286520\exp(-2\gamma\tau)+3827250\exp(-4\gamma\tau)-1234800\exp(-6\gamma\tau))+\gamma^2\tau^2(2976750-10716300\exp(-2\gamma\tau)+13395375\exp(-4\gamma\tau)-5556600\exp(-6\gamma\tau))\big);$$

$$\int \tau^2 P_4^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{132068475\gamma^3}\exp(-3\gamma\tau)\big(146742750-295833384\exp(-2\gamma\tau)+323433000\exp(-4\gamma\tau)-182613200\times \exp(-6\gamma\tau)+41674500\exp(-8\gamma\tau)+\gamma\tau(440228250-1479166920\exp(-2\gamma\tau)+2264031000\exp(-4\gamma\tau)-1643518800\exp(-6\gamma\tau)+458419500\exp(-8\gamma\tau))+\gamma^2\tau^2(660342375-3697917300\exp(-2\gamma\tau)+7924108500\times \exp(-4\gamma\tau)-7395834600\exp(-6\gamma\tau)+2521307250\times \exp(-8\gamma\tau))\big);$$

$$\int \tau^2 P_5^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{26377676325\gamma^3}\exp(-3\gamma\tau)\times \big(41031940950-118171989936\exp(-2\gamma\tau)+193795173000\times \exp(-4\gamma\tau)-182364182000\exp(-6\gamma\tau)+91558876500\times \exp(-8\gamma\tau)-19017860400\exp(-10\gamma\tau)+\gamma\tau(123095822850-590859949680\exp(-2\gamma\tau)+1356566211000\exp(-4\gamma\tau)-1641277638000\exp(-6\gamma\tau)+1007147641500\exp(-8\gamma\tau)-247232185200\exp(-10\gamma\tau))+\gamma^2\tau^2(184643734275-1477149874200\exp(-2\gamma\tau)+4747981738500\exp(-4\gamma\tau)-7385749371000\exp(-6\gamma\tau)+5539312028250\exp(-8\gamma\tau)-1607009203800\exp(-10\gamma\tau))\big).$$

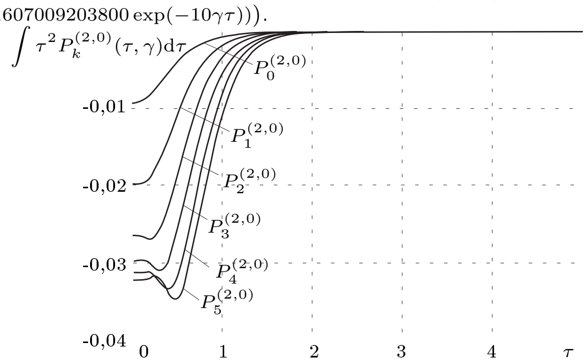


Рис. 1.108. Вид неопределенного интеграла 2-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 2$, $\beta = 0$

[1.109]
$$\int \tau^3 P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}\times (-1)^s\exp\big(-(2s+3)\gamma\tau\big)\left(\frac{\tau^3}{\gamma(2s+3)}+\frac{3\tau^2}{\gamma^2(2s+3)^2}+\frac{6\tau}{\gamma^3(2s+3)^3}+\frac{6}{\gamma^4(2s+3)^4}\right).$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$$\int \tau^3 P_0^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{27\gamma^4}\exp(-3\gamma\tau)(9\gamma^3\tau^3+9\gamma^2\tau^2+6\times \gamma\tau+2);$$

$$\int \tau^3 P_1^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{5625\gamma^4}\exp(-3\gamma\tau)\big(1250-216\times \exp(-2\gamma\tau)+\gamma\tau(3750-1080\exp(-2\gamma\tau)+\gamma^2\tau^2(5625-2700\times \exp(-2\gamma\tau))+\gamma^3\tau^3(5625-4500\exp(-2\gamma\tau)))\big);$$

$$\int \tau^3 P_2^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2701125\gamma^4}\exp(-3\gamma\tau)\big(1200500-518616\exp(-2\gamma\tau)+101250\exp(-4\gamma\tau)+\gamma\tau(3601500-2593080\exp(-2\gamma\tau)+708750\exp(-4\gamma\tau))+\gamma^2\tau^2(5402250-6482700\exp(-2\gamma\tau)+2480625\exp(-4\gamma\tau))+\gamma^3\tau^3(5402250-10804500\exp(-2\gamma\tau)+5788125\exp(-4\gamma\tau))\big);$$

$$\int \tau^3 P_3^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{93767625\gamma^4}\exp(-3\gamma\tau)\big(69457500-54010152\exp(-2\gamma\tau)+24603750\exp(-4\gamma\tau)-4802000\times \exp(-6\gamma\tau)+\gamma\tau(208372500-270050760\exp(-2\gamma\tau)+172226250\exp(-4\gamma\tau)-43218000\exp(-6\gamma\tau))+\gamma^2\tau^2(312558750-675126900\exp(-2\gamma\tau)+602791875\times \exp(-4\gamma\tau)-194481000\exp(-6\gamma\tau))+\gamma^3\tau^3(312558750-1125211500\exp(-2\gamma\tau)+1406514375\exp(-4\gamma\tau)-583443000\exp(-6\gamma\tau))\big);$$

$$\int \tau^3 P_4^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{152539088625\gamma^4}\exp(-3\gamma\tau)\times \big(169487876250-205012535112\exp(-2\gamma\tau)+160099335000\times \exp(-4\gamma\tau)-70306082000\exp(-6\gamma\tau)+13127467500\times \exp(-8\gamma\tau)+\gamma\tau(508463628750-1025062675560\exp(-2\gamma\tau)+1120695345000\exp(-4\gamma\tau)-632754738000\exp(-6\gamma\tau)+144402142500\exp(-8\gamma\tau))+\gamma^2\tau^2(762695443125-2562656688900\exp(-2\gamma\tau)+3922433707500\exp(-4\gamma\tau)-2847396321000\exp(-6\gamma\tau)+794211783750\exp(-8\gamma\tau))+\gamma^3\tau^3(762695443125-4271094481500\exp(-2\gamma\tau)+9152345317500\exp(-4\gamma\tau)-8542188963000\exp(-6\gamma\tau)+2912109873750\exp(-8\gamma\tau))\big);$$

$$\int \tau^3 P_5^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{396060810019875\gamma^4}\exp(-3\gamma\tau)\times \big(616094593364250-1064611457333424\exp(-2\gamma\tau)+1247071938255000\exp(-4\gamma\tau)-912732730910000\times \exp(-6\gamma\tau)+374933599267500\exp(-8\gamma\tau)-65896886286000\exp(-10\gamma\tau)+\gamma\tau(1848283780092750-5323057286667120\exp(-2\gamma\tau)+8729503567785000\times \exp(-4\gamma\tau)-8214594578190000\exp(-6\gamma\tau)+4124269591942500\exp(-8\gamma\tau)-856659521718000\times \exp(-10\gamma\tau))+\gamma^2\tau^2(2772425670139125-13307643216667800\exp(-2\gamma\tau)+30553262487247500\times \exp(-4\gamma\tau)-36965675601855000\exp(-6\gamma\tau)+22683482755683750\exp(-8\gamma\tau)-5568286891167000\times \exp(-10\gamma\tau))+\gamma^3\tau^3(2772425670139125-22179405361113000\exp(-2\gamma\tau)+71290945803577500\times \exp(-4\gamma\tau)-110897026805565000\exp(-6\gamma\tau)+$$

$+ 83172770104173750\exp(-8\gamma\tau) - 24129243195057000\times$
$\times\exp(-10\gamma\tau)))$.

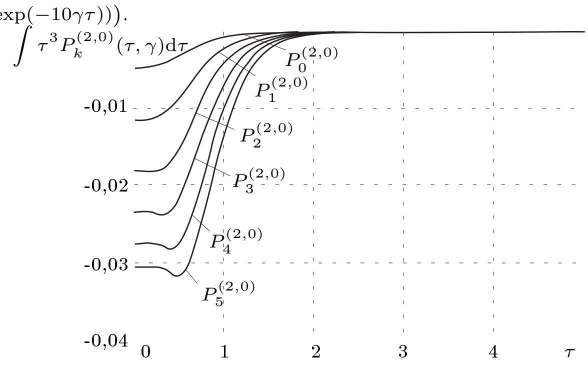


Рис. 1.109. Вид неопределенного интеграла 3-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma=2$, $c=2$, $\alpha=2$, $\beta=0$

[1.110]
$$\int \tau^n P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}\times$$
$$\times(-1)^s\exp\big(-(2s+3)\gamma\tau\big)\sum_{j=0}^{n}\frac{n!\tau^{n-j}}{(n-j)!\Big(\gamma(2s+3)\Big)^{j+1}}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n P_0^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp(-\gamma\tau)\sum_{j=0}^{n}\Big(\frac{2}{3\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n P_1^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-3\gamma\tau)\times$$
$$\times\left(3n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} - \right.$$
$$\left.- n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n P_2^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-3\gamma\tau)\times$$
$$\times\left(6n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} - \right.$$
$$- 20n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} +$$
$$\left.+ 15n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n P_3^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-3\gamma\tau)\times$$
$$\times\left(10n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} - \right.$$
$$- 60n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} +$$
$$+ 105n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}} -$$
$$\left.- 56n!\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(9\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n P_4^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-3\gamma\tau)\times$$
$$\times\left(15n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} - \right.$$
$$- 140n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} +$$
$$+ 420n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}} -$$
$$- 504n!\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(9\gamma\Big)^{j+1}} +$$
$$\left.+ 210n!\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(11\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n P_5^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-3\gamma\tau)\times$$
$$\times\left(21n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} - \right.$$
$$- 280n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} +$$
$$+ 1260n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}} -$$
$$- 2520n!\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(9\gamma\Big)^{j+1}} +$$
$$- 2310n!\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(11\gamma\Big)^{j+1}} -$$
$$\left.- 792n!\exp(-10\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(13\gamma\Big)^{j+1}}\right).$$

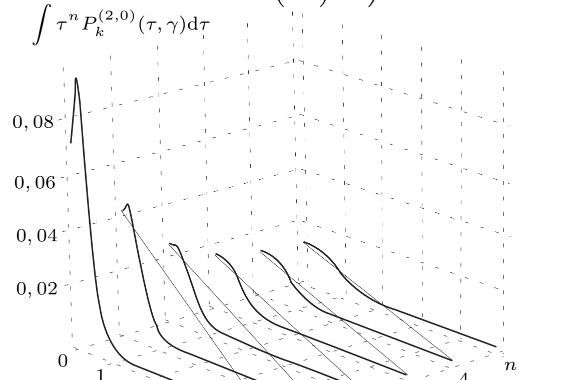


Рис. 1.110. Вид неопределенного интеграла n-ого рода от ортогональных функций Якоби 2-ого порядка; $n=0..5$, $\gamma=2$, $c=2$, $\alpha=2$, $\beta=0$

[1.111]
$$\int P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}\times \frac{2(-1)^s}{c\gamma(2s+\alpha+1)}\exp\bigl(-(2s+\alpha+1)c\gamma\tau/2\bigr).$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int P_0^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c\gamma(\alpha+1)}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr);$$

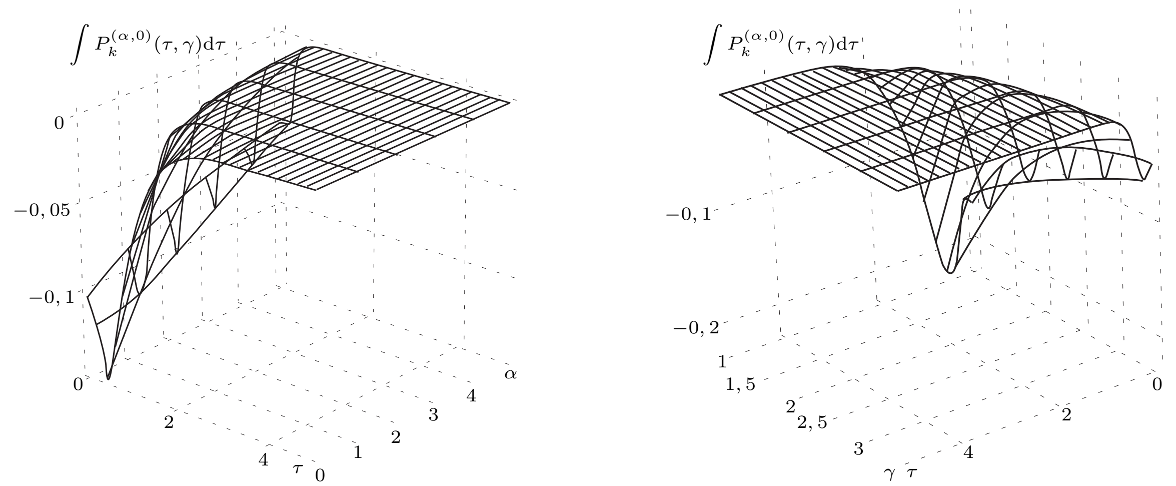
$$\int P_1^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c\gamma(\alpha+3)}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\bigl(\alpha+3-(\alpha+2)\exp(-c\gamma\tau)\bigr);$$

$$\int P_2^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{c\gamma(\alpha+5)}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\bigl(\alpha^2+7\alpha+10-2(\alpha^2+7\alpha+10)\exp(-c\gamma\tau)+(\alpha^2+7\alpha+12)\exp(-2c\gamma\tau)\bigr);$$

$$\int P_3^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3c\gamma(\alpha+7)}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\bigl(\alpha^3+12\alpha^2+41\alpha+42-(3\alpha^3+39\alpha^2+150\alpha+168)\exp(-c\gamma\tau)+(3\alpha^3+42\alpha^2+183\alpha+252)\exp(-2c\gamma\tau)-(\alpha^3+15\alpha^2+74\alpha+120)\exp(-3c\gamma\tau)\bigr);$$

$$\int P_4^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{12c\gamma(\alpha+9)}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\bigl(\alpha^4+18\alpha^3+107\alpha^2+258\alpha+216-(4\alpha^4+80\alpha^3+548\alpha^2+1528\alpha+1440)\exp(-c\gamma\tau)+(6\alpha^4+132\alpha^3+1026\alpha^2+3348\alpha+3888)\exp(-2c\gamma\tau)-(4\alpha^4+96\alpha^3+836\alpha^2+3144\alpha+4320)\exp(-3c\gamma\tau)+(\alpha^4+26\alpha^3+251\alpha^2+1066\alpha+1680)\times\exp(-4c\gamma\tau)\bigr);$$

$$\int P_5^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{60c\gamma(\alpha+11)}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\bigl(\alpha^5+25\alpha^4+225\alpha^3+935\alpha^2+1814\alpha+1320-(5\alpha^5+140\alpha^4+1455\alpha^3+7060\alpha^2+15940\alpha+13200)\exp(-c\gamma\tau)+(10\alpha^5+310\alpha^4+3650\alpha^3+20450\alpha^2+54540\alpha+55440)\exp(-2c\gamma\tau)-(10\alpha^5+340\alpha^4+4470\alpha^3+28460\alpha^2+87920\alpha+105600)\times\exp(-3c\gamma\tau)+(5\alpha^5+185\alpha^4+2685\alpha^3+19135\alpha^2+67030\alpha+92400)\exp(-4c\gamma\tau)-(\alpha^5+40\alpha^4+635\alpha^3+5000\alpha^2+19524\alpha+30240)\exp(-5c\gamma\tau)\bigr).$$



а) б)

Рис. 1.111. Вид неопределенного интеграла от ортогональных функций Якоби 2-ого порядка: а) $\gamma = 2$, $\alpha\in[0;5]$; б) $\gamma\in(1;3,5]$, $\alpha = 1$

[1.112]
$$\int \tau P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}\times (-1)^s\exp\bigl(-(2s+\alpha+1)c\gamma\tau/2\bigr)\times\left(\frac{2\tau}{c\gamma(2s+\alpha+1)}+\frac{4}{c^2\gamma^2(2s+\alpha+1)^2}\right).$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int \tau P_0^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^2\gamma^2(\alpha+1)^2}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times\bigl(\gamma\tau(\alpha+1)+2\bigr);$$

$$\int \tau P_1^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^2\gamma^2(\alpha+1)(\alpha+3)^2}\times\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\bigl(2(\alpha+3)^2-2(\alpha^2+3\alpha+2)\exp(-c\gamma\tau)+\gamma\tau(\alpha^3+7\alpha^2+15\alpha+9-(\alpha^3+6\alpha^2+11\alpha+6)\exp(-c\gamma\tau))\bigr);$$

$$\int \tau P_2^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{c^2\gamma^2(\alpha+1)(\alpha+3)(\alpha+5)^2}\times\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\bigl(2\alpha^4+30\alpha^3+162\alpha^2+370\alpha+300-(4\alpha^4+52\alpha^3+228\alpha^2+380\alpha+200)\exp(-c\gamma\tau)+(2\alpha^4+22\alpha^3+86\alpha^2+138\alpha+72)\exp(-2c\gamma\tau)+\gamma\tau(\alpha^5+16\alpha^4+96\alpha^3+266\alpha^2+335\alpha+150-(\alpha^5+32\alpha^4+192\alpha^3+532\alpha^2+670\alpha+300)\exp(-c\gamma\tau)+(\alpha^5+16\alpha^4+98\alpha^3+284\alpha^2+381\alpha+180)\exp(-2c\gamma\tau))\bigr);$$

$$\int \tau P_3(\alpha,0)(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^2\gamma^2(\alpha+1)^2(\alpha+3)^2(\alpha+5)^2}\times\frac{1}{(\alpha+7)^2}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\Bigl(\binom{\alpha+3}{\alpha}(2\alpha^6+60\alpha^5+734\alpha^4+4680\alpha^3+16382\alpha^2+29820\alpha+22050)-\binom{\alpha+4}{\alpha+1}(6\alpha^6+156\alpha^5+1578\alpha^4+7752\alpha^3+18714\alpha^2+19740\alpha+7350)\exp(-c\gamma\tau)+\binom{\alpha+5}{\alpha+2}(6\alpha^6+132\alpha^5+1098\alpha^4+4344\alpha^3+8538\alpha^2+7812\alpha+2646)\exp(-2c\gamma\tau)-\binom{\alpha+6}{\alpha+3}(2\alpha^6+36\alpha^5+254\alpha^4+888\alpha^3+$$

$$+ 1380\alpha^2 + 1380\alpha + 450) \exp(-3c\gamma\tau)\Big) + \gamma\tau\Big(\binom{\alpha+3}{\alpha}(\alpha^7 + 31 \times$$

$$\times \alpha^6 + 397\alpha^5 + 2707\alpha^4 + 10531\alpha^3 + 23101\alpha^2 + 25935\alpha + 22050) -$$

$$- \binom{\alpha+4}{\alpha+1}(3\alpha^7 + 87\alpha^6 + 1023\alpha^5 + 6243\alpha^4 + 20985\alpha^3 + 37941\alpha^2 +$$

$$+ 33285\alpha + 11025) \exp(-c\gamma\tau) + \binom{\alpha+5}{\alpha+2}(3\alpha^7 + 81\alpha^6 + 879\alpha^5 +$$

$$+ 4917\alpha^4 + 15129\alpha^3 + 25251\alpha^2 + 20853\alpha + 6615) \exp(-2c\gamma\tau) -$$

$$- \binom{\alpha+6}{\alpha+3}(\alpha^7 + 25\alpha^6 + 253\alpha^5 + 1333\alpha^4 + 3907\alpha^3 + 6283\alpha^2 +$$

$$+ 5055\alpha + 1575) \exp(-3c\gamma\tau)\Big);$$

$$\int \tau P_4^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^2\gamma^2(\alpha+1)^2(\alpha+3)^2(\alpha+5)^2} \times$$

$$\times \frac{1}{(\alpha+7)^2(\alpha+9)^2} \exp\big(-c\gamma\tau(\alpha+1)/2\big)\Big(\binom{\alpha+4}{\alpha}(2\alpha^8 + 96\alpha^7 +$$

$$+ 1976\alpha^6 + 22752\alpha^5 + 160076\alpha^4 + 703776\alpha^3 + 1885752\alpha^2 +$$

$$+ 2812320\alpha + 1786050) - \binom{\alpha+5}{\alpha+1}(8\alpha^8 + 352\alpha^7 + 6496\alpha^6 +$$

$$+ 65056\alpha^5 + 381424\alpha^4 + 1312672\alpha^3 + 2504672\alpha^2 + 2308320\alpha +$$

$$+ 793800) \exp(-c\gamma\tau) + \binom{\alpha+6}{\alpha+2}(12\alpha^8 + 480\alpha^7 + 7920\alpha^6 +$$

$$+ 69600\alpha^5 + 351336\alpha^4 + 1026720\alpha^3 + 1669680\alpha^2 + 1360800\alpha +$$

$$+ 428652) \exp(-2c\gamma\tau) - \binom{\alpha+7}{\alpha+3}(8\alpha^8 + 288\alpha^7 + 4256\alpha^6 +$$

$$+ 33504\alpha^5 + 152624\alpha^4 + 408288\alpha^3 + 618912\alpha^2 + 479520\alpha +$$

$$+ 145800) \exp(-3c\gamma\tau) + \binom{\alpha+8}{\alpha+4}(2\alpha^8 + 64\alpha^7 + 856\alpha^6 + 6208\alpha^5 +$$

$$+ 26476\alpha^4 + 67264\alpha^3 + 98072\alpha^2 + 73920\alpha + 22050) \times$$

$$\times \exp(-4c\gamma\tau)\Big) + \gamma\tau\Big(\binom{\alpha+4}{\alpha}(\alpha^9 + 49\alpha^8 + 1036\alpha^7 + 12364\alpha^6 +$$

$$+ 91414\alpha^5 + 431926\alpha^4 + 1294764\alpha^3 + 2349036\alpha^2 + 2299185\alpha +$$

$$+ 893025) - \binom{\alpha+5}{\alpha+1}(4\alpha^9 + 188\alpha^8 + 3776\alpha^7 + 42272\alpha^6 + 288296 \times$$

$$\times \alpha^5 + 1228472\alpha^4 + 3221344\alpha^3 + 4911168\alpha^2 + 23859380\alpha +$$

$$+ 1190700) \exp(-c\gamma\tau) + \binom{\alpha+6}{\alpha+2}(6\alpha^9 + 270\alpha^8 + 5160\alpha^7 +$$

$$+ 54600\alpha^6 + 349668\alpha^5 + 1391700\alpha^4 + 3401640\alpha^3 + 4854600\alpha^2 +$$

$$+ 3616326\alpha + 1071630) \exp(-2c\gamma\tau) - \binom{\alpha+7}{\alpha+3}(4\alpha^9 + 172\alpha^8 +$$

$$+ 3136\alpha^7 + 31648\alpha^6 + 193576\alpha^5 + 738328\alpha^4 + 1738464\alpha^3 +$$

$$+ 2405952\alpha^2 + 1751220\alpha + 510300) \exp(-3c\gamma\tau) + \binom{\alpha+8}{\alpha+4}(\alpha^9 +$$

$$+ 41\alpha^8 + 716\alpha^7 + 6956\alpha^6 + 41174\alpha^5 + 152774\alpha^4 + 351724\alpha^3 +$$

$$+ 478284\alpha^2 + 343665\alpha + 99225) \exp(-4c\gamma\tau)\Big);$$

$$\int \tau P_5^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^2\gamma^2(\alpha+1)^2(\alpha+3)^2(\alpha+5)^2} \times$$

$$\times \frac{1}{(\alpha+7)^2(\alpha+9)^2(\alpha+11)^2} \exp\big(-c\gamma\tau(\alpha+1)/2\big)\Big(\binom{\alpha+5}{\alpha} \times$$

$$\times (2\alpha^{10} + 140\alpha^9 + 4330\alpha^8 + 77840\alpha^7 + 6978440\alpha^6 + 6978440\alpha^5 +$$

$$+ 36738020\alpha^4 + 129455760\alpha^3 + 291833082\alpha^2 + 379583820\alpha +$$

$$+ 216112050) - \binom{\alpha+6}{\alpha+1}(10\alpha^{10} + 660\alpha^9 + 19010\alpha^8 + 313200\alpha^7 +$$

$$+ 3248340\alpha^6 + 21969720\alpha^5 + 96919700\alpha^4 + 270305520\alpha^3 +$$

$$+ 443302690\alpha^2 + 370962900\alpha + 120062250) \exp(-c\gamma\tau) +$$

$$+ \binom{\alpha+7}{\alpha+2}(20\alpha^{10} + 1240\alpha^9 + 33220\alpha^8 + 503200\alpha^7 +$$

$$+ 4734760\alpha^6 + 28629520\alpha^5 + 111281960\alpha^4 + 270544800\alpha^3 +$$

$$+ 387329220\alpha^2 + 290145240\alpha + 86444820) \exp(-2c\gamma\tau) -$$

$$- \binom{\alpha+8}{\alpha+3}(20\alpha^{10} + 1160\alpha^9 + 28900\alpha^8 + 51852240\alpha^7 +$$

$$+ 3511720\alpha^6 + 19550000\alpha^5 + 70171880\alpha^4 + 158746080\alpha^3 +$$

$$+ 213958980\alpha^2 + 153073800\alpha + 44104500) \exp(-3c\gamma\tau) +$$

$$+ \binom{\alpha+9}{\alpha+4}(10\alpha^{10} + 540\alpha^9 + 12530\alpha^8 + 163920\alpha^7 + 1333140\alpha^6 +$$

$$+ 7004520\alpha^5 + 23907380\alpha^4 + 51852240\alpha^3 + 67575010\alpha^2 +$$

$$+ 47147100\alpha + 13340250) \exp(-4c\gamma\tau) - \binom{\alpha+10}{\alpha+5}(2\alpha^{10} + 100 \times$$

$$\times \alpha^9 + 2170\alpha^8 + 26800\alpha^7 + 207556\alpha^6 + 1046680\alpha^5 + 3453380\alpha^4 +$$

$$+ 7287600\alpha^3 + 9296442\alpha^2 + 6384420\alpha + 1786050) \times$$

$$\times \exp(-5c\gamma\tau)\Big) + \gamma\tau\Big(\binom{\alpha+5}{\alpha}(\alpha^{11} + 71\alpha^{10} + 2235\alpha^9 + 488778 \times$$

$$\times \alpha^8 + 488778\alpha^7 + 3939078\alpha^6 + 83096890\alpha^5 + 83096890\alpha^4 +$$

$$+ 210644421\alpha^3 + 335708451\alpha^2 + 297847935\alpha + 108056025) -$$

$$- \binom{\alpha+6}{\alpha+1}(5\alpha^{11} + 345\alpha^{10} + 10495\alpha^9 + 185115\alpha^8 + 2093970\alpha^7 +$$

$$+ 15857370\alpha^6 + 81414430\alpha^5 + 280532310\alpha^4 + 627109625\alpha^3 +$$

$$+ 850435485\alpha^2 + 616475475\alpha + 180093375) \exp(-c\gamma\tau) +$$

$$+ \binom{\alpha+7}{\alpha+2}(10\alpha^{11} + 670\alpha^{10} + 19710\alpha^9 + 334650\alpha^8 + 3625380\alpha^7 +$$

$$+ 26151660\alpha^6 + 127214780\alpha^5 + 413477300\alpha^4 + 870026610\alpha^3 +$$

$$+ 1113395670\alpha^2 + 768585510\alpha + 216112050) \exp(-2c\gamma\tau) -$$

$$- \binom{\alpha+8}{\alpha+3}(5\alpha^{11} + 650\alpha^{10} + 18510\alpha^9 + 303630\alpha^8 + 3173220\alpha^7 +$$

$$+ 22066020\alpha^6 + 103510940\alpha^5 + 324974620\alpha^4 + 662590770\alpha^3 +$$

$$+ 825393330\alpha^2 + 557810550\alpha + 154365750) \exp(-3c\gamma\tau) +$$

$$+ \binom{\alpha+9}{\alpha+4}(5\alpha^{11} + 315\alpha^{10} + 8695\alpha^9 + 138345\alpha^8 + 1404210\alpha^7 +$$

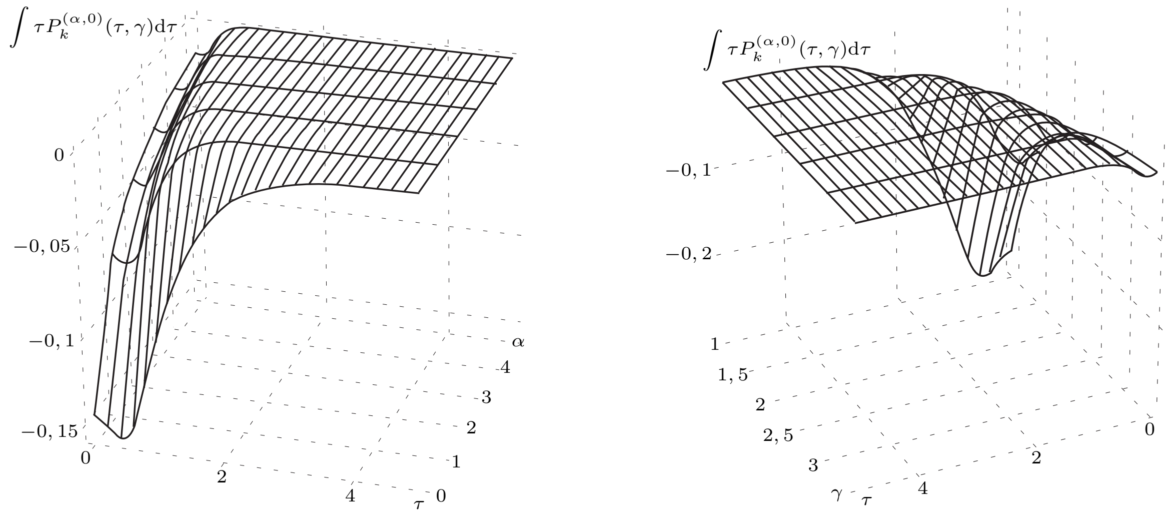
$$+ 9501390\alpha^6 + 43474030\alpha^5 + 133509330\alpha^4 + 267122585\alpha^3 +$$

$$+ 327661095\alpha^2 + 218832075\alpha + 60031125) \exp(-4c\gamma\tau) -$$

$$- \binom{\alpha+10}{\alpha+5}(\alpha^{11} + 61\alpha^{10} + 1635\alpha^9 + 25335\alpha^8 + 251178\alpha^7 +$$

$$+ 1664898\alpha^6 + 7483430\alpha^5 + 22637390\alpha^4 + 44730021\alpha^3 +$$

$$+ 54322641\alpha^2 + 36007335\alpha + 9823275) \exp(-5c\gamma\tau)\Big).$$



а) б)

Рис. 1.112. Вид неопределенного интеграла 1-ого рода от ортогональных функций Якоби 2-ого порядка: а) $\gamma = 2$, $\alpha \in [0;5]$; б) $\gamma \in (1;3,5]$, $\alpha = 1$

[1.113] $$\int \tau^2 P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}\times$$
$$\times(-1)^s\exp\bigl(-(2s+\alpha+1)c\gamma\tau/2\bigr)\left(\frac{2\tau^2}{c\gamma(2s+\alpha+1)}+\right.$$
$$\left.+\frac{8\tau}{c^2\gamma^2(2s+\alpha+1)^2}+\frac{16}{c^3\gamma^3(2s+\alpha+1)^3}\right).$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$$\int \tau^2 P_0^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^3\gamma^3(\alpha+1)^3}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\bigl(8+$$
$$+4c\gamma\tau(\alpha+1)+c^2\gamma^2\tau^2(\alpha+1)^2\bigr);$$

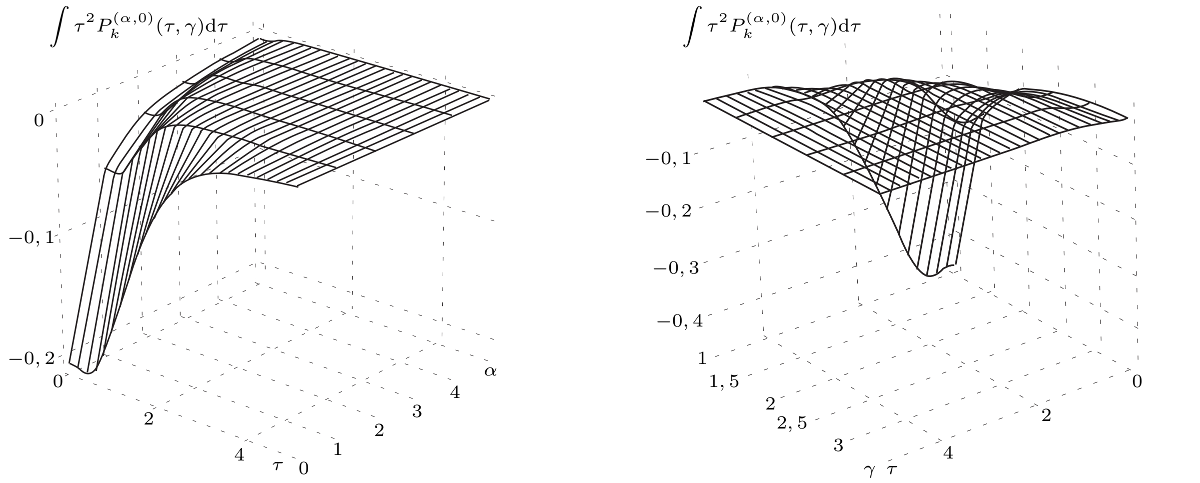
$$\int \tau^2 P_1^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^3\gamma^3(\alpha+1)^2}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\bigl(8+$$
$$+4c\gamma\tau(\alpha+1)+c^2\gamma^2\tau^2(\alpha+1)^2\bigr)+\frac{2}{c^3\gamma^3(\alpha+3)^3}\times$$
$$\times\exp\bigl(-c\gamma\tau(\alpha+3)/2\bigr)\bigr)(\alpha+2)\bigl(8+4c\gamma\tau(\alpha+3)+c^2\gamma^2\tau^2(\alpha+3)^2\bigr);$$

$$\int \tau^2 P_2^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{c^3\gamma^3(\alpha+1)^2}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times$$
$$\times(\alpha+2)\bigl(8+4c\gamma\tau(\alpha+1)+c^2\gamma^2\tau^2(\alpha+1)^2\bigr)+\frac{2}{c^3\gamma^3(\alpha+3)^3}\times$$
$$\times\exp\bigl(-c\gamma\tau(\alpha+3)/2\bigr)(\alpha+2)\bigl(8+4c\gamma\tau(\alpha+3)+c^2\gamma^2\tau^2\times$$
$$\times(\alpha+3)^2\bigr)-\frac{1}{c^3\gamma^3(\alpha+5)^3}\exp\bigl(-c\gamma\tau(\alpha+5)/2\bigr)(\alpha^2+7\alpha+12)\times$$
$$\times\bigl(8+4c\gamma\tau(\alpha+5)+c^2\gamma^2\tau^2(\alpha+5)^2\bigr);$$

$$\int \tau^2 P_3^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^3\gamma^3(\alpha+1)^3}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times$$
$$\times\binom{\alpha+3}{\alpha}\bigl(8+4c\gamma\tau(\alpha+1)+c^2\gamma^2\tau^2(\alpha+1)^2\bigr)+\frac{6}{c^3\gamma^3(\alpha+3)^3}\times$$
$$\times\exp\bigl(-c\gamma\tau(\alpha+3)/2\bigr)\bigr)\binom{\alpha+4}{\alpha+1}\bigl(8+4c\gamma\tau(\alpha+3)+c^2\gamma^2\tau^2\times$$
$$\times(\alpha+3)^2\bigr)-\frac{6}{c^3\gamma^3(\alpha+5)^3}\exp\bigl(-c\gamma\tau(\alpha+5)/2\bigr)\binom{\alpha+5}{\alpha+2}\times$$
$$\times\bigl(8+4c\gamma\tau(\alpha+5)+c^2\gamma^2\tau^2(\alpha+5)^2\bigr)+\frac{2}{c^3\gamma^3(\alpha+7)^3}\times$$
$$\times\exp\bigl(-c\gamma\tau(\alpha+7)/2\bigr)\binom{\alpha+6}{\alpha+3}\bigl(8+4c\gamma\tau(\alpha+7)+c^2\gamma^2\tau^2\times$$
$$\times(\alpha+7)^2\bigr);$$

$$\int \tau^2 P_4^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^3\gamma^3(\alpha+1)^3}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times$$
$$\times\binom{\alpha+4}{\alpha}\bigl(8+4c\gamma\tau(\alpha+1)+c^2\gamma^2\tau^2(\alpha+1)^2\bigr)+\frac{8}{c^3\gamma^3(\alpha+3)^3}\times$$
$$\times\exp\bigl(-c\gamma\tau(\alpha+3)/2\bigr)\binom{\alpha+5}{\alpha+1}\bigl(8+4c\gamma\tau(\alpha+3)+c^2\gamma^2\tau^2\times$$
$$\times(\alpha+3)^2\bigr)-\frac{12}{c^3\gamma^3(\alpha+5)^3}\exp\bigl(-c\gamma\tau(\alpha+5)/2\bigr)\binom{\alpha+6}{\alpha+2}\times$$
$$\times\bigl(8+4c\gamma\tau(\alpha+5)+c^2\gamma^2\tau^2(\alpha+5)^2\bigr)+\frac{8}{c^3\gamma^3(\alpha+7)^3}\times$$
$$\times\exp\bigl(-c\gamma\tau(\alpha+7)/2\bigr)\binom{\alpha+7}{\alpha+3}\bigl(8+4c\gamma\tau(\alpha+7)+c^2\gamma^2\tau^2\times$$
$$\times(\alpha+7)^2\bigr)-\frac{2}{c^3\gamma^3(\alpha+9)^3}\exp\bigl(-c\gamma\tau(\alpha+9)/2\bigr)\binom{\alpha+8}{\alpha+4}\times$$
$$\times\bigl(8+4c\gamma\tau(\alpha+9)+c^2\gamma^2\tau^2(\alpha+9)^2\bigr);$$

$$\int \tau^2 P_5^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^3\gamma^3(\alpha+1)^3}\exp\bigl(-c\gamma\tau(\alpha+1)/2)\bigr)\times$$
$$\times\binom{\alpha+5}{\alpha}\bigl(8+4c\gamma\tau(\alpha+1)+c^2\gamma^2\tau^2(\alpha+1)^2\bigr)+\frac{10}{c^3\gamma^3(\alpha+3)^3}\times$$
$$\times\exp\bigl(-c\gamma\tau(\alpha+3)/2\bigr)\binom{\alpha+6}{\alpha+1}\bigl(8+4c\gamma\tau(\alpha+3)+c^2\gamma^2\tau^2\times$$
$$\times(\alpha+3)^2\bigr)-\frac{20}{c^3\gamma^3(\alpha+5)^3}\exp\bigl(-c\gamma\tau(\alpha+5)/2\bigr)\binom{\alpha+7}{\alpha+2}\times$$
$$\times\bigl(8+4c\gamma\tau(\alpha+5)+c^2\gamma^2\tau^2(\alpha+5)^2\bigr)+\frac{20}{c^3\gamma^3(\alpha+7)^3}\times$$
$$\times\exp\bigl(-c\gamma\tau(\alpha+7)/2\bigr)\binom{\alpha+8}{\alpha+3}\bigl(8+4c\gamma\tau(\alpha+7)+c^2\gamma^2\tau^2\times$$
$$\times(\alpha+7)^2\bigr)-\frac{10}{c^3\gamma^3(\alpha+9)^3}\exp\bigl(-c\gamma\tau(\alpha+9)/2\bigr)\binom{\alpha+9}{\alpha+4}\times$$
$$\times\bigl(8+4c\gamma\tau(\alpha+9)+c^2\gamma^2\tau^2(\alpha+9)^2\bigr)+\frac{2}{c^3\gamma^3(\alpha+11)^3}\times$$
$$\times\exp\bigl(-c\gamma\tau(\alpha+11)/2)\bigr)\binom{\alpha+10}{\alpha+5}\bigl(8+4c\gamma\tau(\alpha+11)+c^2\gamma^2\tau^2\times$$
$$\times(\alpha+11)^2\bigr).$$



а) б)

Рис. 1.113. Вид неопределенного интеграла 2-ого рода от ортогональных функций Якоби 2-ого порядка: а) $\gamma = 2$, $\alpha \in [0; 5]$; б) $\gamma \in (1; 3,5]$, $\alpha = 1$

[1.114]
$$\int \tau^3 P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k} \binom{k}{s}\binom{k+s+\alpha}{s+\alpha}\times$$
$$\times(-1)^s \exp\bigl(-(2s+\alpha+1)c\gamma\tau/2\bigr)\times$$
$$\times\left(\frac{2\tau^3}{c\gamma(2s+\alpha+1)} + \frac{12\tau^2}{c^2\gamma^2(2s+\alpha+1)^2} + \right.$$
$$\left. + \frac{48\tau}{c^3\gamma^3(2s+\alpha+1)^3} + \frac{96}{c^4\gamma^4(2s+\alpha+1)^4}\right).$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$$\int \tau^3 P_0^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^4\gamma^4(\alpha+1)^4}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times$$
$$\times\bigl(48+24c\gamma\tau(\alpha+1)+6c^2\gamma^2\tau^2(\alpha+1)^2+c^3\gamma^3\tau^3(\alpha+1)^3\bigr);$$

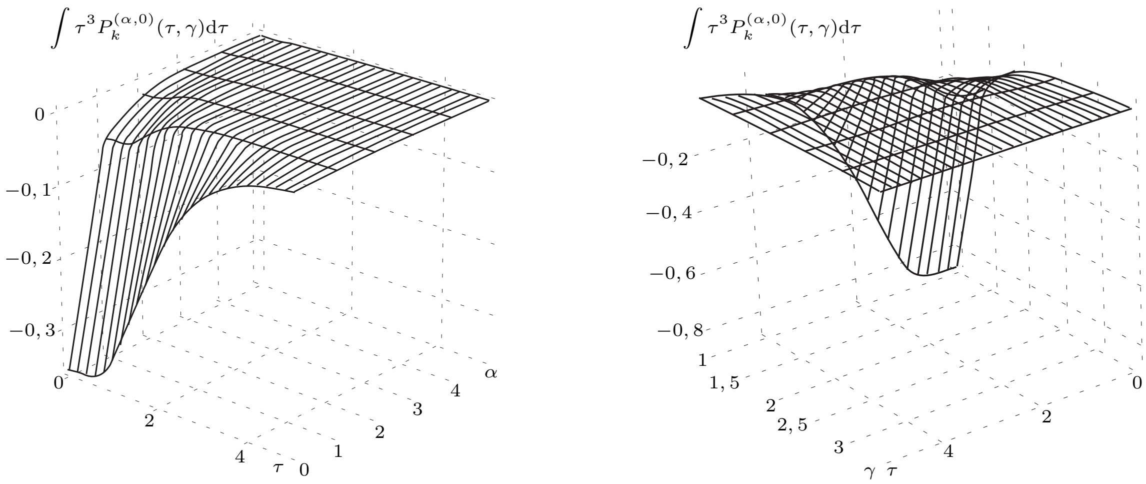
$$\int \tau^3 P_1^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^4\gamma^4(\alpha+1)^3}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times$$
$$\times\bigl(48+24c\gamma\tau(\alpha+1)+6c^2\gamma^2\tau^2(\alpha+1)^2+c^3\gamma^3\tau^3(\alpha+1)^3\bigr)+$$
$$+\frac{2}{c^4\gamma^4(\alpha+3)^4}\exp\Bigl(-c\gamma\tau(\alpha+3)/2\Bigr)(\alpha+2)\bigl(48+24c\gamma\tau(\alpha+3)+$$
$$+6c^2\gamma^2\tau^2(\alpha+3)^2+c^3\gamma^3\tau^3(\alpha+3)^3\bigr);$$

$$\int \tau^3 P_2^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^4\gamma^4(\alpha+1)^4}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times$$
$$\times\binom{\alpha+2}{\alpha}\bigl(48+24c\gamma\tau(\alpha+1)+6c^2\gamma^2\tau^2(\alpha+1)^2+c^3\gamma^3\tau^3\times$$
$$\times(\alpha+1)^3\bigr)+\frac{4}{c^4\gamma^4(\alpha+3)^4}\exp\bigl(-c\gamma\tau(\alpha+3)/2\bigr)\binom{\alpha+3}{\alpha+1}\times$$
$$\times\bigl(48+24c\gamma\tau(\alpha+3)+6c^2\gamma^2\tau^2(\alpha+3)^2+c^3\gamma^3\tau^3(\alpha+3)^3\bigr)-$$
$$-\frac{2}{c^4\gamma^4(\alpha+5)^4}\exp\bigl(-c\gamma\tau(\alpha+5)/2\bigr)\binom{\alpha+4}{\alpha+2}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+5)+6c^2\gamma^2\tau^2(\alpha+5)^2+c^3\gamma^3\tau^3(\alpha+5)^3\bigr);$$

$$\int \tau^3 P_3^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^4\gamma^4(\alpha+1)^4}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times$$
$$\times\binom{\alpha+3}{\alpha}\bigl(48+24c\gamma\tau(\alpha+1)+6c^2\gamma^2\tau^2(\alpha+1)^2+c^3\gamma^3\tau^3\times$$
$$\times(\alpha+1)^3\bigr)+\frac{6}{c^4\gamma^4(\alpha+3)^4}\exp\Bigl(-c\gamma\tau(\alpha+3)/2\Bigr)\binom{\alpha+4}{\alpha+1}\times$$
$$\times\bigl(48+24c\gamma\tau(\alpha+3)+6c^2\gamma^2\tau^2(\alpha+3)^2+c^3\gamma^3\tau^3(\alpha+3)^3\bigr)-$$
$$-\frac{6}{c^4\gamma^4(\alpha+5)^4}\exp\bigl(-c\gamma\tau(\alpha+5)/2\bigr)\binom{\alpha+5}{\alpha+2}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+5)+6c^2\gamma^2\tau^2(\alpha+5)^2+c^3\gamma^3\tau^3(\alpha+5)^3\bigr)+$$
$$+\frac{2}{c^4\gamma^4(\alpha+7)^4}\exp\bigl(-c\gamma\tau(\alpha+7)/2\bigr)\binom{\alpha+6}{\alpha+3}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+7)+6c^2\gamma^2\tau^2(\alpha+7)^2+c^3\gamma^3\tau^3(\alpha+7)^3\bigr);$$

$$\int \tau^3 P_4^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^4\gamma^4(\alpha+1)^4}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times$$
$$\times\binom{\alpha+4}{\alpha}\bigl(48+24c\gamma\tau(\alpha+1)+6c^2\gamma^2\tau^2(\alpha+1)^2+c^3\gamma^3\tau^3\times$$
$$\times(\alpha+1)^3\bigr)+\frac{8}{c^4\gamma^4(\alpha+3)^4}\exp\bigl(-c\gamma\tau(\alpha+3)/2\bigr)\binom{\alpha+5}{\alpha+1}\times$$
$$\times\bigl(48+24c\gamma\tau(\alpha+3)+6c^2\gamma^2\tau^2(\alpha+3)^2+c^3\gamma^3\tau^3(\alpha+3)^3\bigr)-$$
$$-\frac{12}{c^4\gamma^4(\alpha+5)^4}\exp\bigl(-c\gamma\tau(\alpha+5)/2\bigr)\binom{\alpha+6}{\alpha+2}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+5)+6c^2\gamma^2\tau^2(\alpha+5)^2+c^3\gamma^3\tau^3(\alpha+5)^3\bigr)+$$
$$+\frac{8}{c^4\gamma^4(\alpha+7)^4}\exp\bigl(-c\gamma\tau(\alpha+7)/2\bigr)\binom{\alpha+7}{\alpha+3}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+7)+6c^2\gamma^2\tau^2(\alpha+7)^2+c^3\gamma^3\tau^3(\alpha+7)^3\bigr)-$$
$$-\frac{2}{c^4\gamma^4(\alpha+9)^4}\exp\bigl(-c\gamma\tau(\alpha+9)/2\bigr)\binom{\alpha+8}{\alpha+4}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+9)+6c^2\gamma^2\tau^2(\alpha+9)^2+c^3\gamma^3\tau^3(\alpha+9)^3\bigr);$$

$$\int \tau^3 P_5^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^4\gamma^4(\alpha+1)^4}\exp\bigl(-c\gamma\tau(\alpha+1)/2\bigr)\times$$
$$\times\binom{\alpha+5}{\alpha}\bigl(48+24c\gamma\tau(\alpha+1)+6c^2\gamma^2\tau^2(\alpha+1)^2+c^3\gamma^3\tau^3\times$$
$$\times(\alpha+1)^3\bigr)+\frac{10}{c^4\gamma^4(\alpha+3)^4}\exp\bigl(-c\gamma\tau(\alpha+3)/2\bigr)\binom{\alpha+6}{\alpha+1}\times$$
$$\times\bigl(48+24c\gamma\tau(\alpha+3)+6c^2\gamma^2\tau^2(\alpha+3)^2+c^3\gamma^3\tau^3(\alpha+3)^3\bigr)-$$
$$-\frac{20}{c^3\gamma^3(\alpha+5)^3}\exp\bigl(-c\gamma\tau(\alpha+5)/2\bigr)\binom{\alpha+7}{\alpha+2}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+5)+6c^2\gamma^2\tau^2(\alpha+5)^2+c^3\gamma^3\tau^3(\alpha+5)^3\bigr)+$$
$$+\frac{20}{c^4\gamma^4(\alpha+7)^4}\exp\bigl(-c\gamma\tau(\alpha+7)/2\bigr)\binom{\alpha+8}{\alpha+3}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+7)+6c^2\gamma^2\tau^2(\alpha+7)^2+c^3\gamma^3\tau^3(\alpha+7)^3\bigr)-$$
$$-\frac{10}{c^4\gamma^4(\alpha+9)^4}\exp\bigl(-c\gamma\tau(\alpha+9)/2\bigr)\binom{\alpha+9}{\alpha+4}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+9)+6c^2\gamma^2\tau^2(\alpha+9)^2+c^3\gamma^3\tau^3(\alpha+9)^3\bigr)+$$
$$+\frac{2}{c^4\gamma^4(\alpha+11)^4}\exp\bigl(-c\gamma\tau(\alpha+11)/2\bigr)\binom{\alpha+10}{\alpha+5}\bigl(48+24c\gamma\tau\times$$
$$\times(\alpha+11)+6c^2\gamma^2\tau^2(\alpha+11)^2+c^3\gamma^3\tau^3(\alpha+11)^3\bigr).$$



а) **б)**

Рис. 1.114. Вид неопределенного интеграла 3-ого рода от ортогональных функций Якоби 2-ого порядка: а) $\gamma = 2$, $\alpha \in [0;5]$; б) $\gamma \in (1;3,5]$, $\alpha = 1$

[1.115]
$$\int \tau^n P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}\times$$
$$\times(-1)^s \exp\bigl(-(2s+\alpha+1)c\gamma\tau/2\bigr)\times$$
$$\times\sum_{j=0}^{n}\frac{n!\tau^{n-j}}{(n-j)!\Bigl(c\gamma(2s+\alpha+1)/2\Bigr)^{j+1}}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n P_0^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp(-c\gamma\tau(\alpha+1)/2)\times$$
$$\times\sum_{j=0}^{n}\Bigl(\frac{2}{c\gamma(\alpha+1)/2}\Bigr)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n P_1^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau(\alpha+1)/2)\times$$
$$\times\left((\alpha+1)n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+1)/2\Bigr)^{j+1}}-\right.$$
$$\left.-(\alpha+2)n!\exp(-\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+3)/2\Bigr)^{j+1}}\right);$$

$$\int \tau^n P_2^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau(\alpha+1)/2)\times$$
$$\times\left(\binom{\alpha+2}{\alpha}n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+1)/2\Bigr)^{j+1}}-\right.$$
$$-2\binom{\alpha+3}{\alpha+1}n!\exp(-c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+3)/2\Bigr)^{j+1}}+$$
$$\left.+\binom{\alpha+4}{\alpha+2}n!\exp(-2c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+5)/2\Bigr)^{j+1}}\right);$$

$$\int \tau^n P_3^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau(\alpha+1)/2)\times$$
$$\times\left(\binom{\alpha+3}{\alpha}n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+1)/2\Bigr)^{j+1}}-\right.$$
$$-3\binom{\alpha+4}{\alpha+1}n!\exp(-c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+3)/2\Bigr)^{j+1}}+$$
$$+3\binom{\alpha+5}{\alpha+2}n!\exp(-2c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+5)/2\Bigr)^{j+1}}-$$
$$\left.-\binom{\alpha+6}{\alpha+3}n!\exp(-3c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(9c\gamma(\alpha+7)/2\Bigr)^{j+1}}\right);$$

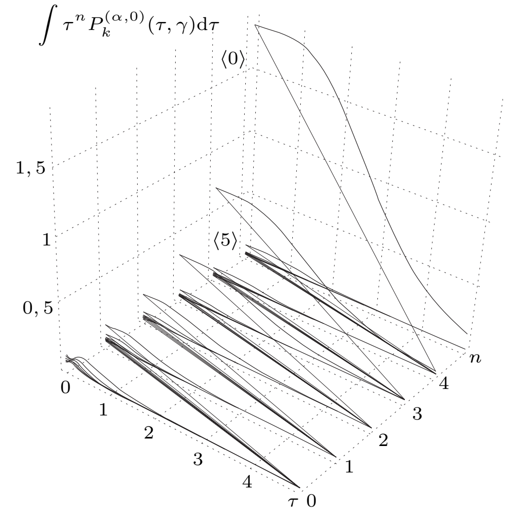
$$\int \tau^n P_4^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau(\alpha+1)/2)\times$$
$$\times\left(\binom{\alpha+4}{\alpha}n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+1)/2\Bigr)^{j+1}}-\right.$$
$$-4\binom{\alpha+5}{\alpha+1}n!\exp(-c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+3)/2\Bigr)^{j+1}}+$$
$$+6\binom{\alpha+6}{\alpha+2}n!\exp(-2c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+5)/2\Bigr)^{j+1}}-$$
$$-4\binom{\alpha+7}{\alpha+3}n!\exp(-3c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+7)/2\Bigr)^{j+1}}+$$
$$\left.+\binom{\alpha+8}{\alpha+4}n!\exp(-4c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+9)/2\Bigr)^{j+1}}\right);$$

$$\int \tau^n P_5^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau(\alpha+1)/2)\times$$
$$\times\left(\binom{\alpha+5}{\alpha}n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+1)/2\Bigr)^{j+1}}-\right.$$
$$-5\binom{\alpha+6}{\alpha+1}n!\exp(-c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+3)/2\Bigr)^{j+1}}+$$
$$+10\binom{\alpha+7}{\alpha+2}n!\exp(-2c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma(\alpha+5)/2\Bigr)^{j+1}}-$$

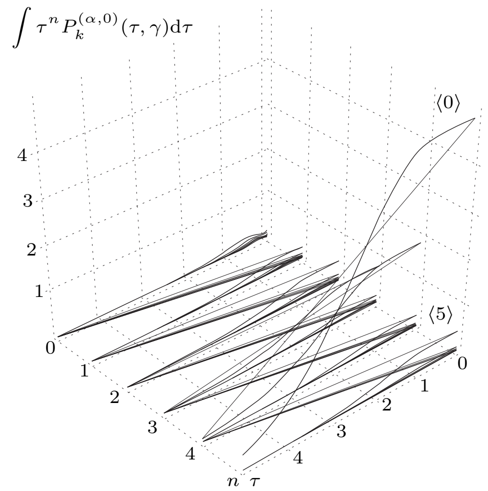
$$- 10\binom{\alpha+8}{\alpha+3} n! \exp(-3c\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\Big(c\gamma(\alpha+7)/2\Big)^{j+1}} +$$

$$+ 5\binom{\alpha+9}{\alpha+4} n! \exp(-4c\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\Big(c\gamma(\alpha+9)/2\Big)^{j+1}} -$$

$$- \binom{\alpha+10}{\alpha+5} n! \exp(-5c\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\Big(c\gamma(\alpha+11)/2\Big)^{j+1}}\Bigg).$$



а)



б)

Рис. 1.115. Вид неопределенного интеграла n-ого рода от ортогональных функций Якоби 2-ого порядка: а) $n = 0..5$, $\gamma = 2$, $\alpha \in [0; 5]$; б) $n = 0..5$, $\gamma \in (1; 3, 5]$, $\alpha = 1$

[1.116] $$\int P_k^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1}{s} \times \frac{(-1)^s}{\gamma(2s+1)} \exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int P_0^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma}\exp(-\gamma\tau);$$

$$\int P_1^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma}\exp(-\gamma\tau)(1-\exp(-2\gamma\tau));$$

$$\int P_2^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3\gamma}\exp(-\gamma\tau)(3-8\exp(-2\gamma\tau)+6\times \exp(-4\gamma\tau));$$

$$\int P_3^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma}\exp(-\gamma\tau)(1-5\exp(-2\gamma\tau)+9\times \exp(-4\gamma\tau)-5\exp(-6\gamma\tau));$$

$$\int P_4^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{5\gamma}\exp(-\gamma\tau)(5-40\exp(-2\gamma\tau)+126\times \exp(-4\gamma\tau)-160\exp(-6\gamma\tau)+70\exp(-8\gamma\tau));$$

$$\int P_5^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3\gamma}\exp(-\gamma\tau)(3-35\exp(-2\gamma\tau)+168\times \exp(-4\gamma\tau)-360\exp(-6\gamma\tau)+350\exp(-8\gamma\tau)-126\times \exp(-10\gamma\tau)).$$

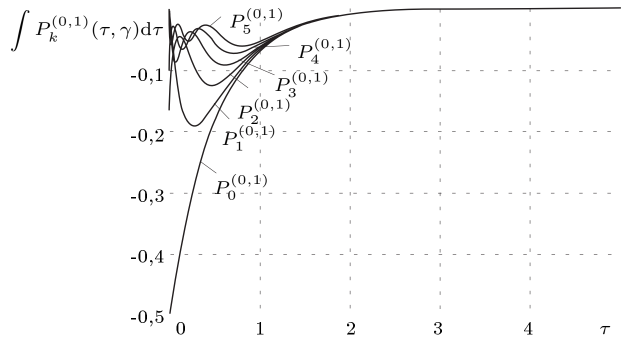


Рис. 1.116. Вид неопределенного интеграла от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.117] $$\int \tau P_k^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1}{s} \times (-1)^s \exp\big(-(2s+1)\gamma\tau\big)\left(\frac{\tau}{\gamma(2s+1)} + \frac{1}{\gamma^2(2s+1)^2}\right).$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int \tau P_0^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^2}\exp(-\gamma\tau)(\gamma\tau+1);$$

$$\int \tau P_1^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3\gamma^2}\exp(-\gamma\tau)\big(3-\exp(-2\gamma\tau)+\gamma\tau\times (3-3\exp(-2\gamma\tau))\big);$$

$$\int \tau P_2^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{45\gamma^2}\exp(-\gamma\tau)\big(45 - 40\exp(-2\gamma\tau) + 18\exp(-4\gamma\tau) + \gamma\tau(45 - 120\exp(-2\gamma\tau) + 90\exp(-4\gamma\tau))\big);$$

$$\int \tau P_3(0,1)(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{105\gamma^2}\exp(-\gamma\tau)\big(105 - 175 \times \exp(-2\gamma\tau) + 189\exp(-4\gamma\tau) - 75\exp(-6\gamma\tau) + \gamma\tau \times (105 - 525\exp(-2\gamma\tau) + 945\exp(-4\gamma\tau) - 525\exp(-6\gamma\tau))\big);$$

$$\int \tau P_4^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1575\gamma^2}\exp(-\gamma\tau)\big(1575 - 4200 \times \exp(-2\gamma\tau) + 7938\exp(-4\gamma\tau) - 7200\exp(-6\gamma\tau) + 2450 \times \exp(-8\gamma\tau) + \gamma\tau(1575 - 12600\exp(-2\gamma\tau) + 39690\exp(-4\gamma\tau) - 50400\exp(-6\gamma\tau) + 22050\exp(-8\gamma\tau))\big);$$

$$\int \tau P_5^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{10395\gamma^2}\exp(-\gamma\tau)\big(10395 - 40425 \times \exp(-2\gamma\tau) + 116424\exp(-4\gamma\tau) - 178200\exp(-6\gamma\tau) + 134750\exp(-8\gamma\tau) - 39690\exp(-10\gamma\tau) + \gamma\tau(10395 - 121275 \times \exp(-2\gamma\tau) + 582120\exp(-4\gamma\tau) - 1247400\exp(-6\gamma\tau) + 1212750\exp(-8\gamma\tau) - 436590\exp(-10\gamma\tau))\big).$$

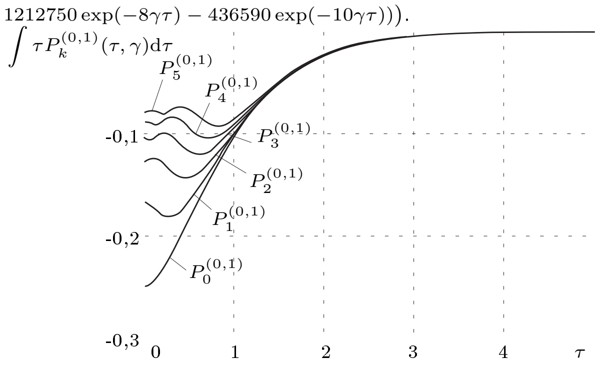


Рис. 1.117. Вид неопределенного интеграла 1-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.118]

$$\int \tau^2 P_k^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}\times (-1)^s\exp\big(-(2s+1)\gamma\tau\big)\left(\frac{\tau^2}{\gamma(2s+1)} + \frac{2\tau}{\gamma^2(2s+1)^2} + \frac{2}{\gamma^3(2s+1)^3}\right).$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$$\int \tau^2 P_0^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^3}\exp(-\gamma\tau)(\gamma^2\tau^2 + 2\gamma\tau + 2);$$

$$\int \tau^2 P_1^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{9\gamma^3}\exp(-\gamma\tau)\big(18 - 2\exp(-2\gamma\tau) + \gamma\tau(18 - 6\exp(-2\gamma\tau)) + \gamma^2\tau^2(9 - 9\exp(-2\gamma\tau))\big);$$

$$\int \tau^2 P_2^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{675\gamma^3}\exp(-\gamma\tau)\big(1350 - 400 \times \exp(-2\gamma\tau) + 108\exp(-4\gamma\tau) + \gamma\tau(1350 - 1200\exp(-2\gamma\tau) + 540\exp(-4\gamma\tau)) + \gamma^2\tau^2(675 - 1800\exp(-2\gamma\tau) + 1350 \times \exp(-4\gamma\tau))\big);$$

$$\int \tau^2 P_3^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{11025\gamma^3}\exp(-\gamma\tau)\big(22050 - 12250 \times \exp(-2\gamma\tau) + 7938\exp(-4\gamma\tau) - 2250\exp(-6\gamma\tau) + \gamma\tau \times (22050 - 36750\exp(-2\gamma\tau) + 39690\exp(-4\gamma\tau) - 15750 \times \exp(-6\gamma\tau)) + \gamma^2\tau^2(11025 - 55125\exp(-2\gamma\tau) + 99225 \times \exp(-4\gamma\tau) - 55125\exp(-6\gamma\tau))\big);$$

$$\int \tau^2 P_4^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{496125\gamma^3}\exp(-\gamma\tau)\big(992250 - 882000\exp(-2\gamma\tau) + 1000188\exp(-4\gamma\tau) - 648000 \times \exp(-6\gamma\tau) + 171500\exp(-8\gamma\tau) + \gamma\tau(992250 - 2646000 \times \exp(-2\gamma\tau) + 5000940\exp(-4\gamma\tau) - 4536000\exp(-6\gamma\tau) + 1543500\exp(-8\gamma\tau)) + \gamma^2\tau^2(496125 - 3969000\exp(-2\gamma\tau) + 12502350\exp(-4\gamma\tau) - 15876000\exp(-6\gamma\tau) + 6945750 \times \exp(-8\gamma\tau))\big);$$

$$\int \tau^2 P_5^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{36018675\gamma^3}\exp(-\gamma\tau)\big(72037350 - 93381750\exp(-2\gamma\tau) + 161363664\exp(-4\gamma\tau) - 176418000 \times \exp(-6\gamma\tau) + 103757500\exp(-8\gamma\tau) - 25004700\exp(-10\gamma\tau) + \gamma\tau(72037350 - 280145250\exp(-2\gamma\tau) + 806818320 \times \exp(-4\gamma\tau) - 1234926000\exp(-6\gamma\tau) + 933817500 \times \exp(-8\gamma\tau) - 275051700\exp(-10\gamma\tau)) + \gamma^2\tau^2 \times (36018675 - 420217875\exp(-2\gamma\tau) + 2017045800\exp(-4\gamma\tau) - 4322241000\exp(-6\gamma\tau) + 4202178750\exp(-8\gamma\tau) - 1512784350\exp(-10\gamma\tau))\big).$$

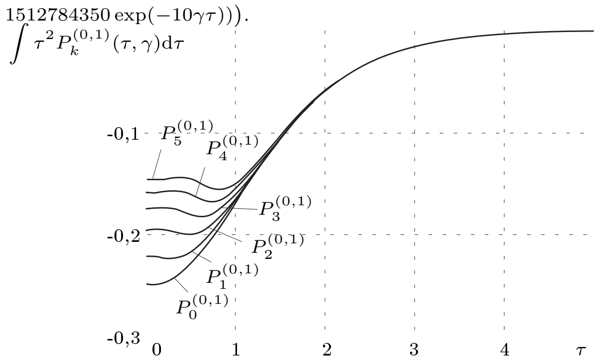


Рис. 1.118. Вид неопределенного интеграла 2-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.119]

$$\int \tau^3 P_k^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}\times (-1)^s\exp\big(-(2s+1)\gamma\tau\big)\left(\frac{\tau^3}{\gamma(2s+1)} + \frac{3\tau^2}{\gamma^2(2s+1)^2} + \frac{6\tau}{\gamma^3(2s+1)^3} + \frac{6}{\gamma^4(2s+1)^4}\right).$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$$\int \tau^3 P_0^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^4}\exp(-\gamma\tau)(\gamma^3\tau^3 + 3\gamma^2\tau^2 + 6\gamma\tau + 6);$$

$$\int \tau^3 P_1^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{9\gamma^4}\exp(-\gamma\tau)\big(54 - 2\exp(-2\gamma\tau) + \gamma\tau(54 - 6\exp(-2\gamma\tau)) + \gamma^2\tau^2(27 - 9\exp(-2\gamma\tau)) + \gamma^3\tau^3 \times (9 - 9\exp(-2\gamma\tau))\big);$$

$$\int \tau^3 P_2^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3375\gamma^4}\exp(-\gamma\tau)\big(20250 - 2000 \times \exp(-2\gamma\tau) + 324\exp(-4\gamma\tau) + \gamma\tau(20250 - 6000\exp(-2\gamma\tau) + 1620\exp(-4\gamma\tau)) + \gamma^2\tau^2(10125 - 9000\exp(-2\gamma\tau) + 4050 \times \exp(-4\gamma\tau)) + \gamma^3\tau^3(3375 - 9000\exp(-2\gamma\tau) + 6750 \times \exp(-4\gamma\tau))\big);$$

$$\int \tau^3 P_3^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{385875\gamma^4}\exp(-\gamma\tau)\big(2315250 -$$

$$- 428750 \exp(-2\gamma\tau) + 166698 \exp(-4\gamma\tau) - 33750 \exp(-6\gamma\tau) +$$
$$+ \gamma\tau(2315250 - 1286250 \exp(-2\gamma\tau) + 833490 \exp(-4\gamma\tau) -$$
$$- 236250 \exp(-6\gamma\tau)) + \gamma^2\tau^2(1157625 - 1929375 \exp(-2\gamma\tau) +$$
$$+ 2083725 \exp(-4\gamma\tau) - 826875 \exp(-6\gamma\tau)) + \gamma^3\tau^3 \times$$
$$\times (385875 - 1929375 \exp(-2\gamma\tau) + 3472875 \exp(-4\gamma\tau) -$$
$$- 1929375 \exp(-6\gamma\tau)));$$

$$\int \tau^3 P_4^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{52093125\gamma^4} \exp(-\gamma\tau)\big(312558750 -$$
$$- 92610000 \exp(-2\gamma\tau) + 63011844 \exp(-4\gamma\tau) - 29160000 \times$$
$$\times \exp(-6\gamma\tau) + 6002500 \exp(-8\gamma\tau) + \gamma\tau(312558750 -$$
$$- 277830000 \exp(-2\gamma\tau) + 315059220 \exp(-4\gamma\tau) - 204120000 \times$$
$$\times \exp(-6\gamma\tau) + 54022500 \exp(-8\gamma\tau)) + \gamma^2\tau^2(156279375 -$$
$$- 416745000 \exp(-2\gamma\tau) + 787648050 \exp(-4\gamma\tau) - 714420000 \times$$
$$\times \exp(-6\gamma\tau) + 243101250 \exp(-8\gamma\tau)) + \gamma^3\tau^3(52093125 -$$
$$- 787648050 \exp(-2\gamma\tau) + 1312746750 \exp(-4\gamma\tau) -$$
$$- 1666980000 \exp(-6\gamma\tau) + 729303750 \exp(-8\gamma\tau)));$$

$$\int \tau^3 P_5^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{41601569625\gamma^4} \exp(-\gamma\tau) \times$$
$$\times \big(249609417750 - 107855921250 \exp(-2\gamma\tau) + 111825019152 \times$$
$$\times \exp(-4\gamma\tau) - 87326910000 \exp(-6\gamma\tau) + 39946637500 \times$$
$$\times \exp(-8\gamma\tau) - 7876480500 \exp(-10\gamma\tau) + \gamma\tau(249609417750 -$$
$$- 323567763750 \exp(-2\gamma\tau) + 559125095760 \exp(-4\gamma\tau) -$$
$$- 611288370000 \exp(-6\gamma\tau) + 359519737500 \exp(-8\gamma\tau) -$$
$$- 86641285500 \exp(-10\gamma\tau)) + \gamma^2\tau^2(124804708875 -$$
$$- 485351645625 \exp(-2\gamma\tau) + 1397812739400 \exp(-4\gamma\tau) -$$
$$- 2139509295000 \exp(-6\gamma\tau) + 1617838818750 \exp(-8\gamma\tau) -$$
$$- 476527070250 \exp(-10\gamma\tau)) + \gamma^3\tau^3(41601569625 -$$
$$- 485351645625 \exp(-2\gamma\tau) + 2329687899000 \exp(-4\gamma\tau) -$$
$$- 4992188355000 \exp(-6\gamma\tau) + 4853516456250 \exp(-8\gamma\tau) -$$
$$- 1747265924250 \exp(-10\gamma\tau))\big).$$

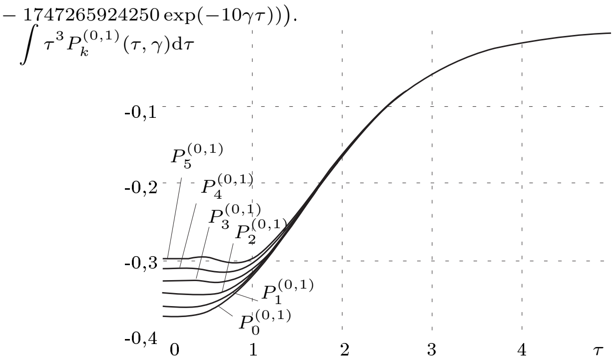


Рис. 1.119. Вид неопределенного интеграла 3-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.120]
$$\int \tau^n P_k^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1}{s} \times$$
$$\times (-1)^s \exp\big(-(2s+1)\gamma\tau\big) \sum_{j=0}^{n} \frac{n!\tau^{n-j}}{(n-j)!\big(\gamma(2s+1)\big)^{j+1}}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n P_0^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma} \exp(-\gamma\tau) \sum_{j=0}^{n} \left(\frac{2}{\gamma}\right)^j \frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n P_1^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n! \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(\gamma\big)^{j+1}} - \right.$$
$$\left. - 3n! \exp(-2\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(3\gamma\big)^{j+1}} \right);$$

$$\int \tau^n P_2^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n! \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(\gamma\big)^{j+1}} - \right.$$
$$- 8n! \exp(-2\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(3\gamma\big)^{j+1}} +$$
$$\left. + 10n! \exp(-4\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(5\gamma\big)^{j+1}} \right);$$

$$\int \tau^n P_3^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n! \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(\gamma\big)^{j+1}} - \right.$$
$$- 15n! \exp(-2\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(3\gamma\big)^{j+1}} +$$
$$+ 45n! \exp(-4\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(5\gamma\big)^{j+1}} -$$
$$\left. - 35n! \exp(-6\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(7\gamma\big)^{j+1}} \right);$$

$$\int \tau^n P_4^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n! \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(3\gamma\big)^{j+1}} - \right.$$
$$- 24n! \exp(-2\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(5\gamma\big)^{j+1}} +$$
$$+ 126n! \exp(-4\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(7\gamma\big)^{j+1}} -$$
$$- 224n! \exp(-6\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(9\gamma\big)^{j+1}} +$$
$$\left. + 126n! \exp(-8\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(11\gamma\big)^{j+1}} \right);$$

$$\int \tau^n P_5^{(0,1)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau) \times$$
$$\times \left(n! \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(\gamma\big)^{j+1}} - \right.$$
$$- 35n! \exp(-2\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(3\gamma\big)^{j+1}} +$$
$$+ 280n! \exp(-4\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(5\gamma\big)^{j+1}} -$$
$$- 840n! \exp(-6\gamma\tau) \sum_{j=0}^{n} \frac{\tau^{n-j}}{(n-j)!\big(7\gamma\big)^{j+1}} +$$

$$+\ 1050n!\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(9\gamma\big)^{j+1}}\ -$$

$$-\ 462n!\exp(-10\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\big(11\gamma\big)^{j+1}}\Bigg).$$

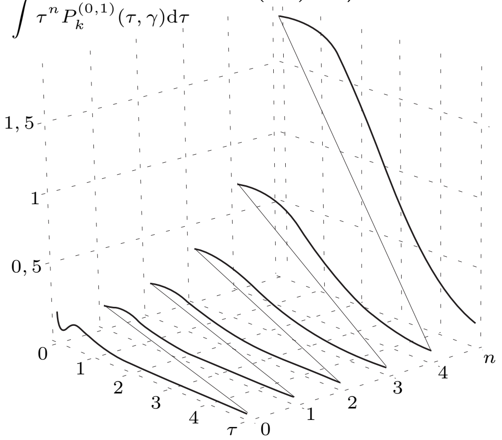


Рис. 1.120. Вид неопределенного интеграла n-ого рода от ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 1$

[1.121]
$$\int P_k^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}\times$$
$$\times\ \frac{(-1)^s}{\gamma(2s+1)}\exp\big(-(2s+1)\gamma\tau\big).$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$\int P_0^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma}\exp(-\gamma\tau);$

$\int P_1^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3\gamma}\exp(-\gamma\tau)(3-4\exp(-2\gamma\tau));$

$\int P_2^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3\gamma}\exp(-\gamma\tau)(3-10\exp(-2\gamma\tau)+9\times$
$\times\exp(-4\gamma\tau));$

$\int P_3^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{5\gamma}\exp(-\gamma\tau)(5-30\exp(-2\gamma\tau)+63\times$
$\times\exp(-4\gamma\tau)-40\exp(-6\gamma\tau));$

$\int P_4^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{15\gamma}\exp(-\gamma\tau)(15-140\exp(-2\gamma\tau)+$
$+504\exp(-4\gamma\tau)-720\exp(-6\gamma\tau)+350\exp(-8\gamma\tau));$

$\int P_5^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{21\gamma}\exp(-\gamma\tau)(21-280\exp(-2\gamma\tau)+$
$+1512\exp(-4\gamma\tau)-3600\exp(-6\gamma\tau)+3850\exp(-8\gamma\tau)-$
$-1512\exp(-10\gamma\tau)).$

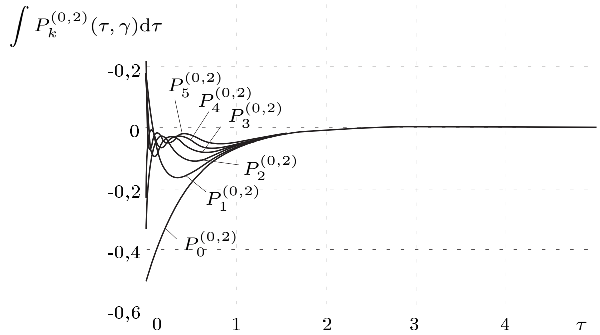


Рис. 1.121. Вид неопределенного интеграла от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.122]
$$\int \tau P_k^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}\times$$
$$\times(-1)^s\exp\big(-(2s+1)\gamma\tau\big)\left(\frac{\tau}{\gamma(2s+1)}+\frac{1}{\gamma^2(2s+1)^2}\right).$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$\int \tau P_0^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^2}\exp(-\gamma\tau)(\gamma\tau+1);$

$\int \tau P_1^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{9\gamma^2}\exp(-\gamma\tau)\big(9-4\exp(-2\gamma\tau)+\gamma\tau(9-$
$-12\exp(-2\gamma\tau))\big);$

$\int \tau P_2^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{45\gamma^2}\exp(-\gamma\tau)\big(45-50\exp(-2\gamma\tau)+$
$+27\exp(-4\gamma\tau)+\gamma\tau(45-150\exp(-2\gamma\tau)+135\exp(-4\gamma\tau))\big);$

$\int \tau P_3(0,2)(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{175\gamma^2}\exp(-\gamma\tau)\big(175-350\times$
$\times\exp(-2\gamma\tau)+441\exp(-4\gamma\tau)-200\exp(-6\gamma\tau)+\gamma\tau(175-$
$-1050\exp(-2\gamma\tau)+2205\exp(-4\gamma\tau)-1400\exp(-6\gamma\tau))\big);$

$\int \tau P_4^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{4725\gamma^2}\exp(-\gamma\tau)\big(4725-14700\times$
$\times\exp(-2\gamma\tau)+31752\exp(-4\gamma\tau)-32400\exp(-6\gamma\tau)+$
$+12250\exp(-8\gamma\tau)+\gamma\tau(4725-44100\exp(-2\gamma\tau)+$
$+158760\exp(-4\gamma\tau)-226800\exp(-6\gamma\tau)+110250\exp(-8\gamma\tau))\big);$

$\int \tau P_5^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{72765\gamma^2}\exp(-\gamma\tau)\big(72765-$
$-323400\exp(-2\gamma\tau)+1047816\exp(-4\gamma\tau)-1782000\times$
$\times\exp(-6\gamma\tau)+1482250\exp(-8\gamma\tau)-476280\exp(-10\gamma\tau)+\gamma\tau\times$
$\times(72765-970200\exp(-2\gamma\tau)+5239080\exp(-4\gamma\tau)-12474000\times$
$\times\exp(-6\gamma\tau)+13340250\exp(-8\gamma\tau)-5239080\exp(-10\gamma\tau))\big).$

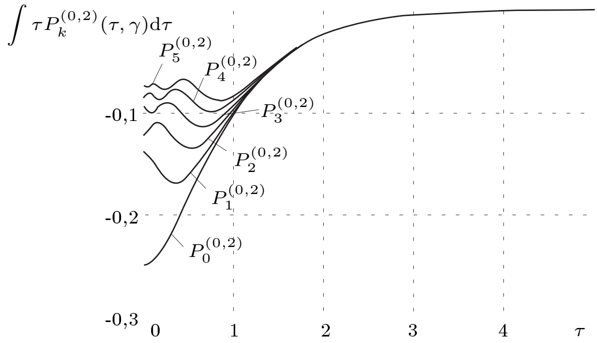


Рис. 1.122. Вид неопределенного интеграла 1-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.123]
$$\int \tau^2 P_k^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k} \binom{k}{s}\binom{k+s+2}{s}\times$$
$$\times (-1)^s \exp\bigl(-(2s+1)\gamma\tau\bigr)\left(\frac{\tau^2}{\gamma(2s+1)}+\right.$$
$$\left.+\frac{2\tau}{\gamma^2(2s+1)^2}+\frac{2}{\gamma^3(2s+1)^3}\right).$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$\int \tau^2 P_0^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^3}\exp(-\gamma\tau)(\gamma^2\tau^2 + 2\gamma\tau + 2);$

$\int \tau^2 P_1^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{27\gamma^3}\exp(-\gamma\tau)\bigl(54 - 8\exp(-2\gamma\tau) +$
$+ \gamma\tau(54 - 24\exp(-2\gamma\tau) + \gamma^2\tau^2(27 - 36\exp(-2\gamma\tau)))\bigr);$

$\int \tau^2 P_2^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{675\gamma^3}\exp(-\gamma\tau)\bigl(1350 - 500\times$
$\times\exp(-2\gamma\tau) + 162\exp(-4\gamma\tau) + \gamma\tau(1350 - 1500\exp(-2\gamma\tau) +$
$+ 810\exp(-4\gamma\tau)) + \gamma^2\tau^2(675 - 2250\exp(-2\gamma\tau) + 2025\times$
$\times\exp(-4\gamma\tau))\bigr);$

$\int \tau^2 P_3^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{18375\gamma^3}\exp(-\gamma\tau)\bigl(36750 - 24500\times$
$\times\exp(-2\gamma\tau) + 18522\exp(-4\gamma\tau) - 6000\exp(-6\gamma\tau) + \gamma\tau(36750 -$
$- 73500\exp(-2\gamma\tau) + 92610\exp(-4\gamma\tau) - 42000\exp(-6\gamma\tau)) +$
$+ \gamma^2\tau^2(18375 - 110250\exp(-2\gamma\tau) + 231525\exp(-4\gamma\tau) -$
$- 147000\exp(-6\gamma\tau))\bigr);$

$\int \tau^2 P_4^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1488375\gamma^3}\exp(-\gamma\tau)\bigl(2976750 -$
$- 3087000\exp(-2\gamma\tau) + 4000752\exp(-4\gamma\tau) - 2916000\times$
$\times\exp(-6\gamma\tau) + 857500\exp(-8\gamma\tau) + \gamma\tau(2976750 - 9261000\times$
$\times\exp(-2\gamma\tau) + 20003760\exp(-4\gamma\tau) - 20412000\exp(-6\gamma\tau) +$
$+ 7717500\exp(-8\gamma\tau)) + \gamma^2\tau^2(1488375 - 13891500\exp(-2\gamma\tau) +$
$+ 50009400\exp(-4\gamma\tau) - 71442000\exp(-6\gamma\tau) + 34728750\times$
$\times\exp(-8\gamma\tau))\bigr);$

$\int \tau^2 P_5^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{252130725\gamma^3}\exp(-\gamma\tau)\bigl(504261450 -$
$- 747054000\exp(-2\gamma\tau) + 1452272976\exp(-4\gamma\tau) -$
$- 1764180000\exp(-6\gamma\tau) + 1141332500\exp(-8\gamma\tau) -$
$- 300056400\exp(-10\gamma\tau) + \gamma\tau(504261450 - 2241162000\times$
$\times\exp(-2\gamma\tau) + 7261364880\exp(-4\gamma\tau) - 12349260000\times$
$\times\exp(-6\gamma\tau) + 10271992500\exp(-8\gamma\tau) - 3300620400\times$
$\times\exp(-10\gamma\tau)) + \gamma^2\tau^2(252130725 - 3361743000\exp(-2\gamma\tau) +$
$+ 18153412200\exp(-4\gamma\tau) - 43222410000\exp(-6\gamma\tau) +$
$+ 46223966250\exp(-8\gamma\tau) - 18153412200\exp(-10\gamma\tau))\bigr).$

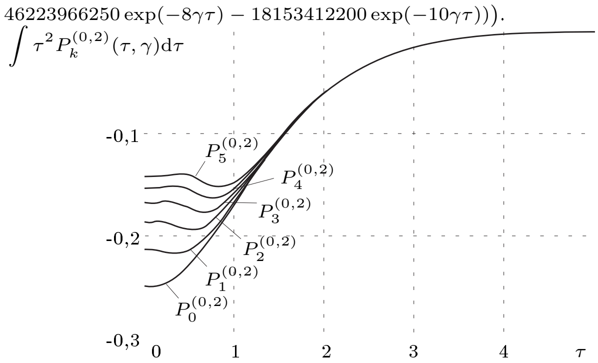


Рис. 1.123. Вид неопределенного интеграла 2-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.124]
$$\int \tau^3 P_k^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k} \binom{k}{s}\binom{k+s+2}{s}\times$$
$$\times (-1)^s \exp\bigl(-(2s+1)\gamma\tau\bigr)\left(\frac{\tau^3}{\gamma(2s+1)}+\frac{3\tau^2}{\gamma^2(2s+1)^2}+\right.$$
$$\left.+\frac{6\tau}{\gamma^3(2s+1)^3}+\frac{6}{\gamma^4(2s+1)^4}\right).$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$\int \tau^3 P_0^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{\gamma^4}\exp(-\gamma\tau)(\gamma^3\tau^3 + 3\gamma^2\tau^2 + 6\gamma\tau + 6);$

$\int \tau^3 P_1^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{27\gamma^4}\exp(-\gamma\tau)\bigl(162 - 8\exp(-2\gamma\tau) +$
$+ \gamma\tau(162 - 24\exp(-2\gamma\tau) + \gamma^2\tau^2(81 - 36\exp(-2\gamma\tau)) + \gamma^3\tau^3\times$
$\times(27 - 36\exp(-2\gamma\tau))\bigr);$

$\int \tau^3 P_2^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3375\gamma^4}\exp(-\gamma\tau)\bigl(20250 - 2500\times$
$\times\exp(-2\gamma\tau) + 486\exp(-4\gamma\tau) + \gamma\tau(20250 - 7500\exp(-2\gamma\tau) +$
$+ 2430\exp(-4\gamma\tau)) + \gamma^2\tau^2(10125 - 11250\exp(-2\gamma\tau) + 6075\times$
$\times\exp(-4\gamma\tau)) + \gamma^3\tau^3(3375 - 11250\exp(-2\gamma\tau) + 10125\times$
$\times\exp(-4\gamma\tau))\bigr);$

$\int \tau^3 P_3^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{643125\gamma^4}\exp(-\gamma\tau)\bigl(3858750 -$
$- 857500\exp(-2\gamma\tau) + 388962\exp(-4\gamma\tau) - 90000\exp(-6\gamma\tau) +$
$+ \gamma\tau(3858750 - 2572500\exp(-2\gamma\tau) + 1944810\exp(-4\gamma\tau) -$
$- 630000\exp(-6\gamma\tau)) + \gamma^2\tau^2(1929375 - 3858750\exp(-2\gamma\tau) +$
$+ 4862025\exp(-4\gamma\tau) - 2205000\exp(-6\gamma\tau)) + \gamma^3\tau^3(643125 -$
$- 3858750\exp(-2\gamma\tau) + 8103375\exp(-4\gamma\tau) - 5145000\times$
$\times\exp(-6\gamma\tau))\bigr);$

$\int \tau^3 P_4^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{156279375\gamma^4}\exp(-\gamma\tau)\bigl(937676250 -$
$- 324135000\exp(-2\gamma\tau) + 252047376\exp(-4\gamma\tau) - 131220000\times$
$\times\exp(-6\gamma\tau) + 30012500\exp(-8\gamma\tau) + \gamma\tau(937676250 -$
$- 972405000\exp(-2\gamma\tau) + 1260236880\exp(-4\gamma\tau) - 918540000\times$
$\times\exp(-6\gamma\tau) + 270112500\exp(-8\gamma\tau)) + \gamma^2\tau^2(468838125 -$
$- 1458607500\exp(-2\gamma\tau) + 3150592200\exp(-4\gamma\tau) -$
$- 3214890000\exp(-6\gamma\tau) + 1215506250\exp(-8\gamma\tau)) + \gamma^3\tau^3\times$
$\times(156279375 - 787648050\exp(-2\gamma\tau) + 5250987000\times$
$\times\exp(-4\gamma\tau) - 7501410000\exp(-6\gamma\tau) + 3646518750\times$
$\times\exp(-8\gamma\tau))\bigr);$

$$\int \tau^3 P_5^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{291210987375\gamma^4}\exp(-\gamma\tau)\times$$
$$\times\big(1747265924250 - 862847370000\exp(-2\gamma\tau) +$$
$$+ 1006425172368\exp(-4\gamma\tau) - 873269100000\exp(-6\gamma\tau) +$$
$$+ 439413012500\exp(-8\gamma\tau) - 94517766000\exp(-10\gamma\tau) +$$
$$+ \gamma\tau(1747265924250 - 2588542110000\exp(-2\gamma\tau) +$$
$$+ 5032125861840\exp(-4\gamma\tau) - 6112883700000\exp(-6\gamma\tau) +$$
$$+ 3954717112500\exp(-8\gamma\tau) - 1039695426000\exp(-10\gamma\tau)) +$$
$$+ \gamma^2\tau^2(873632962125 - 3882813165000\exp(-2\gamma\tau) +$$
$$+ 12580314654600\exp(-4\gamma\tau) - 21395092950000\exp(-6\gamma\tau) +$$
$$+ 17796227006250\exp(-8\gamma\tau) - 5718324843000\exp(-10\gamma\tau)) +$$
$$+ \gamma^3\tau^3(291210987375 - 3882813165000\exp(-2\gamma\tau) +$$
$$+ 20967191091000\exp(-4\gamma\tau) - 49921883550000\exp(-6\gamma\tau) +$$
$$+ 53388681018750\exp(-8\gamma\tau) - 20967191091000\exp(-10\gamma\tau))\big).$$

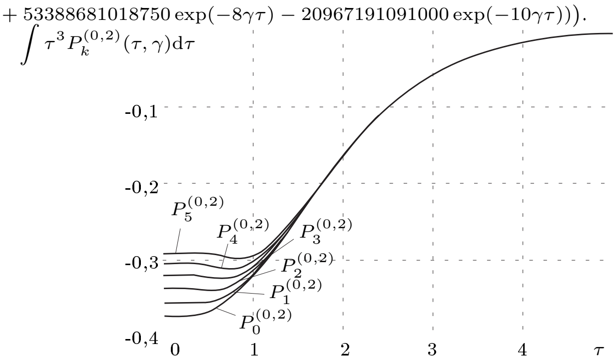


Рис. 1.124. Вид неопределенного интеграла 3-ого рода от ортогональных функций Якоби 0-5 порядков; $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.125]
$$\int \tau^n P_k^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}\times$$
$$\times(-1)^s\exp\big(-(2s+1)\gamma\tau\big)\sum_{j=0}^{n}\frac{n!\tau^{n-j}}{(n-j)!\Big(\gamma(2s+1)\Big)^{j+1}}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n P_0^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp(-\gamma\tau)\sum_{j=0}^{n}\Big(\frac{2}{\gamma}\Big)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n P_1^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\gamma\Big)^{j+1}} - \right.$$
$$\left. - 4n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n P_2^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\gamma\Big)^{j+1}} - \right.$$
$$- 10n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} +$$
$$\left. + 15n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n P_3^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\gamma\Big)^{j+1}} - \right.$$
$$- 18n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} +$$
$$+ 63n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} -$$
$$\left. - 56n!\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n P_4^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} - \right.$$
$$- 28n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} +$$
$$+ 168n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}} -$$
$$- 336n!\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(9\gamma\Big)^{j+1}} +$$
$$\left. + 210n!\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(11\gamma\Big)^{j+1}}\right);$$

$$\int \tau^n P_5^{(0,2)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-\gamma\tau)\times$$
$$\times\left(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(\gamma\Big)^{j+1}} - \right.$$
$$- 40n!\exp(-2\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(3\gamma\Big)^{j+1}} +$$
$$+ 360n!\exp(-4\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(5\gamma\Big)^{j+1}} -$$
$$- 1200n!\exp(-6\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7\gamma\Big)^{j+1}} +$$
$$+ 1650n!\exp(-8\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(9\gamma\Big)^{j+1}} -$$
$$\left. - 792n!\exp(-10\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(11\gamma\Big)^{j+1}}\right).$$

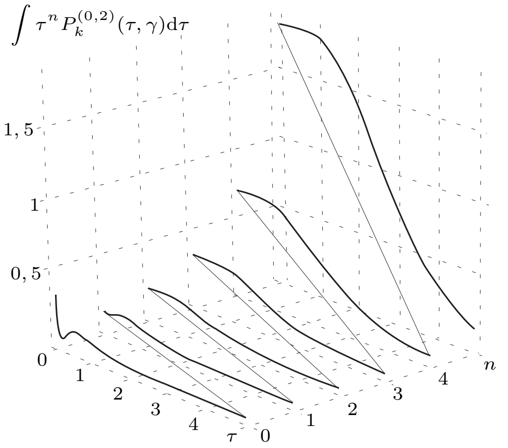


Рис. 1.125. Вид неопределенного интеграла n-ого рода от ортогональных функций Якоби 2-ого порядка; $n = 0..5$, $\gamma = 2$, $c = 2$, $\alpha = 0$, $\beta = 2$

[1.126]
$$\int P_k^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}\times$$
$$\times\,\frac{2(-1)^s}{c\gamma(2s+1)}\exp\bigl(-(2s+1)c\gamma\tau/2\bigr).$$

Частные случаи для неопределенного интеграла от функций 0-5 порядков:

$$\int P_0^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c\gamma}\exp\bigl(-(2s+1)c\gamma\tau/2\bigr);$$

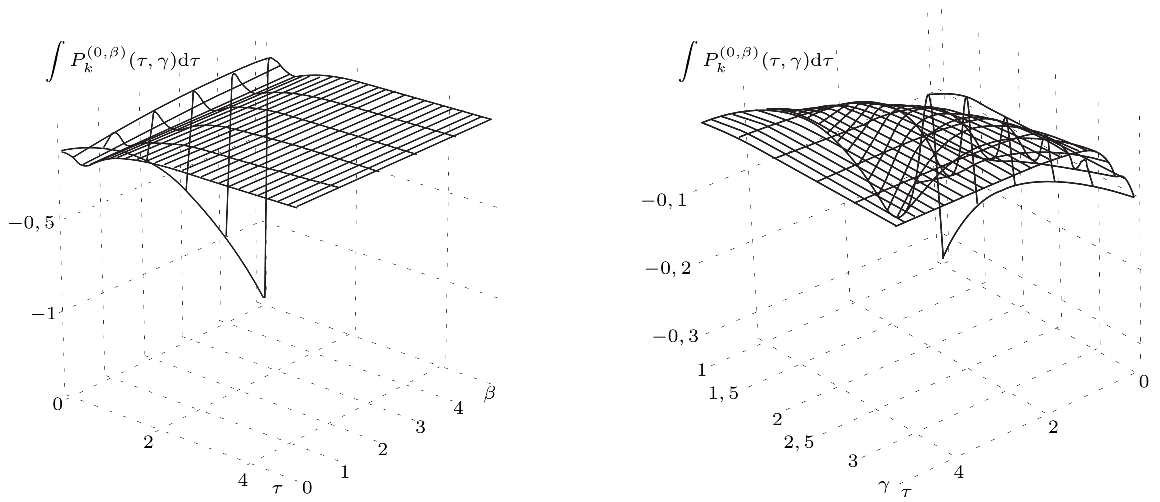
$$\int P_1^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{3c\gamma}\exp\bigl(-(2s+1)c\gamma\tau/2\bigr)\bigl(3-(\beta+2)\times$$
$$\times\exp(-c\gamma\tau)\bigr);$$

$$\int P_2^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{15c\gamma}\exp\bigl(-(2s+1)c\gamma\tau/2\bigr)\bigl(30-20\times$$
$$\times(\beta+3)\exp(-c\gamma\tau)+3(\beta^2+7\beta+12)\exp(-2c\gamma\tau)\bigr);$$

$$\int P_3^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{105c\gamma}\exp\bigl(-(2s+1)c\gamma\tau/2\bigr)\bigl(210-210\times$$
$$\times(\beta+4)\exp(-c\gamma\tau)+63(\beta^2+9\beta+20)\exp(-2c\gamma\tau)-5\times$$
$$\times(\beta^3+15\beta^2+74\beta+120)\exp(-3c\gamma\tau)\bigr);$$

$$\int P_4^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3780c\gamma}\exp\bigl(-(2s+1)c\gamma\tau/2\bigr)\times$$
$$\times\bigl(7560-10080(\beta+5)\exp(-c\gamma\tau)+4536(\beta^2+11\beta+30)\times$$
$$\times\exp(-2c\gamma\tau)-720(\beta^3+18\beta^2+107\beta+210)\exp(-3c\gamma\tau)+35\times$$
$$\times(\beta^4+26\beta^3+251\beta^2+1066\beta+1680)\exp(-4c\gamma\tau)\bigr);$$

$$\int P_5^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{41580c\gamma}\exp\bigl(-(2s+1)c\gamma\tau/2\bigr)\times$$
$$\times\bigl(83160-138600(\beta+6)\exp(-c\gamma\tau)+83160(\beta^2+13\beta+42)\times$$
$$\times\exp(-2c\gamma\tau)-19800(\beta^3+21\beta^2+146\beta+336)\exp(-3c\gamma\tau)+$$
$$+1925(\beta^4+30\beta^3+335\beta^2+1650\beta+3024)\exp(-4c\gamma\tau)-63\times$$
$$\times(\beta^5+40\beta^4+635\beta^3+5000\beta^2+19524\beta+30240)\exp(-5c\gamma\tau)\bigr).$$



а) б)

Рис. 1.126. Вид неопределенного интеграла от ортогональных функций Якоби 2-ого порядка: а) $\gamma = 2$, $\beta \in [0;5]$; б) $\gamma \in (1;3,5]$, $\beta = 1$

[1.127]
$$\int \tau P_k^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}\times$$
$$\times(-1)^s\exp\bigl(-(2s+1)c\gamma\tau/2\bigr)\left(\frac{2\tau}{c\gamma(2s+1)}+\right.$$
$$\left.+\frac{4}{c^2\gamma^2(2s+1)^2}\right).$$

Частные случаи для неопределенного интеграла 1-ого рода от функций 0-5 порядков:

$$\int \tau P_0^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^2\gamma^2}\exp\Bigl(-\frac{c\gamma\tau}{2}\Bigr)(\gamma\tau+2);$$

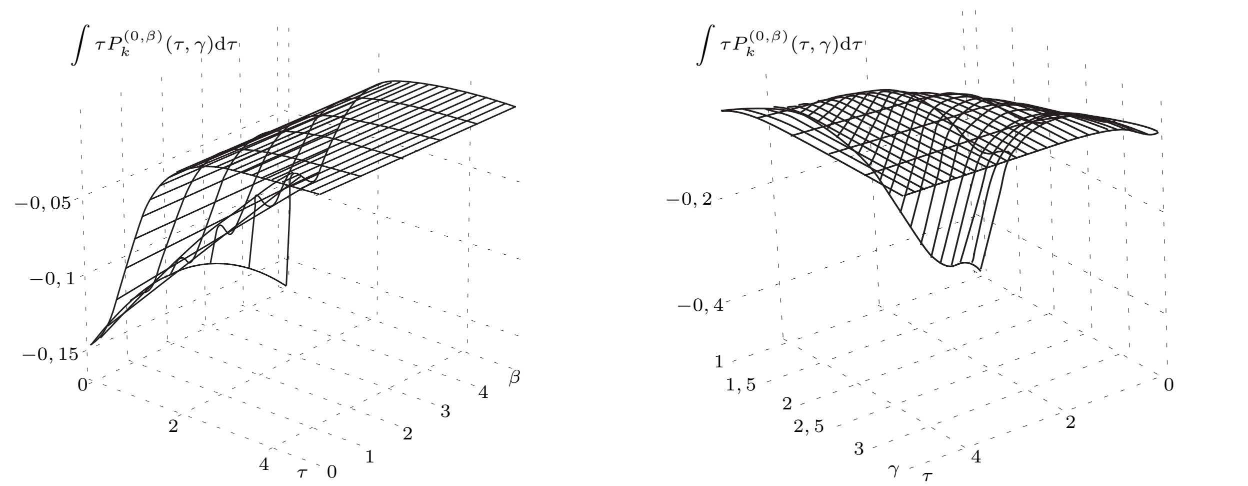
$$\int \tau P_1^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{9c^2\gamma^2}\exp\Bigl(-\frac{c\gamma\tau}{2}\Bigr)\bigl(18-2(\beta+2)\times$$
$$\times\exp(-c\gamma\tau)+\gamma\tau(9-3(\beta+2)\exp(-c\gamma\tau))\bigr);$$

$$\int \tau P_2^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{225c^2\gamma^2}\exp\Big(-\frac{c\gamma\tau}{2}\Big)\big(450(c\gamma\tau+2) - 100(\beta+3)(3c\gamma\tau+2)\exp(-c\gamma\tau) + 9(\beta^2+7\beta+12)(5c\gamma\tau+2)\times \exp(-2c\gamma\tau)\big);$$

$$\int \tau P_3^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3675c^2\gamma^2}\exp\Big(-\frac{c\gamma\tau}{2}\Big)\big(7350(c\gamma\tau+2) - 2450(\beta+4)(3c\gamma\tau+2)\exp(-c\gamma\tau) + 441(\beta^2+9\beta+20)\times (5c\gamma\tau+2)\exp(-2c\gamma\tau) - 25(\beta^3+15\beta^2+74\beta+120)(7c\gamma\tau+2)\times \exp(-3c\gamma\tau)\big);$$

$$\int \tau P_4^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{1190700c^2\gamma^2}\exp\Big(-\frac{c\gamma\tau}{2}\Big)\times \big(2381400(c\gamma\tau+2) - 1058400(\beta+5)(3c\gamma\tau+2)\exp(-c\gamma\tau) + 285768(\beta^2+11\beta+30)(5c\gamma\tau+2)\exp(-2c\gamma\tau) - 32400(\beta^3+18\times \beta^2+107\beta+210)(7c\gamma\tau+2)\exp(-3c\gamma\tau) + 1225(\beta^4+26\beta^3 + 251\beta^2+1066\beta+1680)(9c\gamma\tau+2)\exp(-4c\gamma\tau)\big);$$

$$\int \tau P_5^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{28814940c^2\gamma^2}\exp\Big(-\frac{c\gamma\tau}{2}\Big)\times \big(57629880(c\gamma\tau+2) - 32016600(\beta+6)(3c\gamma\tau+2)\exp(-c\gamma\tau) + 11525976(\beta^2+13\beta+42)(5c\gamma\tau+2)\exp(-2c\gamma\tau) - 1960200\times (\beta^3+21\beta^2+146\beta+336)(7c\gamma\tau+2)\exp(-3c\gamma\tau) + 148225(\beta^4 + 30\beta^3+335\beta^2+1650\beta+3024)(9c\gamma\tau+2)\exp(-4c\gamma\tau) - 3969\times (\beta^5+40\beta^4+635\beta^3+5000\beta^2+19524\beta+30240)(11c\gamma\tau+2)\times \exp(-5c\gamma\tau)\big).$$



а) **б)**

Рис. 1.127. Вид неопределенного интеграла 1-ого рода от ортогональных функций Якоби 2-ого порядка: а) $\gamma = 2$, $\beta \in [0;5]$; б) $\gamma \in (1;3,5]$, $\beta = 1$

[1.128]
$$\int \tau^2 P_k^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}\times (-1)^s\exp\Big(-\frac{(2s+1)}{2}c\gamma\tau\Big)\bigg(\frac{2\tau^2}{c\gamma(2s+1)} + \frac{8\tau}{c^2\gamma^2(2s+1)^2} + \frac{16}{c^3\gamma^3(2s+1)^3}\bigg).$$

Частные случаи для неопределенного интеграла 2-ого рода от функций 0-5 порядков:

$$\int \tau^2 P_0^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{c^3\gamma^3}\exp\Big(-\frac{c\gamma\tau}{2}\Big)(8+4c\gamma\tau+c^2\gamma^2\tau^2);$$

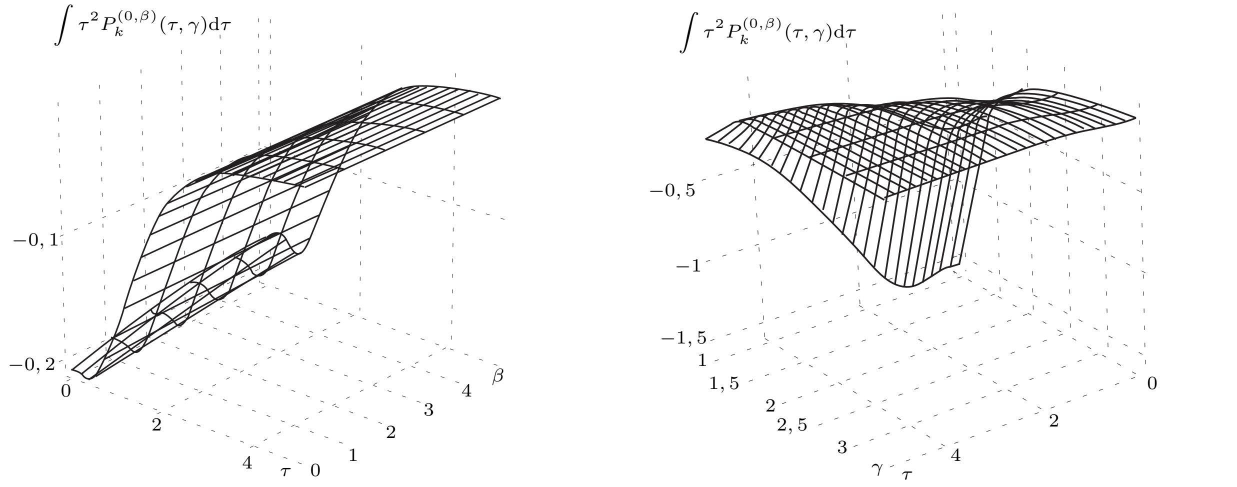
$$\int \tau^2 P_1^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{27c^3\gamma^3}\exp\Big(-\frac{c\gamma\tau}{2}\Big)\big(54(c^2\gamma^2\tau^2 + 4c\gamma\tau+8) - 2(\beta+2)(9c^2\gamma^2\tau^2+12c\gamma\tau+8)\exp(-c\gamma\tau)\big);$$

$$\int \tau^2 P_2^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{3375c^3\gamma^3}\exp\Big(-\frac{c\gamma\tau}{2}\Big)\big(6750(c^2\gamma^2\tau^2 + 4c\gamma\tau+8) - 500(\beta+3)(9c^2\gamma^2\tau^2+12c\gamma\tau+8)\exp(-c\gamma\tau) + 27(\beta^2+7\beta+12)(25c^2\gamma^2\tau^2+20c\gamma\tau+8)\exp(-2c\gamma\tau)\big);$$

$$\int \tau^2 P_3^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{385875c^3\gamma^3}\exp\Big(-\frac{c\gamma\tau}{2}\Big)\times \big(771750(c^2\gamma^2\tau^2+4c\gamma\tau+8) - 85750(\beta+4)(9c^2\gamma^2\tau^2 + 12c\gamma\tau+8)\exp(-c\gamma\tau) + 9261(\beta^2+9\beta+20)(25c^2\gamma^2\tau^2 + 20c\gamma\tau+8)\exp(-2c\gamma\tau) - 375(\beta^3+15\beta^2+74\beta+120)\times (49c^2\gamma^2\tau^2+28c\gamma\tau+8)\exp(-3c\gamma\tau)\big);$$

$$\int \tau^2 P_4^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{375070500c^3\gamma^3}\exp\Big(-\frac{c\gamma\tau}{2}\Big)\times \big(750141000(c^2\gamma^2\tau^2+4c\gamma\tau+8) - 111132000(\beta+5)(9c^2\gamma^2\tau^2 + 12c\gamma\tau+8)\exp(-c\gamma\tau) + 18003384(\beta^2+11\beta+30)(25c^2\gamma^2\tau^2 + 20c\gamma\tau+8)\exp(-2c\gamma\tau) - 1458000(\beta^3+18\beta^2+107\beta+210)\times (49c^2\gamma^2\tau^2+28c\gamma\tau+8)\exp(-3c\gamma\tau) + 42875(\beta^4+26\beta^3+251\times \beta^2+1066\beta+1680)(81c^2\gamma^2\tau^2+36c\gamma\tau+8)\exp(-4c\gamma\tau)\big);$$

$$\int \tau^2 P_5^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{99843767100c^3\gamma^3}\exp\Big(-\frac{c\gamma\tau}{2}\Big)\times \big(199687534200(c^2\gamma^2\tau^2+4c\gamma\tau+8) - 36979173000(\beta+6)\times (9c^2\gamma^2\tau^2+12c\gamma\tau+8)\exp(-c\gamma\tau) + 7987501368(\beta^2+13\beta + 42)(25c^2\gamma^2\tau^2+20c\gamma\tau+8)\exp(-2c\gamma\tau) - 970299000(\beta^3+21\times \beta^2+146\beta+336)(49c^2\gamma^2\tau^2+28c\gamma\tau+8)\exp(-3c\gamma\tau) + 57066625(\beta^4+30\beta^3+335\beta^2+1650\beta+3024)(81c^2\gamma^2\tau^2 + 36c\gamma\tau+8)\exp(-4c\gamma\tau) - 12502359(\beta^5+40\beta^4+635\beta^3+5000\times \beta^2+19524\beta+30240)(121c^2\gamma^2\tau^2+44c\gamma\tau+8)\exp(-5c\gamma\tau)\big).$$



а) б)

Рис. 1.128. Вид неопределенного интеграла 2-ого рода от ортогональных функций Якоби 2-ого порядка: а) $\gamma = 2$, $\beta \in [0; 5]$; б) $\gamma \in (1; 3, 5]$, $\beta = 1$

[1.129]
$$\int \tau^3 P_k^{(0,\beta)}(\tau,\gamma)d\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}\times (-1)^s \exp\bigl(-(2s+1)c\gamma\tau/2\bigr)\left(\frac{2\tau^3}{c\gamma(2s+1)}+\frac{12\tau^2}{c^2\gamma^2(2s+1)^2}+\frac{48\tau}{c^3\gamma^3(2s+1)^3}+\frac{96}{c^4\gamma^4(2s+1)^4}\right).$$

Частные случаи для неопределенного интеграла 3-ого рода от функций 0-5 порядков:

$$\int \tau^3 P_0^{(0,\beta)}(\tau,\gamma)d\tau = -\frac{2}{c^4\gamma^4}\exp\Bigl(-\frac{c\gamma\tau}{2}\Bigr)\bigl(48+24c\gamma\tau+6c^2\gamma^2\tau^2+c^3\gamma^3\tau^3\bigr);$$

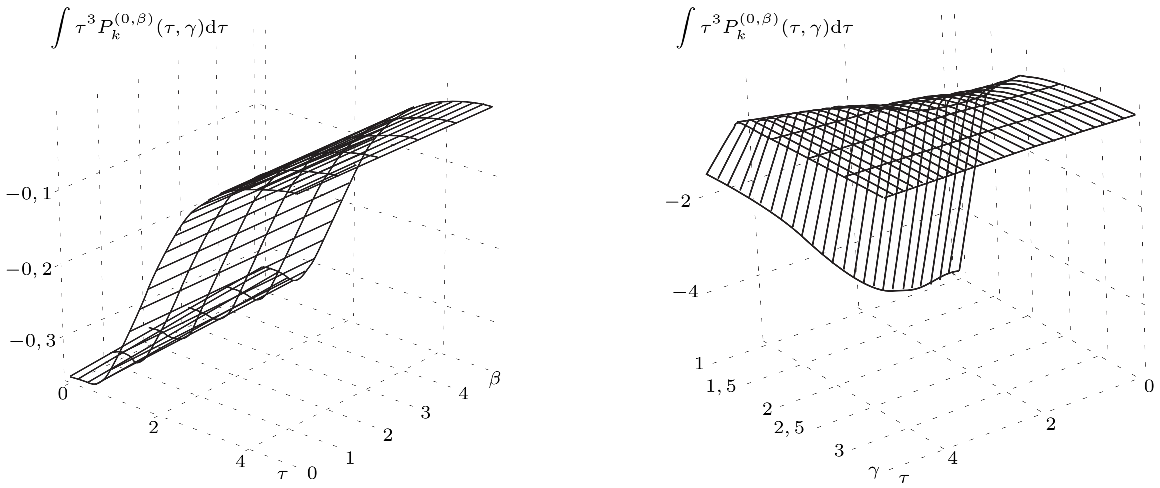
$$\int \tau^3 P_1^{(0,\beta)}(\tau,\gamma)d\tau = -\frac{2}{27c^4\gamma^4}\exp\Bigl(-\frac{c\gamma\tau}{2}\Bigr)\bigl(54(c^3\gamma^3\tau^3+6c^2\gamma^2\tau^2+24c\gamma\tau+48)-2(\beta+2)(9c^3\gamma^3\tau^3+18c^2\gamma^2\tau^2+24c\gamma\tau+16)\exp(-c\gamma\tau)\bigr);$$

$$\int \tau^3 P_2^{(0,\beta)}(\tau,\gamma)d\tau = -\frac{1}{16875c^4\gamma^4}\exp\Bigl(-\frac{c\gamma\tau}{2}\Bigr)\times \bigl(33750(c^3\gamma^3\tau^3+6c^2\gamma^2\tau^2+24c\gamma\tau+48)-2500(\beta+3)\times (9c^3\gamma^3\tau^3+18c^2\gamma^2\tau^2+24c\gamma\tau+16)\exp(-c\gamma\tau)+27(\beta^2+7\beta+12)(125c^3\gamma^3\tau^3+150c^2\gamma^2\tau^2+120c\gamma\tau+48)\exp(-2c\gamma\tau)\bigr);$$

$$\int \tau^3 P_3^{(0,\beta)}(\tau,\gamma)d\tau = -\frac{1}{13505625c^4\gamma^4}\exp\Bigl(-\frac{c\gamma\tau}{2}\Bigr)\times \bigl(27011250(c^3\gamma^3\tau^3+6c^2\gamma^2\tau^2+24c\gamma\tau+48)-3001250(\beta+4)\times (9c^3\gamma^3\tau^3+18c^2\gamma^2\tau^2+24c\gamma\tau+16)\exp(-c\gamma\tau)+64827(\beta^2+9\beta+20)(125c^3\gamma^3\tau^3+150c^2\gamma^2\tau^2+120c\gamma\tau+48)\exp(-2c\gamma\tau)-1875(\beta^3+15\beta^2+74\beta+120)(343c^3\gamma^3\tau^3+294c^2\gamma^2\tau^2+168c\gamma\tau+48)\exp(-3c\gamma\tau)\bigr);$$

$$\int \tau^3 P_4^{(0,\beta)}(\tau,\gamma)d\tau = -\frac{1}{39382402500c^4\gamma^4}\exp\Bigl(-\frac{c\gamma\tau}{2}\Bigr)\times \bigl(78764805000(c^3\gamma^3\tau^3+6c^2\gamma^2\tau^2+24c\gamma\tau+48)-11668860000\times (\beta+5)(9c^3\gamma^3\tau^3+18c^2\gamma^2\tau^2+24c\gamma\tau+16)\exp(-c\gamma\tau)+378071064(\beta^2+11\beta+30)(125c^3\gamma^3\tau^3+150c^2\gamma^2\tau^2+120c\gamma\tau+48)\exp(-2c\gamma\tau)-21870000(\beta^3+18\beta^2+107\beta+210)\times (343c^3\gamma^3\tau^3+294c^2\gamma^2\tau^2+168c\gamma\tau+48)\exp(-3c\gamma\tau)+1500625(\beta^4+26\beta^3+251\beta^2+1066\beta+1680)(243c^3\gamma^3\tau^3+162c^2\gamma^2\tau^2+72c\gamma\tau+16)\exp(-4c\gamma\tau)\bigr);$$

$$\int \tau^3 P_5^{(0,\beta)}(\tau,\gamma)d\tau = -\frac{1}{9609962583375c^4\gamma^4}\exp\Bigl(-\frac{c\gamma\tau}{2}\Bigr)\times \bigl(19219925166750(c^3\gamma^3\tau^3+6c^2\gamma^2\tau^2+24c\gamma\tau+48)-3559245401250(\beta+6)(9c^3\gamma^3\tau^3+18c^2\gamma^2\tau^2+24c\gamma\tau+16)\times \exp(-c\gamma\tau)+307518802668(\beta^2+13\beta+42)(125c^3\gamma^3\tau^3+150c^2\gamma^2\tau^2+120c\gamma\tau+48)\exp(-2c\gamma\tau)-80049667500(\beta^3+21\times \beta^2+146\beta+336)(343c^3\gamma^3\tau^3+294c^2\gamma^2\tau^2+168c\gamma\tau+48)\times \exp(-3c\gamma\tau)+43941301250(\beta^4+30\beta^3+335\beta^2+1650\beta+3024)(243c^3\gamma^3\tau^3+162c^2\gamma^2\tau^2+72c\gamma\tau+16)\exp(-4c\gamma\tau)-1312746750(\beta^5+40\beta^4+635\beta^3+5000\beta^2+19524\beta+30240)(1331c^3\gamma^3\tau^3+726c^2\gamma^2\tau^2+264c\gamma\tau+48)\exp(-5c\gamma\tau)\bigr).$$



а) **б)**

Рис. 1.129. Вид неопределенного интеграла 3-ого рода от ортогональных функций Якоби 2-ого порядка: а) $\gamma = 2$, $\beta \in [0; 5]$; б) $\gamma \in (1; 3,5]$, $\beta = 1$

[1.130]
$$\int \tau^n P_k^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}\times$$
$$\times (-1)^s \exp\bigl(-(2s+1)c\gamma\tau/2\bigr)\times$$
$$\times \sum_{j=0}^{n}\frac{n!\tau^{n-j}}{(n-j)!\Bigl(c\gamma(2s+1)/2\Bigr)^{j+1}}.$$

Частные случаи для неопределенного интеграла n-ого рода от функций 0-5 порядков:

$$\int \tau^n P_0^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2n!}{\gamma}\exp(-c\gamma\tau/2)\times$$
$$\times \sum_{j=0}^{n}\Bigl(\frac{2}{c\gamma/2}\Bigr)^j\frac{\tau^{n-j}}{(n-j)!};$$

$$\int \tau^n P_1^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau/2)\times$$
$$\times\Biggl(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma/2\Bigr)^{j+1}} -$$
$$-(\beta+2)n!\exp(-\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(3c\gamma/2\Bigr)^{j+1}}\Biggr);$$

$$\int \tau^n P_2^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau/2)\times$$
$$\times\Biggl(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma/2\Bigr)^{j+1}} -$$
$$-2(\beta+3)n!\exp(-c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(3c\gamma/2\Bigr)^{j+1}} +$$
$$+\binom{\beta+4}{2}n!\exp(-2c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(5c\gamma/2\Bigr)^{j+1}}\Biggr);$$

$$\int \tau^n P_3^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau/2)\times$$
$$\times\Biggl(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma/2\Bigr)^{j+1}} -$$
$$-3(\beta+4)n!\exp(-c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(3c\gamma/2\Bigr)^{j+1}} +$$
$$+3\binom{\beta+5}{2}n!\exp(-2c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(5c\gamma/2\Bigr)^{j+1}} -$$
$$-\binom{\beta+6}{3}n!\exp(-3c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(7c\gamma/2\Bigr)^{j+1}}\Biggr);$$

$$\int \tau^n P_4^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau/2)\times$$
$$\times\Biggl(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma/2\Bigr)^{j+1}} -$$
$$-4(\beta+5)n!\exp(-c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(3c\gamma/2\Bigr)^{j+1}} +$$
$$+6\binom{\beta+6}{2}n!\exp(-2c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(5c\gamma/2\Bigr)^{j+1}} -$$
$$-4\binom{\beta+7}{3}n!\exp(-3c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(7c\gamma/2\Bigr)^{j+1}} +$$
$$+\binom{\alpha+8}{4}n!\exp(-4c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(9c\gamma/2\Bigr)^{j+1}}\Biggr);$$

$$\int \tau^n P_5^{(0,\beta)}(\tau,\gamma)\mathrm{d}\tau = -\exp(-c\gamma\tau/2)\times$$
$$\times\Biggl(n!\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(c\gamma/2\Bigr)^{j+1}} -$$
$$-5(\beta+6)n!\exp(-c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(3c\gamma/2\Bigr)^{j+1}} +$$
$$+10\binom{\beta+7}{2}n!\exp(-2c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Bigl(5c\gamma/2\Bigr)^{j+1}} -$$

$$- 10\binom{\beta+8}{3} n!\exp(-3c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(7c\gamma/2\Big)^{j+1}} +$$
$$+ 5\binom{\beta+9}{4} n!\exp(-4c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(9c\gamma/2\Big)^{j+1}} -$$
$$- \binom{\beta+10}{5} n!\exp(-5c\gamma\tau)\sum_{j=0}^{n}\frac{\tau^{n-j}}{(n-j)!\Big(11c\gamma/2\Big)^{j+1}}\Bigg).$$

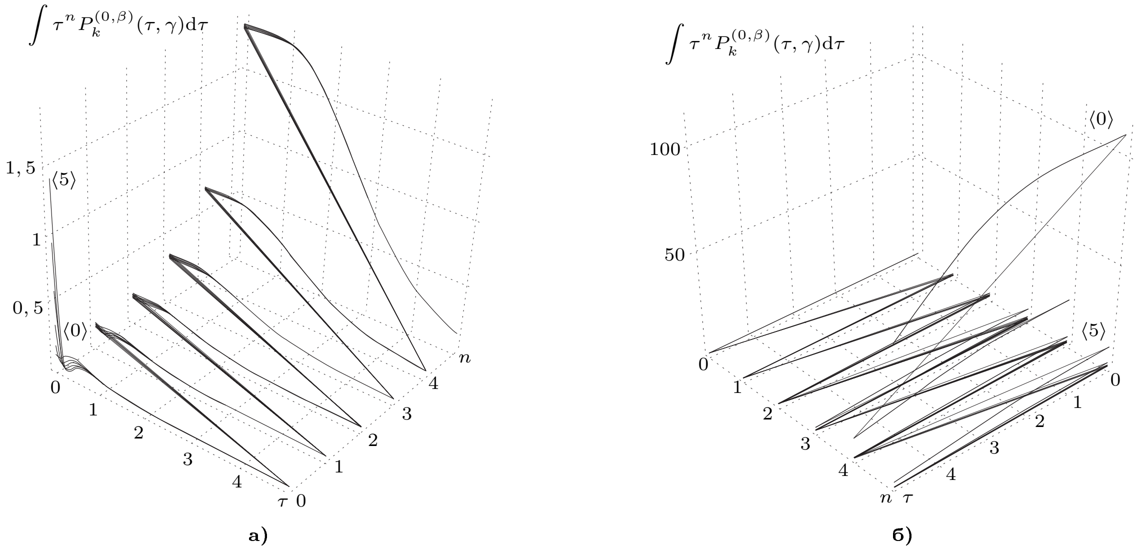


Рис. 1.130. Вид неопределенного интеграла n-ого рода от ортогональных функций Якоби 2-ого порядка: а) $n = 0..5$, $\gamma = 2$, $\beta \in [0;5]$; б) $n = 0..5$, $\gamma \in (1;3,5]$, $\beta = 1$

Глава 2

Основные и расширенные свойства во временной области

Определение.

Для ортогональных функций, определенных в Главе 1, определены основные свойства [13].

Вид функций $\psi_k(\tau,\gamma)$	Норма $\left\|\psi_k\right\|^2$	Вес $\mu^{\{\psi_k(\tau,\gamma)\}}(\tau,\gamma)$	Значение в «нуле» $\psi_k(0,\gamma)$
$P_k^{(\alpha,0)}(\tau,\gamma)$	$\dfrac{1}{c\gamma(2k+\alpha+1)}$	1	$(-1)^k$
$P_k^{(0,\beta)}(\tau,\gamma)$	$\dfrac{1}{c\gamma(2k+\beta+1)}$	$\big(1-\exp(-c\gamma\tau)\big)^\beta$	$(-1)^k\dbinom{k+\beta}{k}$
$L_k^{(\alpha)}(\tau,\gamma)$	$\dfrac{(k+\alpha)!}{k!\gamma^{\alpha+1}}$	τ^α	$\dbinom{k+\alpha}{k}$

и выявлено расширенное свойство [5]

$$\int_0^\infty \frac{\partial\psi_k(\tau,\gamma)}{\partial\tau}d\tau = -\psi_k(0,\gamma).$$

Таблица 2.1. Основные и расширенные свойства во временной области

Вид многочлена $\psi_k(\tau,\gamma)$	Весовая функция $\mu^{\{\psi_k(\tau,\gamma)\}}$	Значение в "нуле" $\psi_k(0,\gamma)$	Интегральная характеристика $\int\limits_0^\infty \frac{\partial\psi_k(\tau,\gamma)}{\partial\tau}\mathrm{d}\tau$
$L_k(\tau,\gamma)$	1	1	-1
$L_k^{(1)}(\tau,\gamma)$	τ	$k+1$	$-k-1$
$L_k^{(2)}(\tau,\gamma)$	τ^2	$\frac{(k+1)(k+2)}{2}$	$-\frac{(k+1)(k+2)}{2}$
$L_k^{(\alpha)}(\tau,\gamma)$	τ^α	$\binom{k+\alpha}{\alpha}$	$-\binom{k+\alpha}{\alpha}$
$P_k^{(-1/2,0)}(\tau,\gamma)$	1	$(-1)^k$	$(-1)^{k+1}$
$Leg_k(\tau,\gamma)$	1	$(-1)^k$	$(-1)^{k+1}$
$P_k^{(1/2,0)}(\tau,\gamma)$	1	$(-1)^k$	$(-1)^{k+1}$
$P_k^{(1,0)}(\tau,\gamma)$	1	$(-1)^k$	$(-1)^{k+1}$
$P_k^{(2,0)}(\tau,\gamma)$	1	$(-1)^k$	$(-1)^{k+1}$
$P_k^{(\alpha,0)}(\tau,\gamma)$	1	$(-1)^k$	$(-1)^{k+1}$
$P_k^{(0,1)}(\tau,\gamma)$	$1-\exp(-c\gamma\tau)$	$(-1)^k(k+1)$	$(-1)^{k+1}(k+1)$
$P_k^{(0,2)}(\tau,\gamma)$	$\big(1-\exp(-c\gamma\tau)\big)^2$	$(-1)^k\frac{(k+1)(k+2)}{2}$	$(-1)^{k+1}\frac{(k+1)(k+2)}{2}$
$P_k^{(0,\beta)}(\tau,\gamma)$	$\big(1-\exp(-c\gamma\tau)\big)^\beta$	$(-1)^k\binom{k+\beta}{\beta}$	$(-1)^{k+1}\binom{k+\beta}{\beta}$

Глава 3

Основные и расширенные соотношения ортогональности во временной области

Определение.

Основное соотношение ортогональности имеет значения только по главной диагонали

$$\int_0^\infty \psi_k(\tau,\gamma)\psi_\nu(\tau,\gamma)\mu^{\{\psi_\nu(\tau,\gamma)\}}(\tau,\gamma)d\tau = g_{k,\nu}(\gamma),$$

$$G(\gamma) = diag\left\{g_{0,0}(\gamma), g_{1,1}(\gamma), ..., g_{K,K}(\gamma)\right\}.$$

Значения диагональных элементов матрицы $g_{k,\nu}(\gamma)$ приведены в Главе 1.

В отличие от основного соотношения в расширенном соотношении ортогональности [5]

$$\int_0^\infty \vartheta_k(\tau,\gamma)\psi_\nu(\tau,\gamma)\mu^{\{\psi_\nu(\tau,\gamma)\}}(\tau,\gamma)d\tau = h_{k,\nu}(\gamma) \quad (k = 0..K, \nu = 0..K)$$

элементы располагаются по нескольким смежным диагоналям, например,

$$H(\gamma) = \begin{pmatrix} h_{0,0}(\gamma) & h_{1,0}(\gamma) & 0 & 0 & \dots & 0 \\ h_{0,1}(\gamma) & h_{1,1}(\gamma) & h_{2,1}(\gamma) & 0 & \dots & 0 \\ 0 & h_{1,2}(\gamma) & h_{2,2}(\gamma) & h_{3,2}(\gamma) & \dots & 0 \\ 0 & 0 & h_{2,3}(\gamma) & h_{3,3}(\gamma) & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & h_{K-1,K}(\gamma) \\ 0 & 0 & 0 & \dots & h_{K,K-1}(\gamma) & h_{K,K}(\gamma) \end{pmatrix}.$$

3.1 Основные соотношения ортогональности

[3.1] $$\int_0^\infty L_s(\tau,\gamma)L_k(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \frac{1}{\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.1]} = \frac{1}{\gamma}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

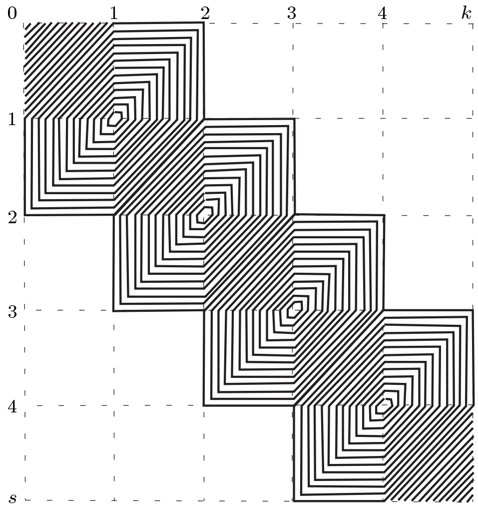


Рис. 3.1. Графическое представление соотношения [3.1] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.2]
$$\int\limits_0^\infty L_s^{(1)}(\tau,\gamma)L_k^{(1)}(\tau,\gamma)\mu^{\{L_s^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{k+1}{\gamma^2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[3.2]} = \frac{1}{\gamma^2}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{pmatrix}.$$

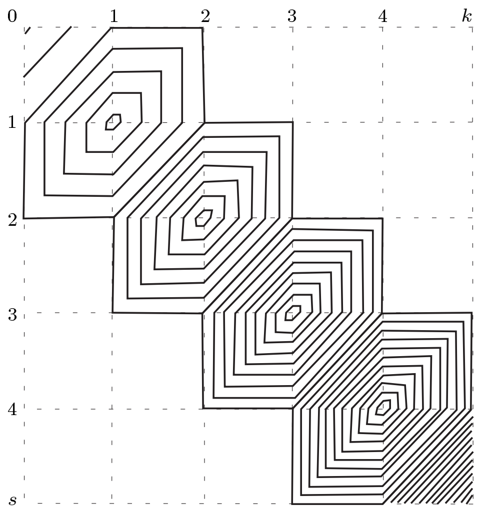


Рис. 3.2. Графическое представление соотношения [3.2] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.3]
$$\int\limits_0^\infty L_s^{(2)}(\tau,\gamma)L_k^{(2)}(\tau,\gamma)\mu^{\{L_s^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{(k+1)(k+2)}{\gamma^3}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[3.3]} = \frac{1}{\gamma^3}\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 30 & 0 \\ 0 & 0 & 0 & 0 & 0 & 42 \end{pmatrix}.$$

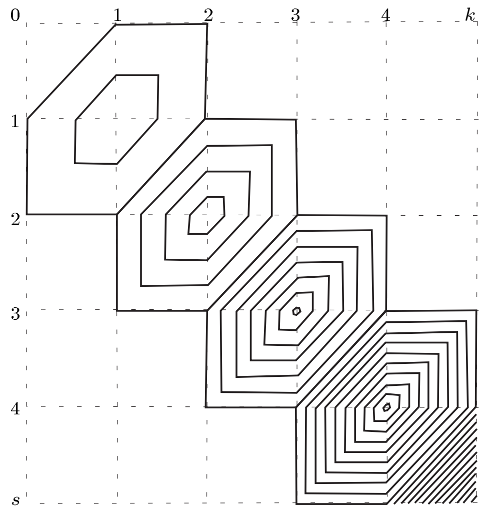


Рис. 3.3. Графическое представление соотношения [3.3] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.4]
$$\int\limits_0^\infty L_s^{(\alpha)}(\tau,\gamma)L_k^{(\alpha)}(\tau,\gamma)\mu^{\{L_s^{(\alpha)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{(k+\alpha)!}{k!\gamma^{\alpha+1}}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[3.4]} = \frac{1}{\gamma^{\alpha+1}} \times$$
$$\times \begin{pmatrix} \alpha! & 0 & 0 & 0 & 0 & 0 \\ 0 & (\alpha+1)! & 0 & 0 & 0 & 0 \\ 0 & 0 & (\alpha+2)! & 0 & 0 & 0 \\ 0 & 0 & 0 & (\alpha+3)! & 0 & 0 \\ 0 & 0 & 0 & 0 & (\alpha+4)! & 0 \\ 0 & 0 & 0 & 0 & 0 & (\alpha+5)! \end{pmatrix}.$$

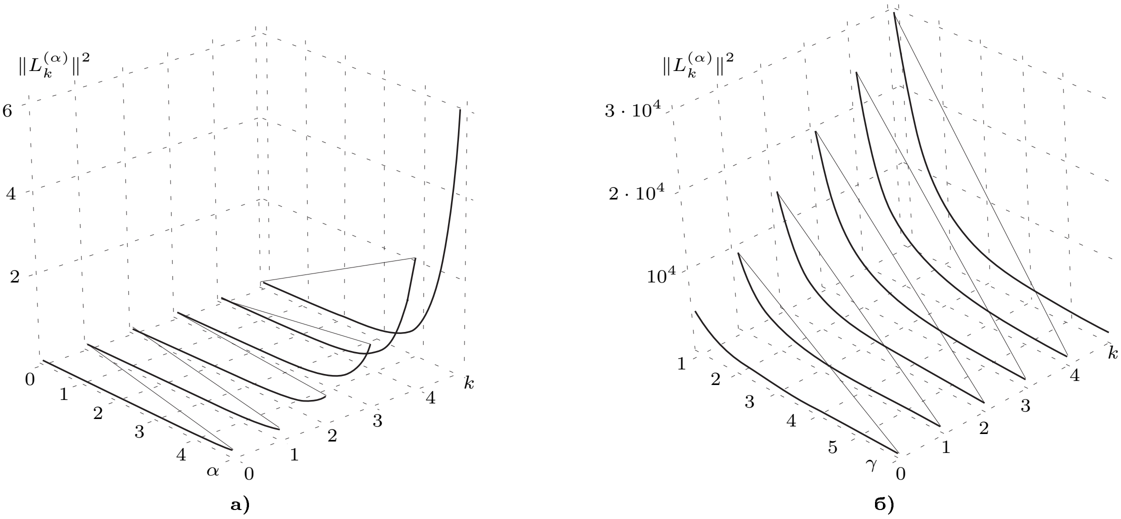


Рис. 3.4. Графическое представление соотношения [3.4] при $k = 0..5$ и $k = s$: а) $\gamma = 1$, $\alpha \in [0; 5]$; б) $\gamma \in [1; 6]$, $\alpha = 1$

[3.5] $$\int\limits_0^\infty P_s^{(-1/2,0)}(\tau,\gamma)P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{1}{\gamma(4k+1)}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.5]} = \frac{1}{\gamma}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/17 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/21 \end{pmatrix}.$$

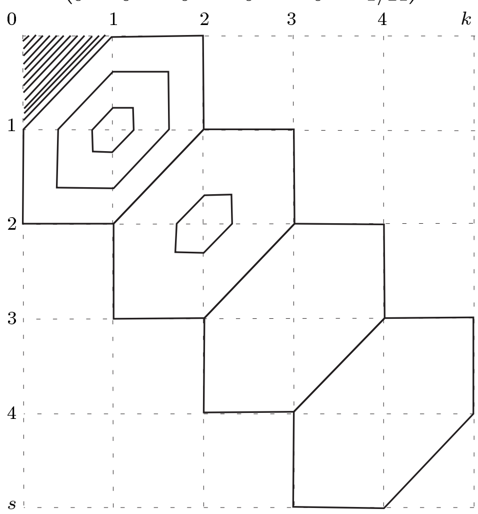


Рис. 3.5. Графическое представление соотношения [3.5] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.6] $$\int\limits_0^\infty Leg_s(\tau,\gamma)Leg_k(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{1}{2\gamma(2k+1)}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.6]} = \frac{1}{2\gamma}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/11 \end{pmatrix}.$$

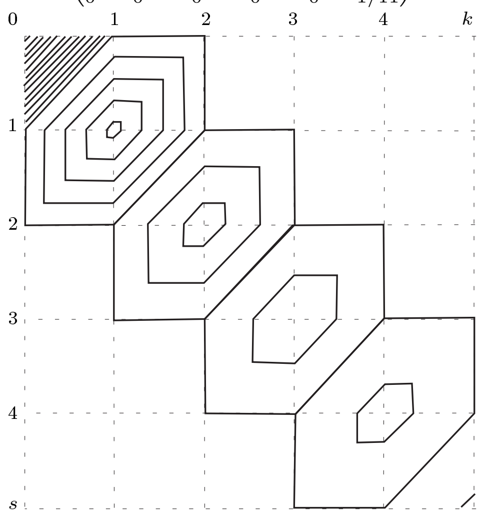


Рис. 3.6. Графическое представление соотношения [3.6] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.7] $$\int_0^\infty P_s^{(1/2,0)}(\tau,\gamma)P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{1}{\gamma(4k+3)}, & \text{если } k=s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.7]} = \frac{1}{\gamma}\begin{pmatrix} 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/15 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/19 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/23 \end{pmatrix}.$$

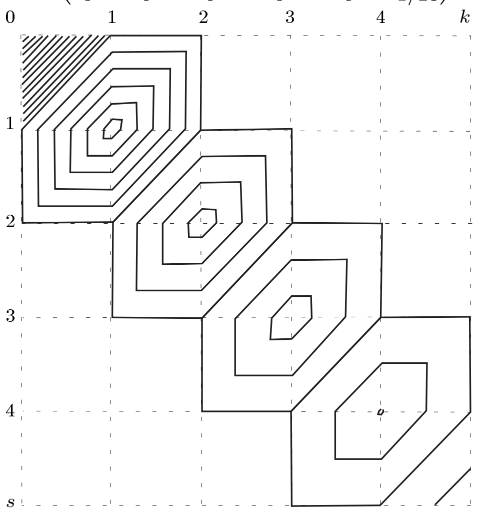


Рис. 3.7. Графическое представление соотношения [3.7] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.8] $$\int_0^\infty P_s^{(1,0)}(\tau,\gamma)P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{1}{2\gamma(k+1)}, & \text{если } k=s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.8]} = \frac{1}{2\gamma}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/6 \end{pmatrix}.$$

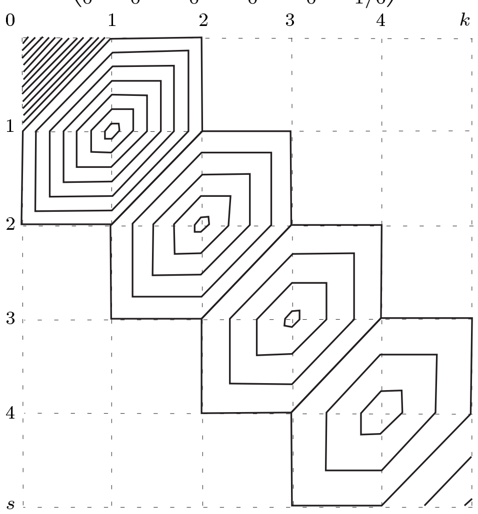


Рис. 3.8. Графическое представление соотношения [3.8] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.9] $$\int_0^\infty P_s^{(2,0)}(\tau,\gamma)P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{1}{2\gamma(2k+3)}, & \text{если } k=s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.9]} = \frac{1}{2\gamma}\begin{pmatrix} 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/13 \end{pmatrix}.$$

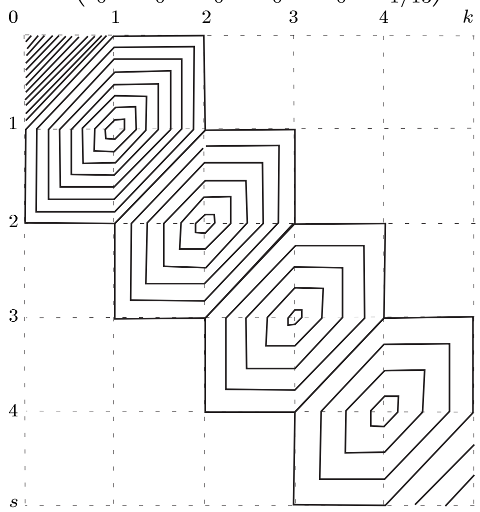


Рис. 3.9. Графическое представление соотношения [3.9] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

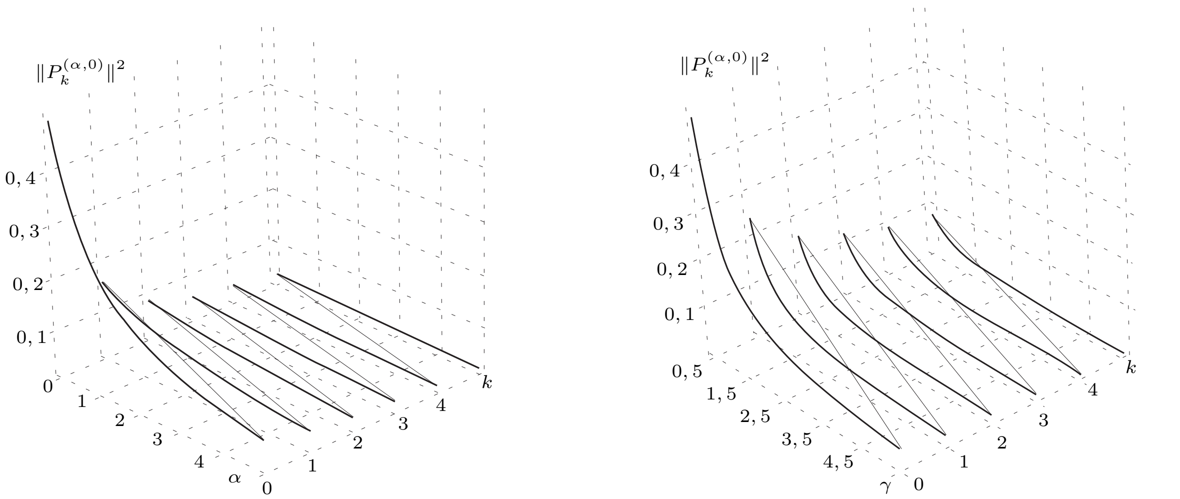
[3.10] $$\int_0^\infty P_s^{(\alpha,0)}(\tau,\gamma)P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{1}{c\gamma(2k+\alpha+1)}, & \text{если } k=s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.10]} = \frac{1}{c\gamma} \times$$

$$\times \begin{pmatrix} \frac{1}{(\alpha+1)} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(\alpha+3)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{(\alpha+5)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{(\alpha+7)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{(\alpha+9)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{(\alpha+11)} \end{pmatrix}.$$



а) б)

Рис. 3.10. Графическое представление соотношения [3.10] при $k = 0..5$ и $k = s$: а) $\gamma = 1$, $c = 2$, $\alpha \in [0; 5]$; б) $\gamma \in [0,5; 5,5]$, $c = 2$, $\alpha = 1$

[3.11]

$$\int_0^\infty P_s^{(0,1)}(\tau,\gamma)P_k^{(0,1)}(\tau,\gamma)\mu^{\{P_s^{(0,1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{1}{4\gamma(k+1)}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.11]} = \frac{1}{4\gamma}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/6 \end{pmatrix}.$$

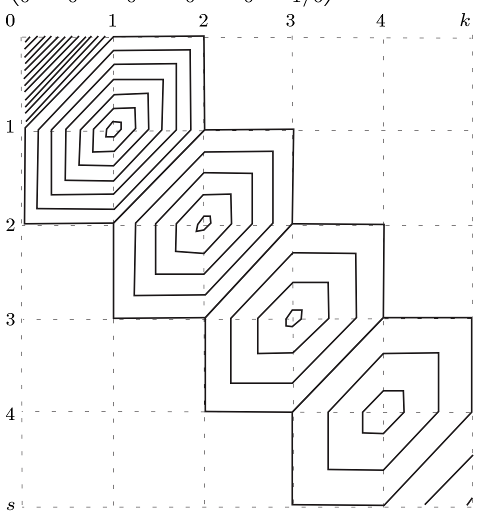


Рис. 3.11. Графическое представление соотношения [3.11] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.12]

$$\int_0^\infty P_s^{(0,2)}(\tau,\gamma)P_k^{(0,2)}(\tau,\gamma)\mu^{\{P_s^{(0,2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{1}{2\gamma(2k+3)}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.12]} = \frac{1}{2\gamma}\begin{pmatrix} 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/13 \end{pmatrix}.$$

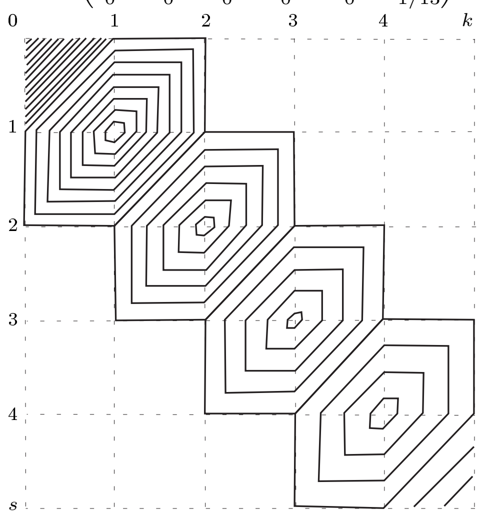


Рис. 3.12. Графическое представление соотношения [3.12] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.13] $$\int\limits_0^\infty P_s^{(0,\beta)}(\tau,\gamma)P_k^{(0,\beta)}(\tau,\gamma)\mu^{\{P_s^{(0,\beta)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{1}{c\gamma(2k+\beta+1)}, & \text{если } k=s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k=0..5$; $s=0..5$:

$$\mathcal{M}_{[3.13]} = \frac{1}{c\gamma} \times \\ \times \begin{pmatrix} \frac{1}{(\beta+1)} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(\beta+3)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{(\beta+5)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{(\beta+7)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{(\beta+9)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{(\beta+11)} \end{pmatrix}.$$

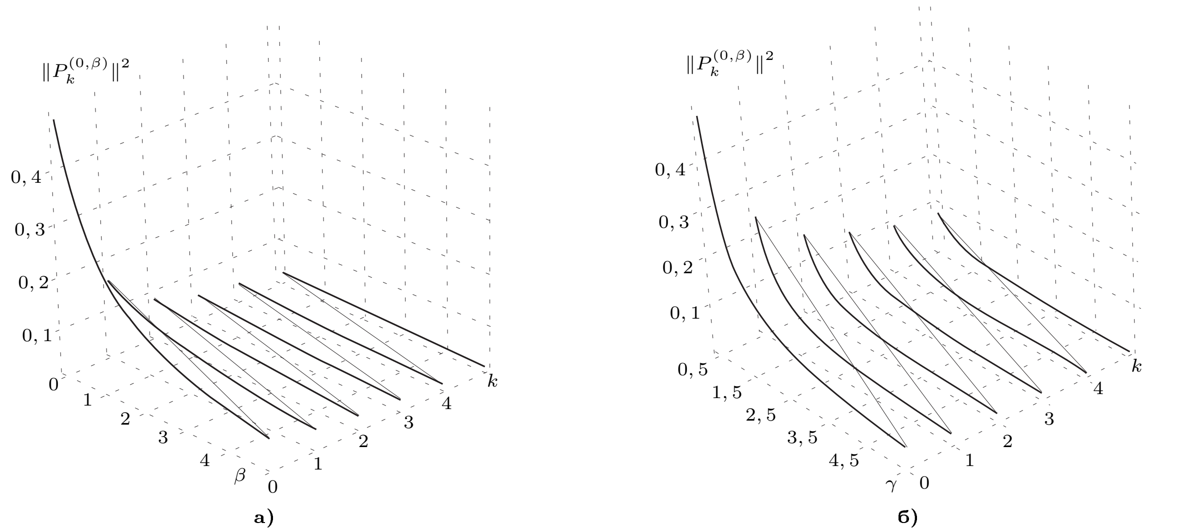


Рис. 3.13. Графическое представление соотношения [3.13] при $k=0..5$ и $k=s$: а) $\gamma=1$, $c=2$, $\alpha\in[0;5]$; б) $\gamma\in[0,5;5,5]$, $c=2$, $\alpha=1$

3.2 Расширенные соотношения ортогональности

[3.14] $$\int\limits_0^\infty L_s^{(1)}(\tau,\gamma)L_k(\tau,\gamma)\mu^{\{L_s^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{k+1}{\gamma^2}, & \text{если } k=s; \\ -\dfrac{k}{\gamma^2}, & \text{если } k=s+1; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k=0..5$; $s=0..5$:

$$\mathcal{M}_{[3.14]} = \frac{1}{\gamma^2}\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -2 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 0 & 4 & -4 & 0 \\ 0 & 0 & 0 & 0 & 5 & -5 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{pmatrix}.$$

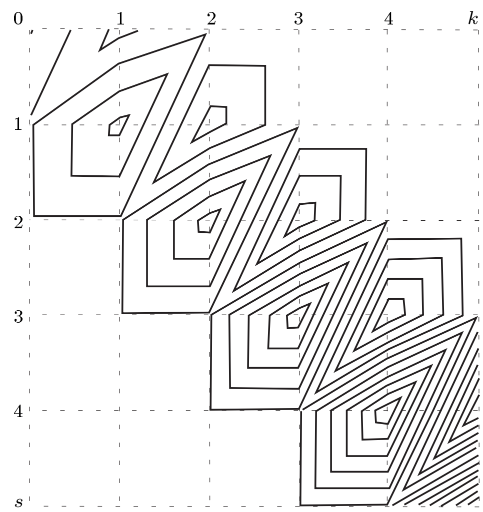


Рис. 3.14. Графическое представление соотношения [3.14] при $k=0..5$, $s=0..5$; $\gamma=1$

[3.15] $$\int\limits_0^\infty \tau L_s(\tau,\gamma)L_k(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} -\dfrac{k+1}{\gamma^2}, & \text{если } k = s-1; \\ \dfrac{2k+1}{\gamma^2}, & \text{если } k = s; \\ -\dfrac{k}{\gamma^2}, & \text{если } k = s+1; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.15]} = \frac{1}{\gamma^2}\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -2 & 0 & 0 & 0 \\ 0 & -2 & 5 & -3 & 0 & 0 \\ 0 & 0 & -3 & 7 & -4 & 0 \\ 0 & 0 & 0 & -4 & 9 & -5 \\ 0 & 0 & 0 & 0 & -5 & 11 \end{pmatrix}.$$

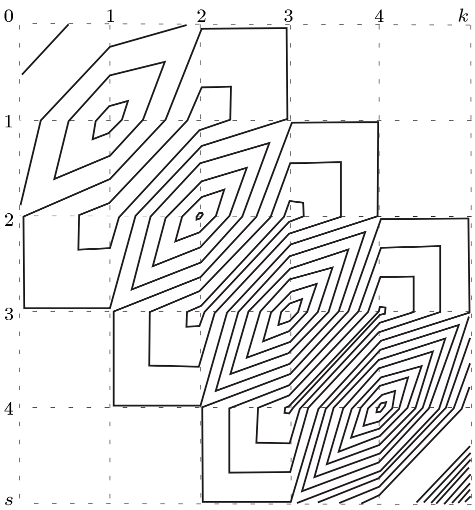


Рис. 3.15. Графическое представление соотношения [3.15] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.16] $$\int\limits_0^\infty L_s^{(2)}(\tau,\gamma)L_k^{(1)}(\tau,\gamma)\mu^{\{L_s^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{(k+1)(k+2)}{\gamma^3}, & \text{если } k = s; \\ -\dfrac{k(k+1)}{\gamma^3}, & \text{если } k = s+1; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.16]} = \frac{1}{\gamma^3}\begin{pmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 6 & -6 & 0 & 0 & 0 \\ 0 & 0 & 12 & -12 & 0 & 0 \\ 0 & 0 & 0 & 20 & -20 & 0 \\ 0 & 0 & 0 & 0 & 30 & -30 \\ 0 & 0 & 0 & 0 & 0 & 42 \end{pmatrix}.$$

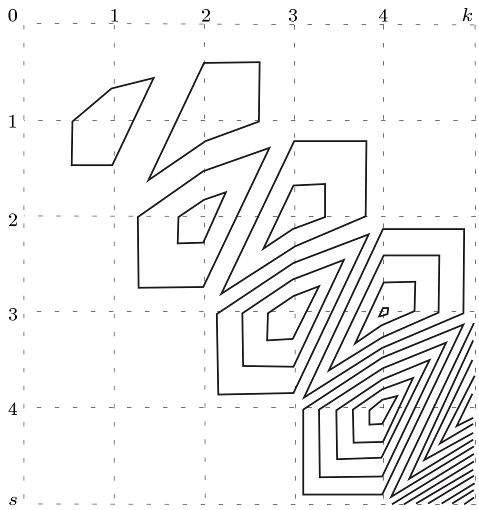


Рис. 3.16. Графическое представление соотношения [3.16] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.17] $$\int\limits_0^\infty \tau L_s^{(1)}(\tau,\gamma)L_k(\tau,\gamma)\mu^{\{L_s^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} -\dfrac{(k+1)(k+2)}{\gamma^3}, & \text{если } k = s-1; \\ \dfrac{(3k+2)(k+1)}{\gamma^3}, & \text{если } k = s; \\ -\dfrac{(3k+1)k}{\gamma^3}, & \text{если } k = s+1; \\ \dfrac{k(k-1)}{\gamma^3}, & \text{если } k = s+2; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.17]} = \frac{1}{\gamma^3}\begin{pmatrix} 2 & -4 & 2 & 0 & 0 & 0 \\ -2 & 10 & -14 & 6 & 0 & 0 \\ 0 & -6 & 24 & -30 & 0 & 0 \\ 0 & 0 & -12 & 44 & -52 & 0 \\ 0 & 0 & 0 & -20 & 70 & -80 \\ 0 & 0 & 0 & 0 & -30 & 102 \end{pmatrix}.$$

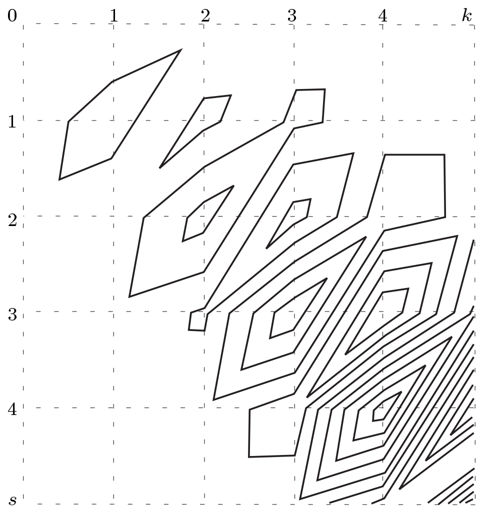


Рис. 3.17. Графическое представление соотношения [3.17] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.18] $$\int_0^\infty L_s(\tau,\gamma)\frac{\partial L_k(\tau,\gamma)}{\partial\tau}\mathrm{d}\tau = \begin{cases} -1, & \text{если } k > s; \\ -\dfrac{1}{2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.18]} = \begin{pmatrix} -1/2 & -1 & -1 & -1 & -1 & -1 \\ 0 & -1/2 & -1 & -1 & -1 & -1 \\ 0 & 0 & -1/2 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1/2 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1/2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1/2 \end{pmatrix}.$$

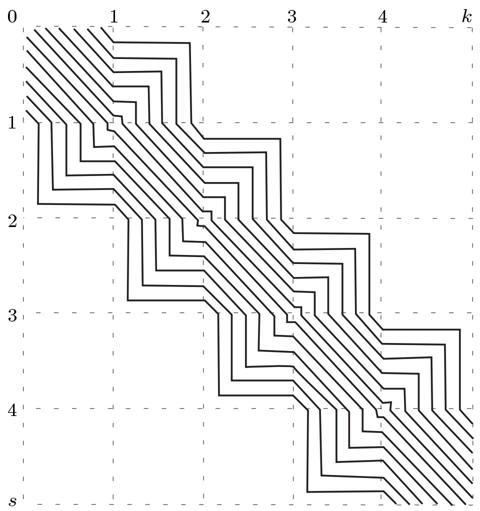


Рис. 3.18. Графическое представление соотношения [3.18] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.19] $$\int_0^\infty L_s(\tau,\gamma)\Big(\int L_k(\tau,\gamma)\mathrm{d}\tau\Big)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{2}{\gamma^2}, & \text{если } k = s; \\ \dfrac{4(-1)^{k+s}}{\gamma^2}, & \text{если } k < s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.19]} = \frac{1}{\gamma^2}\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ -4 & 2 & 0 & 0 & 0 & 0 \\ 4 & -4 & 2 & 0 & 0 & 0 \\ -4 & 4 & -4 & 2 & 0 & 0 \\ 4 & -4 & 4 & -4 & 2 & 0 \\ -4 & 4 & -4 & 4 & -4 & 2 \end{pmatrix}.$$

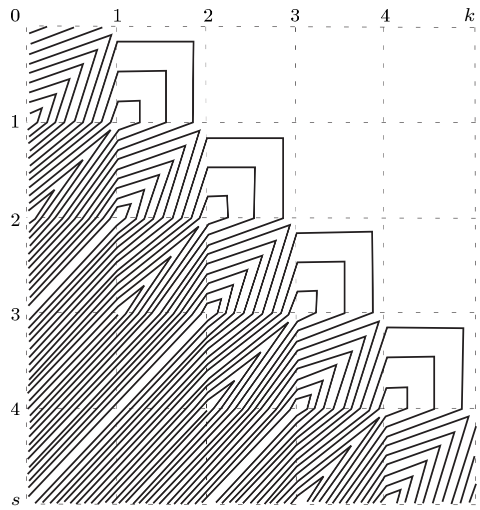


Рис. 3.19. Графическое представление соотношения [3.19] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.20] $$\int_0^\infty L_s(\tau,\gamma)\Big(\int \tau L_k(\tau,\gamma)\mathrm{d}\tau\Big)\mathrm{d}\tau =$$

$$= \begin{cases} -\dfrac{2k}{\gamma^3}, & \text{если } k = s+1; \\ \dfrac{2(4k+1)}{\gamma^3}, & \text{если } k = s; \\ -\dfrac{2(7k+3)}{\gamma^3}, & \text{если } k = s-1; \\ \dfrac{2(8k+4)(-1)^{k+s}}{\gamma^3}, & k < s-1 \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.20]} = \frac{1}{\gamma^3}\begin{pmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ -6 & 10 & -4 & 0 & 0 & 0 \\ 8 & -20 & 18 & -6 & 0 & 0 \\ -8 & 24 & -34 & 26 & -8 & 0 \\ 8 & -24 & 40 & -48 & 34 & -10 \\ -8 & 24 & -40 & 56 & -62 & 42 \end{pmatrix}.$$

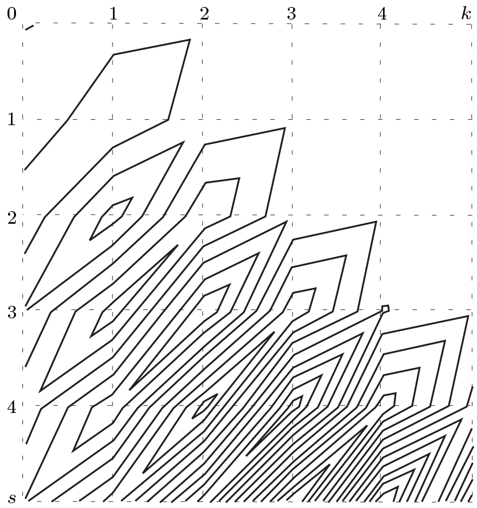


Рис. 3.20. Графическое представление соотношения [3.20] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.21] $$\int\limits_0^\infty L_s(\tau,\gamma) L_k^{(1)}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{1}{\gamma}, & \text{если } k \geq s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.21]} = \frac{1}{\gamma}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

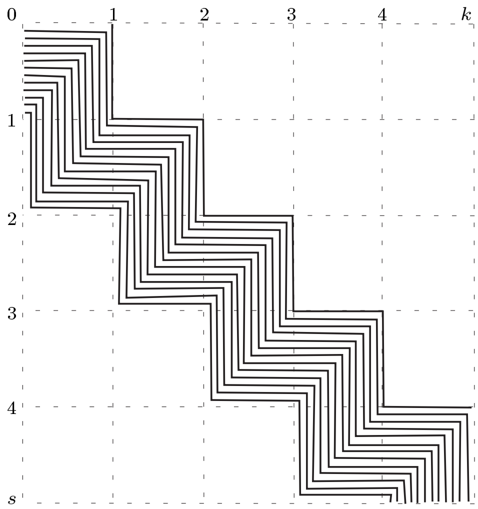


Рис. 3.21. Графическое представление соотношения [3.21] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.22] $$\int\limits_0^\infty \tau L_s(\tau,\gamma) L_k^{(1)}(\tau,\gamma)\mathrm{d}\tau = $$

$$= \begin{cases} \dfrac{k+1}{\gamma^2}, & \text{если } k = s; \\ -\dfrac{k+1}{\gamma^2}, & \text{если } k = s-1; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.22]} = \frac{1}{\gamma^2}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & -2 & 3 & 0 & 0 & 0 \\ 0 & 0 & -3 & 4 & 0 & 0 \\ 0 & 0 & 0 & -4 & 5 & 0 \\ 0 & 0 & 0 & 0 & -5 & 6 \end{pmatrix}.$$

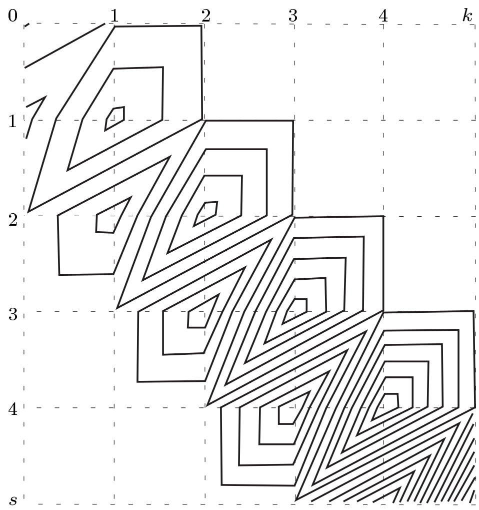


Рис. 3.22. Графическое представление соотношения [3.22] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.23] $$\int\limits_0^\infty L_s(\tau,\gamma) \frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial \tau}\mathrm{d}\tau = $$

$$= \begin{cases} -\dfrac{2(k-s)+1}{2}, & \text{если } k > s; \\ -\dfrac{1}{2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.23]} = \begin{pmatrix} -1/2 & -3/2 & -5/2 & -7/2 & -9/2 & -11/2 \\ 0 & -1/2 & -3/2 & -5/2 & -7/2 & -9/2 \\ 0 & 0 & -1/2 & -3/2 & -5/2 & -7/2 \\ 0 & 0 & 0 & -1/2 & -3/2 & -5/2 \\ 0 & 0 & 0 & 0 & -1/2 & -5/2 \\ 0 & 0 & 0 & 0 & 0 & -1/2 \end{pmatrix}.$$

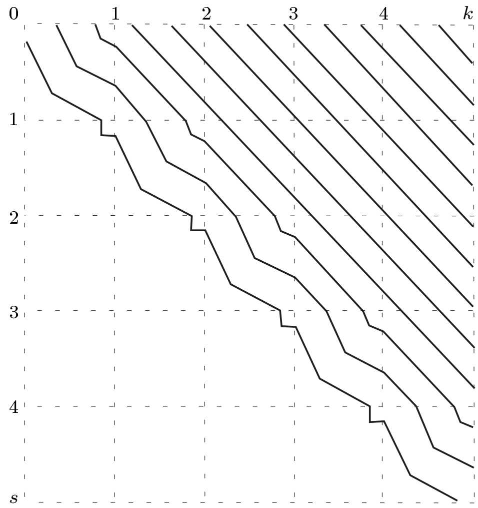


Рис. 3.23. Графическое представление соотношения [3.23] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.24]
$$\int\limits_0^\infty L_s(\tau,\gamma)L_k^{(2)}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{k-s+1}{\gamma}, & \text{если } k > s; \\ \dfrac{1}{\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.24]} = \frac{1}{\gamma}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

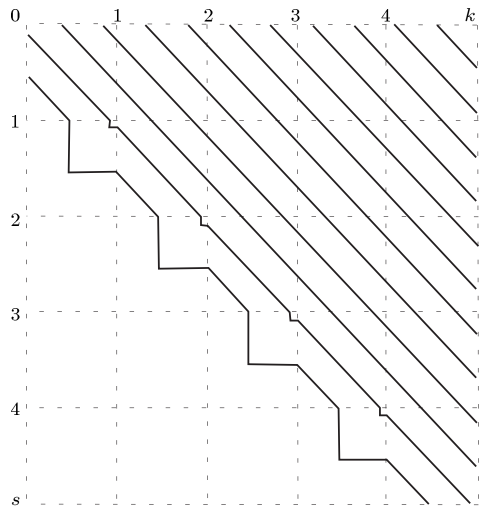


Рис. 3.24. Графическое представление соотношения [3.24] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.25]
$$\int\limits_0^\infty \tau L_s(\tau,\gamma)L_k^{(2)}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} \dfrac{1}{\gamma^2}, & \text{если } k \geq s; \\ -\dfrac{k+1}{\gamma^2}, & \text{если } k = s-1; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.25]} = \frac{1}{\gamma^2}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 1 & 1 & 1 \\ 0 & 0 & 0 & -4 & 1 & 1 \\ 0 & 0 & 0 & 0 & -5 & 1 \end{pmatrix}.$$

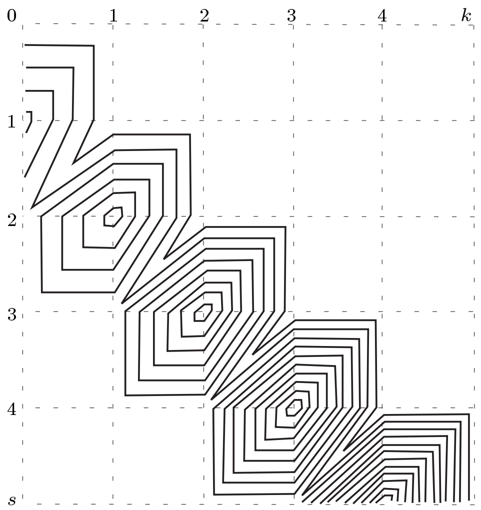


Рис. 3.25. Графическое представление соотношения [3.25] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.26]
$$\int\limits_0^\infty L_s(\tau,\gamma)\frac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau}\mathrm{d}\tau = \begin{cases} -\dfrac{(k-s+1)^2}{2}, & \text{если } k > s;\\ -\dfrac{1}{2}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.26]} = \begin{pmatrix} -1/2 & -2 & -9/2 & -8 & -25/2 & -18\\ 0 & -1/2 & -2 & -9/2 & -8 & -25/2\\ 0 & 0 & -1/2 & -2 & -9/2 & -8\\ 0 & 0 & 0 & -1/2 & -2 & -9/2\\ 0 & 0 & 0 & 0 & -1/2 & -2\\ 0 & 0 & 0 & 0 & 0 & -1/2 \end{pmatrix}.$$

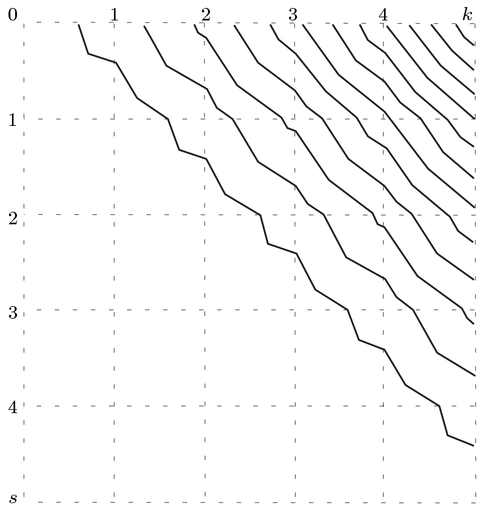


Рис. 3.26. Графическое представление соотношения [3.26] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.27]
$$\int\limits_0^\infty L_s^{(1)}(\tau,\gamma)\frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial\tau}\mu^{\{L_s^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \begin{cases} -\dfrac{s+1}{\gamma}, & \text{если } k > s;\\ -\dfrac{k+1}{\gamma}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.27]} = \frac{1}{\gamma}\begin{pmatrix} -1/2 & -1 & -1 & -1 & -1 & -1\\ 0 & -1 & -2 & -2 & -2 & -2\\ 0 & 0 & -3/2 & -3 & -3 & -3\\ 0 & 0 & 0 & -2 & -4 & -4\\ 0 & 0 & 0 & 0 & -5/2 & -5\\ 0 & 0 & 0 & 0 & 0 & -3 \end{pmatrix}.$$

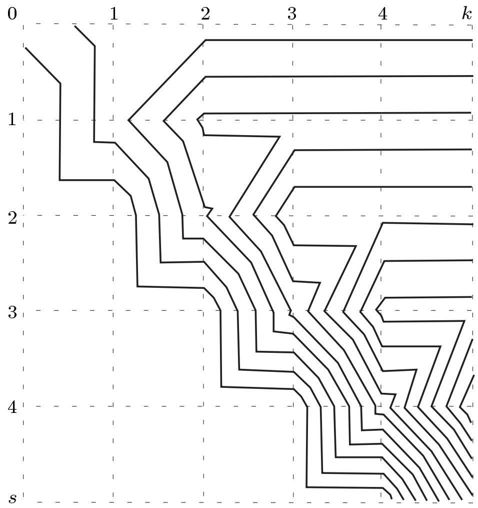


Рис. 3.27. Графическое представление соотношения [3.27] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.28] $$\int\limits_0^\infty \tau L_s^{(1)}(\tau,\gamma)\frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial\tau}\mu^{\{L_s^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} -\dfrac{(s+1)(s+2)}{2\gamma^2}, & \text{если } k = s+1; \\ -\dfrac{k+1}{\gamma^2}, & \text{если } k = s; \\ -\dfrac{s(s+1)}{2\gamma^2}, & \text{если } k = s-1; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.28]} = \frac{1}{\gamma^2}\begin{pmatrix} -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -2 & -3 & 0 & 0 & 0 \\ 0 & 3 & -3 & -6 & 0 & 0 \\ 0 & 0 & 6 & -4 & -10 & 0 \\ 0 & 0 & 0 & 10 & -5 & -15 \\ 0 & 0 & 0 & 0 & 15 & -6 \end{pmatrix}.$$

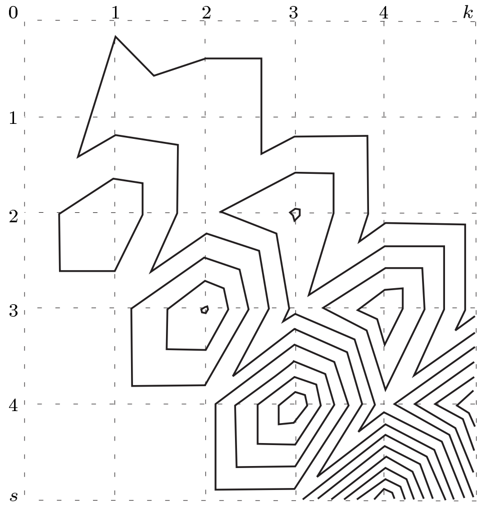


Рис. 3.28. Графическое представление соотношения [3.28] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.29] $$\int\limits_0^\infty L_s^{(2)}(\tau,\gamma)\frac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau}\mu^{\{L_s^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} -\dfrac{(s+1)(s+2)}{\gamma^2}, & \text{если } k > s; \\ -\dfrac{(k+1)(k+2)}{2\gamma^2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.29]} = \frac{1}{\gamma^2}\begin{pmatrix} -1 & -2 & -2 & -2 & -2 & -2 \\ 0 & -3 & -6 & -6 & -6 & -6 \\ 0 & 0 & -6 & -12 & -12 & -12 \\ 0 & 0 & 0 & -10 & -20 & -20 \\ 0 & 0 & 0 & 0 & -15 & -30 \\ 0 & 0 & 0 & 0 & 0 & -21 \end{pmatrix}.$$

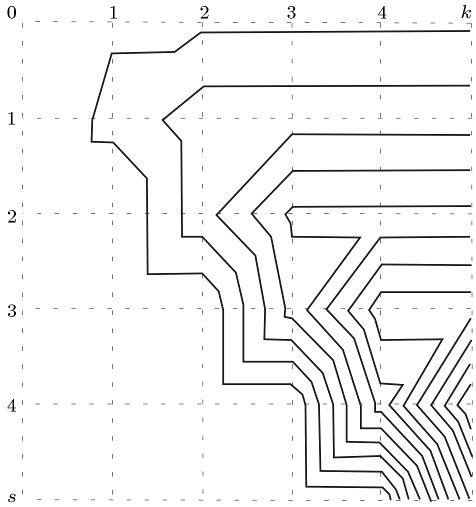


Рис. 3.29. Графическое представление соотношения [3.29] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.30] $$\int\limits_0^\infty \tau L_s^{(2)}(\tau,\gamma)\frac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau}\mu^{\{L_s^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} -\dfrac{(s+1)(s+2)(s+3)}{2\gamma^3}, & \text{если } k = s+1; \\ -\dfrac{3(k+1)(k+2)}{2\gamma^3}, & \text{если } k = s; \\ -\dfrac{s(s+1)(s+2)}{2\gamma^3}, & \text{если } k = s-1; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.30]} = \frac{1}{\gamma^3}\begin{pmatrix} -3 & -3 & 0 & 0 & 0 & 0 \\ 3 & -9 & -12 & 0 & 0 & 0 \\ 0 & 12 & -18 & -30 & 0 & 0 \\ 0 & 0 & 30 & -30 & -60 & 0 \\ 0 & 0 & 0 & 60 & -45 & -105 \\ 0 & 0 & 0 & 0 & 105 & -63 \end{pmatrix}.$$

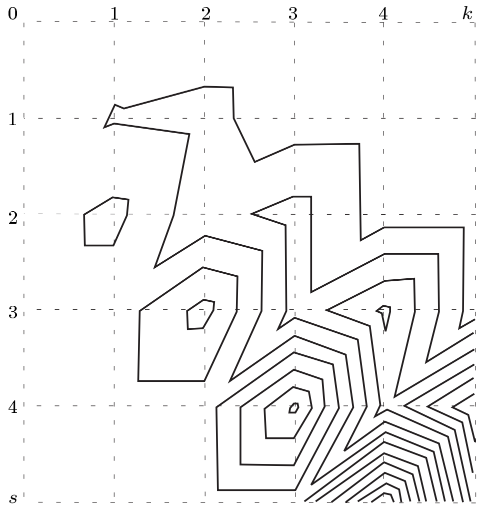


Рис. 3.30. Графическое представление соотношения [3.30] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.31] $$\int\limits_0^\infty P_s^{(0,1)}(\tau,\gamma)Leg_k(\tau,\gamma)\mu^{\{P_s^{(0,1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{1}{4(2k+1)\gamma}, & \text{если } k = s+1; \\ \dfrac{1}{4(2k+1)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.31]} = \frac{1}{\gamma}\begin{pmatrix} 1/4 & 1/12 & 0 & 0 & 0 & 0 \\ 0 & 1/12 & 1/20 & 0 & 0 & 0 \\ 0 & 0 & 1/20 & 1/28 & 0 & 0 \\ 0 & 0 & 0 & 1/28 & 1/36 & 0 \\ 0 & 0 & 0 & 0 & 1/36 & 1/44 \\ 0 & 0 & 0 & 0 & 0 & 1/44 \end{pmatrix}.$$

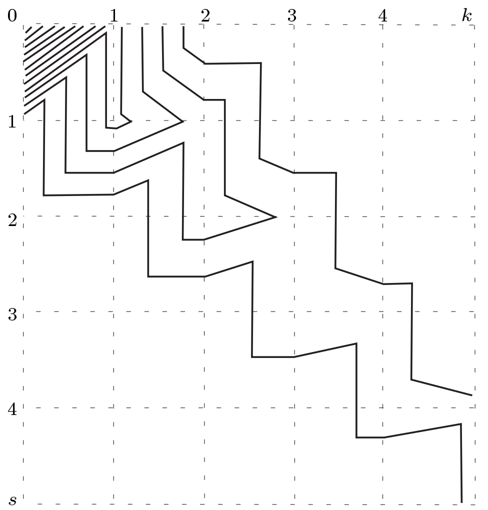


Рис. 3.31. Графическое представление соотношения [3.31] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.32] $$\int\limits_0^\infty P_s^{(0,2)}(\tau,\gamma) Leg_k(\tau,\gamma)\mu^{\{P_s^{(0,2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{k-1}{4(2k-1)(2k+1)\gamma}, & \text{если } k = s+2; \\ \dfrac{1}{4(2k+1)\gamma}, & \text{если } k = s+1; \\ \dfrac{k+2}{4(2k+1)(2k+3)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.32]} = \frac{1}{\gamma}\begin{pmatrix} 1/6 & 1/12 & 1/60 & 0 & 0 & 0 \\ 0 & 1/20 & 1/20 & 1/70 & 0 & 0 \\ 0 & 0 & 1/35 & 1/28 & 1/84 & 0 \\ 0 & 0 & 0 & 5/252 & 1/36 & 1/99 \\ 0 & 0 & 0 & 0 & 7/572 & 1/44 \\ 0 & 0 & 0 & 0 & 0 & 2/195 \end{pmatrix}.$$

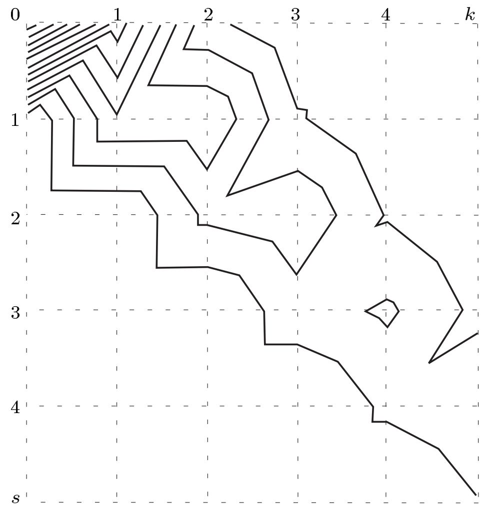


Рис. 3.32. Графическое представление соотношения [3.32] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.33] $$\int\limits_0^\infty P_s^{(0,2)}(\tau,\gamma) P_k^{(0,1)}(\tau,\gamma)\mu^{\{P_s^{(0,2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{k}{4(k+1)(2k+1)\gamma}, & \text{если } k = s+1; \\ \dfrac{k+2}{4(k+1)(2k+3)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$

$$\mathcal{M}_{[3.33]} = \frac{1}{\gamma}\begin{pmatrix} 1/6 & 1/24 & 0 & 0 & 0 & 0 \\ 0 & 3/40 & 1/30 & 0 & 0 & 0 \\ 0 & 0 & 1/21 & 3/112 & 0 & 0 \\ 0 & 0 & 0 & 5/144 & 1/45 & 0 \\ 0 & 0 & 0 & 0 & 3/110 & 5/264 \\ 0 & 0 & 0 & 0 & 0 & 7/312 \end{pmatrix}$$

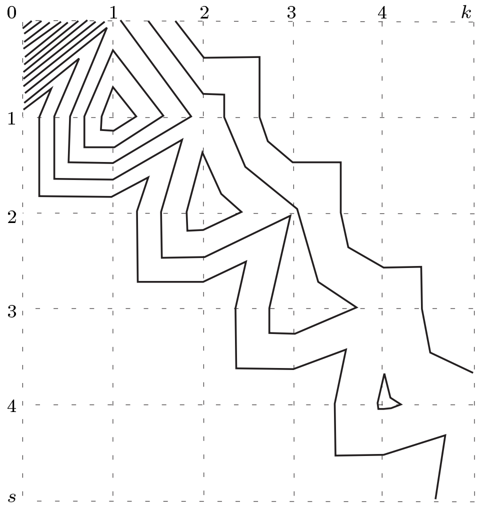


Рис. 3.33. Графическое представление соотношения [3.33] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.34] $$\int\limits_0^\infty P_s^{(-1/2,0)}(\tau,\gamma)\frac{\partial P_k^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\mathrm{d}\tau =$$

$$= \begin{cases} -(-1)^{k+s}, & \text{если } k > s; \\ -\dfrac{1}{2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$

$$\mathcal{M}_{[3.34]} = \begin{pmatrix} -1/2 & 1 & -1 & 1 & -1 & 1 \\ 0 & -1/2 & 1 & -1 & 1 & -1 \\ 0 & 0 & -1/2 & 1 & -1 & 1 \\ 0 & 0 & 0 & -1/2 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1/2 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1/2 \end{pmatrix}$$

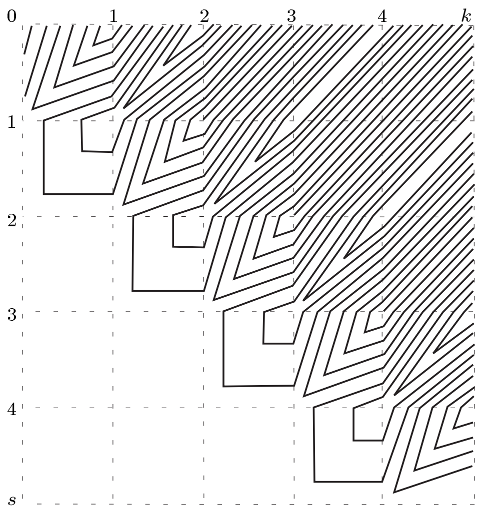


Рис. 3.34. Графическое представление соотношения [3.34] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.35] $$\int\limits_0^\infty Leg_s(\tau,\gamma)\frac{\partial Leg_k(\tau,\gamma)}{\partial\tau}\mathrm{d}\tau =$$

$$= \begin{cases} -(-1)^{k+s}, & \text{если } k > s; \\ -\dfrac{1}{2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$

$$\mathcal{M}_{[3.35]} = \begin{pmatrix} -1/2 & 1 & -1 & 1 & -1 & 1 \\ 0 & -1/2 & 1 & -1 & 1 & -1 \\ 0 & 0 & -1/2 & 1 & -1 & 1 \\ 0 & 0 & 0 & -1/2 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1/2 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1/2 \end{pmatrix}$$

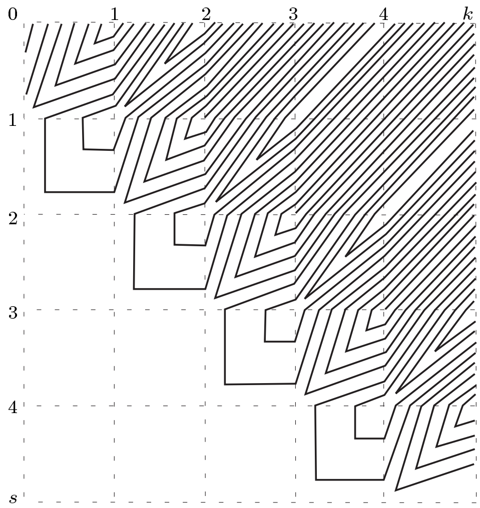


Рис. 3.35. Графическое представление соотношения [3.35] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.36] $$\int\limits_0^\infty P_s^{(1/2,0)}(\tau,\gamma)\frac{\partial P_k^{(1/2,0)}(\tau,\gamma)}{\partial\tau}\mathrm{d}\tau =$$

$$= \begin{cases} -(-1)^{k+s}, & \text{если } k > s; \\ -\dfrac{1}{2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$

$$\mathcal{M}_{[3.36]} = \begin{pmatrix} -1/2 & 1 & -1 & 1 & -1 & 1 \\ 0 & -1/2 & 1 & -1 & 1 & -1 \\ 0 & 0 & -1/2 & 1 & -1 & 1 \\ 0 & 0 & 0 & -1/2 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1/2 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1/2 \end{pmatrix}$$

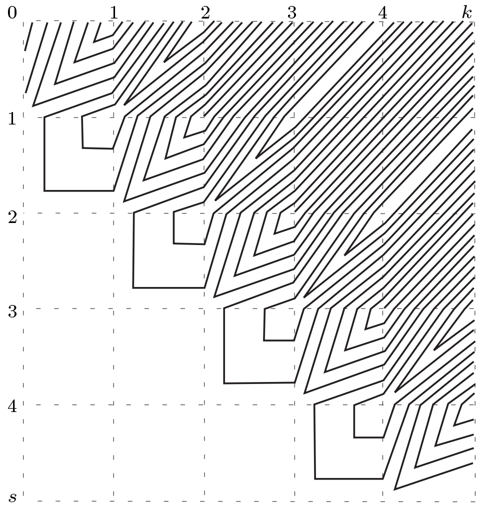


Рис. 3.36. Графическое представление соотношения [3.36] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.37] $$\int\limits_0^\infty P_s^{(1,0)}(\tau,\gamma)\frac{\partial P_k^{(1,0)}(\tau,\gamma)}{\partial \tau}\mathrm{d}\tau =$$

$$= \begin{cases} -(-1)^{k+s}, & \text{если } k > s; \\ -\dfrac{1}{2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$

$$\mathcal{M}_{[3.37]} = \begin{pmatrix} -1/2 & 1 & -1 & 1 & -1 & 1 \\ 0 & -1/2 & 1 & -1 & 1 & -1 \\ 0 & 0 & -1/2 & 1 & -1 & 1 \\ 0 & 0 & 0 & -1/2 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1/2 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1/2 \end{pmatrix}$$

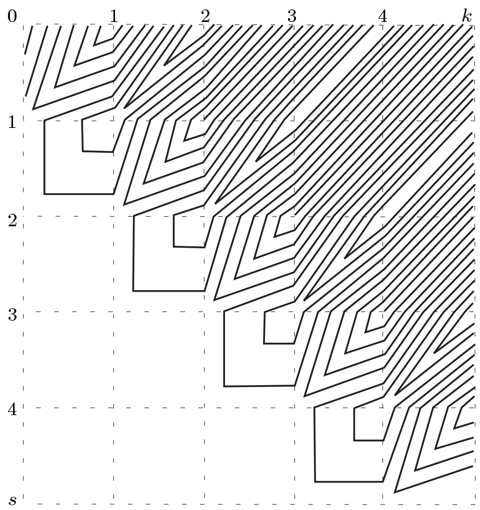


Рис. 3.37. Графическое представление соотношения [3.37] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.38] $$\int\limits_0^\infty P_s^{(2,0)}(\tau,\gamma)\frac{\partial P_k^{(2,0)}(\tau,\gamma)}{\partial \tau}\mathrm{d}\tau =$$

$$= \begin{cases} -(-1)^{k+s}, & \text{если } k > s; \\ -\dfrac{1}{2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$

$$\mathcal{M}_{[3.38]} = \begin{pmatrix} -1/2 & 1 & -1 & 1 & -1 & 1 \\ 0 & -1/2 & 1 & -1 & 1 & -1 \\ 0 & 0 & -1/2 & 1 & -1 & 1 \\ 0 & 0 & 0 & -1/2 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1/2 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1/2 \end{pmatrix}$$

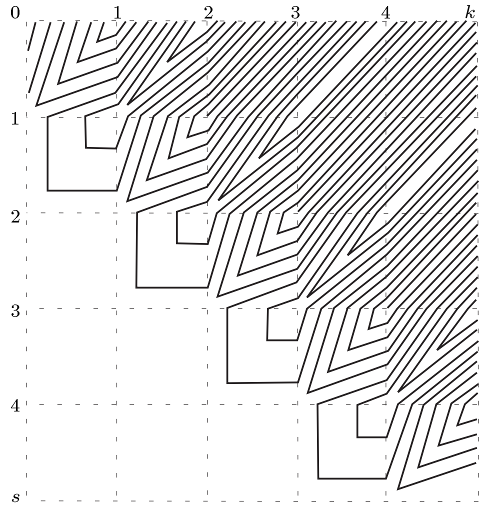


Рис. 3.38. Графическое представление соотношения [3.38] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.39] $$\int\limits_0^\infty P_s^{(\alpha,0)}(\tau,\gamma)\frac{\partial P_k^{(\alpha,0)}(\tau,\gamma)}{\partial \tau}\mathrm{d}\tau =$$

$$= \begin{cases} -(-1)^{k+s}, & \text{если } k > s; \\ -\dfrac{1}{2}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

[3.40] $$\int\limits_0^\infty P_s^{(-1/2,0)}(\tau,\gamma)\Big(\int P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau\Big)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{2}{(4k+1)^2\gamma^2}, & \text{если } k = s; \\ \dfrac{4}{(4s+1)(4k+1)\gamma^2}, & \text{если } k < s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$\mathcal{M}_{[3.40]} = \dfrac{1}{\gamma^2} \times$

$$\times \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 4/5 & 2/25 & 0 & 0 & 0 & 0 \\ 4/9 & 4/45 & 2/81 & 0 & 0 & 0 \\ 4/13 & 4/65 & 4/117 & 2/169 & 0 & 0 \\ 4/17 & 4/85 & 4/153 & 4/221 & 2/289 & 0 \\ 4/21 & 4/105 & 4/189 & 4/273 & 4/357 & 2/441 \end{pmatrix}.$$

Рис. 3.39. Графическое представление соотношения [3.40] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.41] $$\int\limits_0^{\infty} Leg_s(\tau,\gamma)\Big(\int Leg_k(\tau,\gamma)\mathrm{d}\tau\Big)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{1}{2(2k+1)^2\gamma^2}, & \text{если } k = s; \\ \dfrac{1}{(2s+1)(2k+1)\gamma^2}, & \text{если } k < s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.41]} = \frac{1}{\gamma^2} \begin{pmatrix} 1/2 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/18 & 0 & 0 & 0 & 0 \\ 1/5 & 1/15 & 1/50 & 0 & 0 & 0 \\ 1/7 & 1/21 & 1/35 & 1/98 & 0 & 0 \\ 1/9 & 1/27 & 1/45 & 1/63 & 1/162 & 0 \\ 1/11 & 1/33 & 1/55 & 1/77 & 1/99 & 1/242 \end{pmatrix}.$$

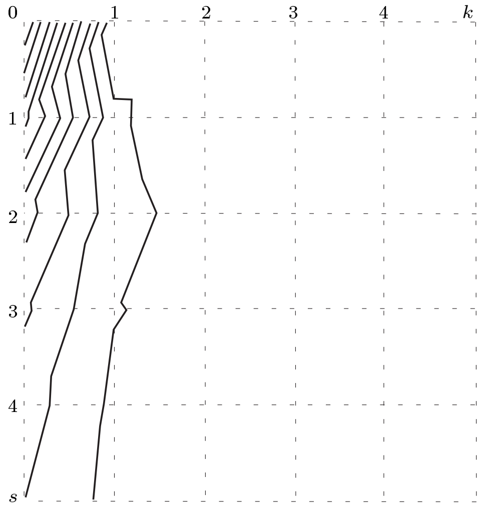


Рис. 3.40. Графическое представление соотношения [3.41] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.42] $$\int\limits_0^{\infty} P_s^{(1/2,0)}(\tau,\gamma)\Big(\int P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau\Big)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{2}{(4k+3)^2\gamma^2}, & \text{если } k = s; \\ \dfrac{4}{(4s+3)(4k+3)\gamma^2}, & \text{если } k < s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.42]} = \frac{1}{\gamma^2} \times$$

$$\times \begin{pmatrix} 2/9 & 0 & 0 & 0 & 0 & 0 \\ 4/21 & 2/49 & 0 & 0 & 0 & 0 \\ 4/33 & 4/77 & 2/121 & 0 & 0 & 0 \\ 4/45 & 4/105 & 4/165 & 2/225 & 0 & 0 \\ 4/57 & 4/133 & 4/209 & 4/285 & 2/361 & 0 \\ 4/69 & 4/161 & 4/253 & 4/345 & 4/437 & 2/529 \end{pmatrix}.$$

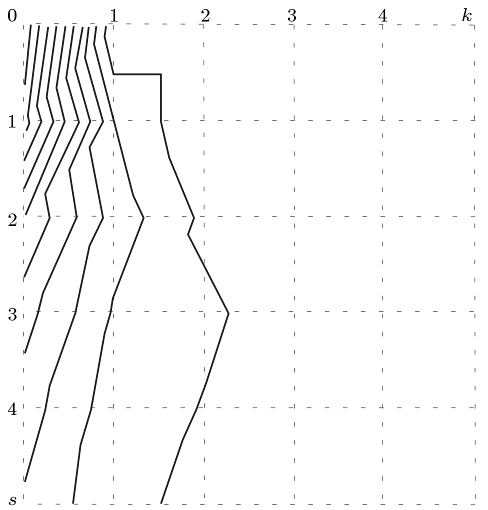


Рис. 3.41. Графическое представление соотношения [3.42] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.43] $$\int\limits_0^\infty P_s^{(1,0)}(\tau,\gamma)\Big(\int P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau\Big)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{1}{2(k+1)^2\gamma^2}, & \text{если } k = s; \\ \dfrac{1}{(s+1)(k+1)\gamma^2}, & \text{если } k < s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.43]} = \frac{1}{\gamma^2}\begin{pmatrix} 1/2 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/8 & 0 & 0 & 0 & 0 \\ 1/3 & 1/6 & 1/18 & 0 & 0 & 0 \\ 1/4 & 1/8 & 1/12 & 1/32 & 0 & 0 \\ 1/5 & 1/10 & 1/15 & 1/20 & 1/50 & 0 \\ 1/6 & 1/12 & 1/18 & 1/24 & 1/30 & 1/72 \end{pmatrix}.$$

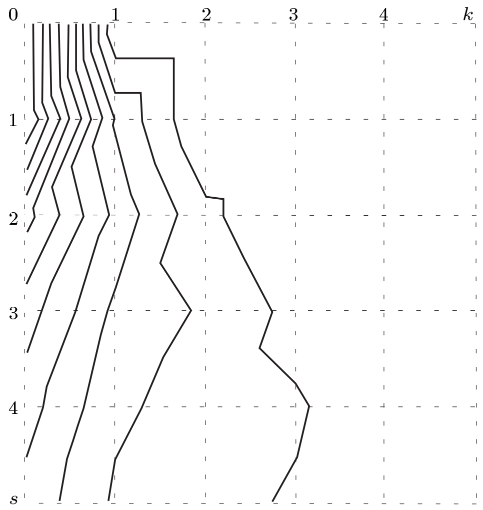


Рис. 3.42. Графическое представление соотношения [3.43] при $k = 0..5,\ s = 0..5;\ \gamma = 1$

[3.44] $$\int\limits_0^\infty P_s^{(2,0)}(\tau,\gamma)\Big(\int P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau\Big)\mathrm{d}\tau =$$

$$= \begin{cases} \dfrac{1}{2(2k+3)^2\gamma^2}, & \text{если } k = s; \\ \dfrac{1}{(2s+3)(2k+3)\gamma^2}, & \text{если } k < s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5;\ s = 0..5$:

$$\mathcal{M}_{[3.44]} = \frac{1}{\gamma^2}\begin{pmatrix} 1/18 & 0 & 0 & 0 & 0 & 0 \\ 1/15 & 1/50 & 0 & 0 & 0 & 0 \\ 1/21 & 1/35 & 1/98 & 0 & 0 & 0 \\ 1/27 & 1/45 & 1/63 & 1/162 & 0 & 0 \\ 1/33 & 1/55 & 1/77 & 1/99 & 1/242 & 0 \\ 1/39 & 1/65 & 1/91 & 1/117 & 1/143 & 1/338 \end{pmatrix}.$$

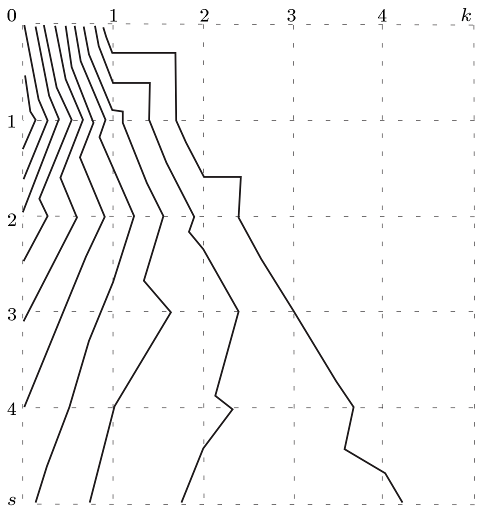


Рис. 3.43. Графическое представление соотношения [3.44] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

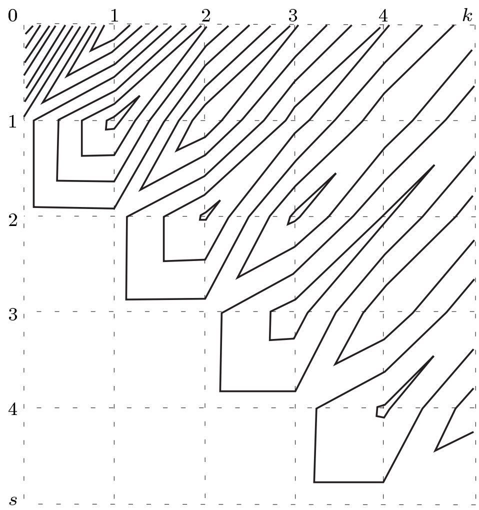


Рис. 3.44. Графическое представление соотношения [3.46] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.45]
$$\int_0^\infty P_s^{(\alpha,0)}(\tau,\gamma)\Big(\int P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau\Big)\mathrm{d}\tau =$$
$$= \begin{cases} \dfrac{2}{(2k+\alpha+1)^2c^2\gamma^2}, & \text{если } k = s; \\ \dfrac{4}{(2s+3)(2k+3)c^2\gamma^2}, & \text{если } k < s; \\ 0, & \text{иначе.} \end{cases}$$

[3.46]
$$\int_0^\infty Leg_s(\tau,\gamma)P_k^{(0,1)}(\tau,\gamma)\mathrm{d}\tau =$$
$$= \begin{cases} \dfrac{(-1)^{k+s}}{2\gamma(k+1)}, & \text{если } k > s; \\ \dfrac{1}{2\gamma(k+1)}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[3.46]} = \frac{1}{\gamma}\begin{pmatrix} 1/2 & -1/4 & 1/6 & -1/8 & 1/10 & -1/12 \\ 0 & 1/4 & -1/6 & 1/8 & -1/10 & 1/12 \\ 0 & 0 & 1/6 & 1/8 & -1/10 & 1/12 \\ 0 & 0 & 0 & 1/8 & -1/10 & 1/12 \\ 0 & 0 & 0 & 0 & 1/10 & 1/12 \\ 0 & 0 & 0 & 0 & 0 & 1/12 \end{pmatrix}.$$

[3.47]
$$\int_0^\infty Leg_s(\tau,\gamma)\frac{\partial P_k^{(0,1)}(\tau,\gamma)}{\partial\tau}\mathrm{d}\tau =$$
$$= \begin{cases} -(-1)^{k+s}\left(\dfrac{2k+1}{2(k+1)} - \right. & \\ \left. \dfrac{2s(s+1)-2k(k+1)}{2(k+1)}\right), & \text{если } k > s; \\ -\dfrac{2k+1}{2(k+1)}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$\mathcal{M}_{[3.47]} =$
$$= \begin{pmatrix} -1/2 & 7/4 & -17/6 & 31/8 & -49/10 & 71/12 \\ 0 & -3/4 & 13/6 & -27/8 & 9/2 & -67/12 \\ 0 & 0 & -5/6 & 19/8 & -37/10 & 59/12 \\ 0 & 0 & 0 & -7/8 & 5/2 & -47/12 \\ 0 & 0 & 0 & 0 & -9/10 & 31/12 \\ 0 & 0 & 0 & 0 & 0 & -11/12 \end{pmatrix}.$$

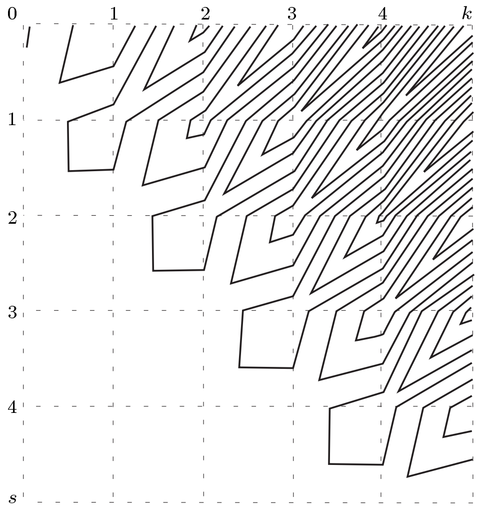


Рис. 3.45. Графическое представление соотношения [3.47] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.48] $$\int\limits_0^\infty Leg_s(\tau,\gamma) P_k^{(0,2)}(\tau,\gamma)\mathrm{d}\tau =$$

$$= \begin{cases} (-1)^{k+s}\left(\dfrac{(k+1)(k+2)}{2\gamma(k+1)(k+2)} - \right. \\ \left. -\dfrac{s(s+1)}{2\gamma(k+1)(k+2)}\right), & \text{если } k > s; \\ \dfrac{1}{\gamma(k+2)}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.48]} = \frac{1}{\gamma} \times$$
$$\times \begin{pmatrix} 1/2 & -1/2 & 1/2 & -1/2 & 1/2 & -1/2 \\ 0 & 1/3 & -5/12 & 9/20 & -7/15 & 10/21 \\ 0 & 0 & 1/4 & -7/20 & 2/5 & -3/7 \\ 0 & 0 & 0 & 1/5 & -3/10 & 5/14 \\ 0 & 0 & 0 & 0 & 1/6 & -11/42 \\ 0 & 0 & 0 & 0 & 0 & 1/7 \end{pmatrix}.$$

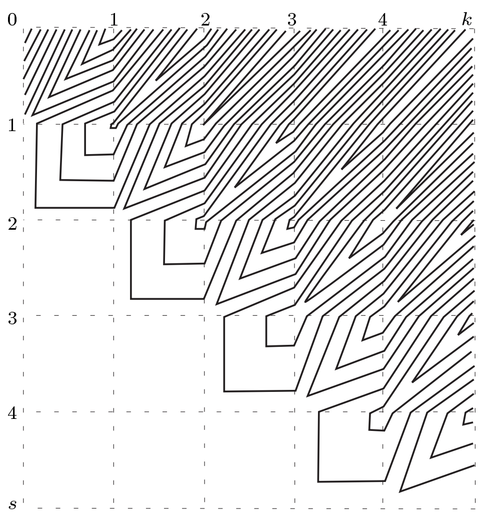


Рис. 3.46. Графическое представление соотношения [3.48] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[3.49] $$\int\limits_0^\infty Leg_s(\tau,\gamma) \frac{\partial P_k^{(0,2)}(\tau,\gamma)}{\partial\tau}\mathrm{d}\tau =$$

$$= \begin{cases} -(-1)^{k+s}\left(\dfrac{(k(k+3)+1)}{2} - \right. \\ -\dfrac{(2k(k+3)+3)s(s+1)}{2(k+1)(k+2)} - \\ \left. -\dfrac{s^2(s+1)^2}{2(k+1)(k+2)}\right), & \text{если } k > s; \\ -\dfrac{(2k+1)}{(k+2)}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[3.49]} =$$
$$= \begin{pmatrix} -1/2 & 5/2 & -11/2 & 19/2 & -29/2 & 41/2 \\ 0 & -1 & 15/4 & -153/20 & 63/5 & -130/7 \\ 0 & 0 & -5/4 & 91/20 & -46/5 & 15 \\ 0 & 0 & 0 & -7/5 & 51/10 & -145/14 \\ 0 & 0 & 0 & 0 & -3/2 & 11/2 \\ 0 & 0 & 0 & 0 & 0 & -11/7 \end{pmatrix}.$$

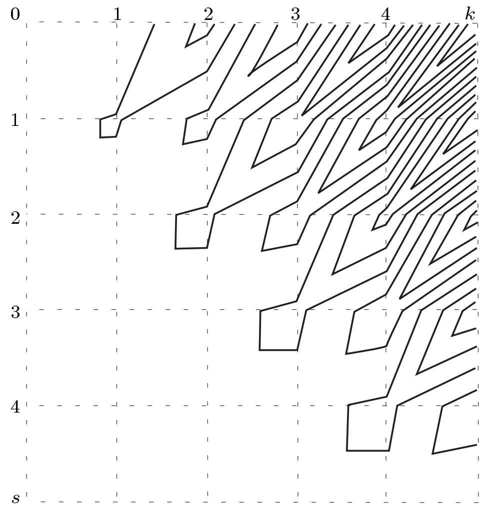


Рис. 3.47. Графическое представление соотношения [3.49] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

Глава 4

Фазовые представления ортогональных функций

Определение.

В общем виде определим фазовые ортогональные функции во временной области как [5]

$$\Phi_k^{\{\psi_k(\tau,\gamma)\}}(j\tau) = \psi_k(\tau,\gamma) + j\frac{\partial\psi_k(\tau,\gamma)}{\partial\tau}.$$

Фазовое представление ортогональных функций во временной области обладает следующими основными свойствами:

$$\begin{cases} \mathrm{Re}\Phi_k^{\{\psi_k(\tau,\gamma)\}}(0) = \psi_k(0,\gamma); \\ \int_0^\infty \mathrm{Im}\Phi_k^{\{\psi_k(\tau,\gamma)\}}(j\tau)d\tau = -\psi_k(0,\gamma). \end{cases}$$

[4.1] $\Phi_k^{\{L_k(\tau,\gamma\}}(j\tau) = L_k(\tau,\gamma) + j\frac{\partial L_k(\tau,\gamma)}{\partial\tau}.$

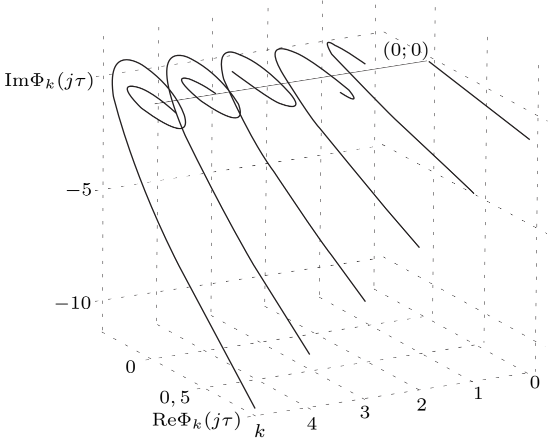


Рис. 4.1. Вид фазового представления ортогональных функций Лагерра 0-5 порядков; $\gamma = 2$

[4.2] $\Phi_k^{\{L_k^{(1)}(\tau,\gamma\}}(j\tau) = L_k^{(1)}(\tau,\gamma) + j\frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial\tau}.$

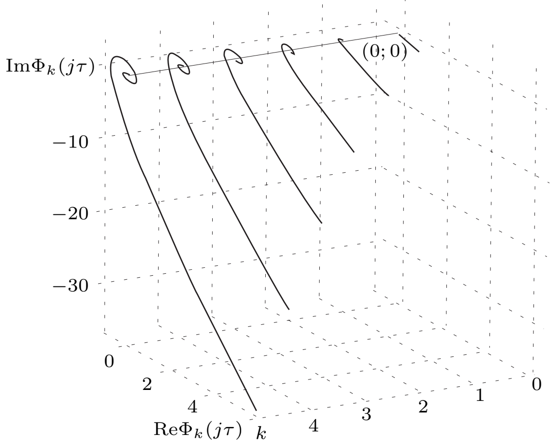


Рис. 4.2. Вид фазового представления ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 2$, $\alpha = 1$

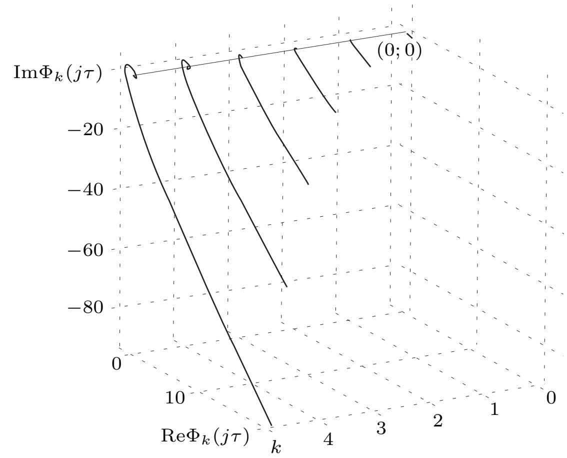
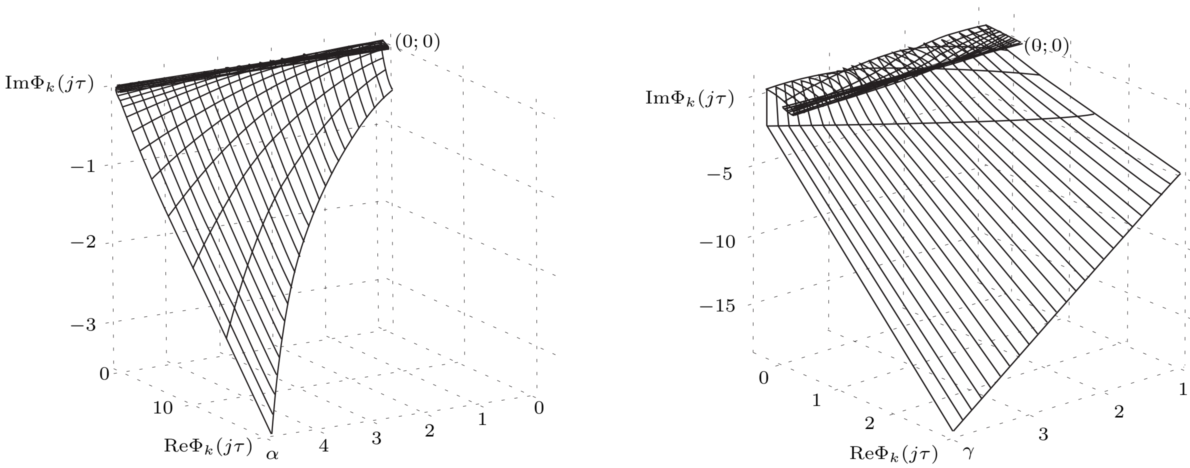


Рис. 4.3. Вид фазового представления ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 2$, $\alpha = 2$

[4.3] $\Phi_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\tau) = L_k^{(2)}(\tau,\gamma) + j\,\dfrac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau}.$

[4.4] $\Phi_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\tau) = L_k^{(\alpha)}(\tau,\gamma) + j\,\dfrac{\partial L_k^{(\alpha)}(\tau,\gamma)}{\partial\tau}.$



а) **б)**

Рис. 4.4. Вид фазового представления ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 2$, $\alpha \in [0;5]$; б) $\gamma \in [1;4]$, $\alpha = 1$

[4.5] $\Phi_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\tau) =$

$$= P_k^{(-1/2,0)}(\tau,\gamma) + j\,\frac{\partial P_k^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}.$$

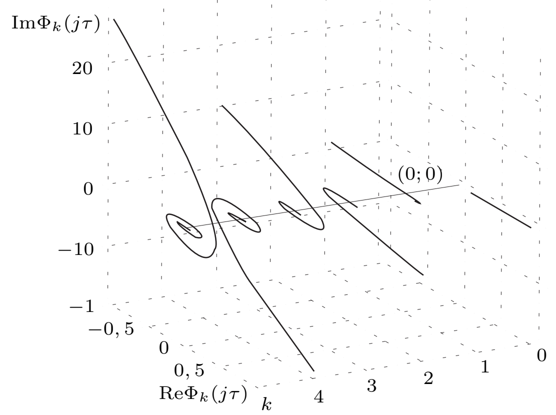


Рис. 4.5. Вид фазового представления ортогональных функций Якоби 0-5 порядков; $\gamma = 0,5$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

[**4.6**] $\Phi_k^{\{Leg_k(\tau,\gamma)\}}(j\tau) = Leg_k(\tau,\gamma) + j\,\dfrac{\partial Leg_k(\tau,\gamma)}{\partial\tau}.$

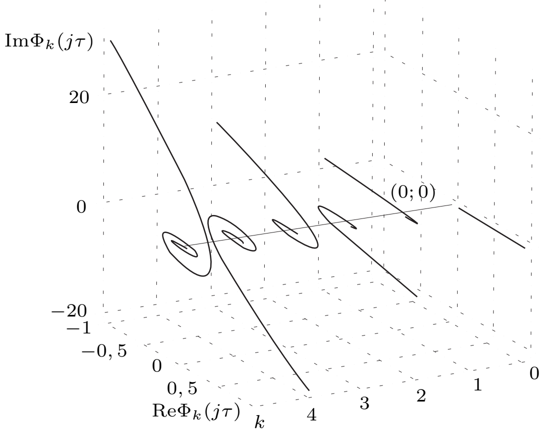


Рис. 4.6. Вид фазового представления ортогональных функций Лежандра 0-5 порядков; $\gamma = 1$, $c = 2$

[**4.7**] $\Phi_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\tau) =$

$$= P_k^{(1/2,0)}(\tau,\gamma) + j\,\frac{\partial P_k^{(1/2,0)}(\tau,\gamma)}{\partial\tau}.$$

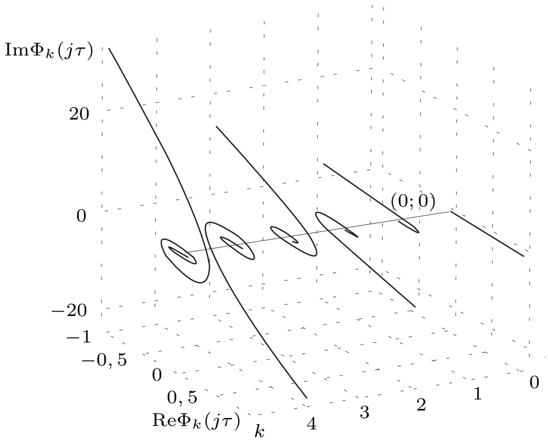


Рис. 4.7. Вид фазового представления ортогональных функций Якоби 0-5 порядков; $\gamma = 0,5$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[**4.8**] $\Phi_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\tau) = P_k^{(1,0)}(\tau,\gamma) + j\,\dfrac{\partial P_k^{(1,0)}(\tau,\gamma)}{\partial\tau}.$

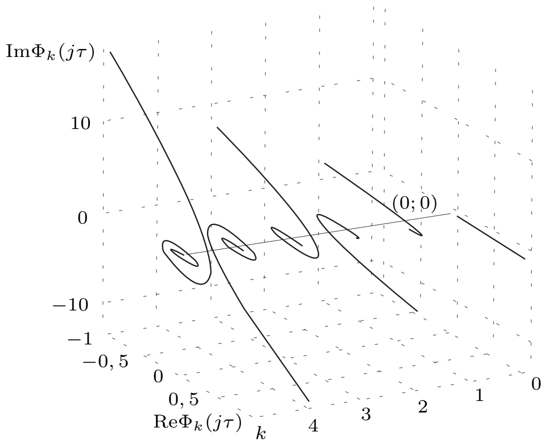


Рис. 4.8. Вид фазового представления ортогональных функций Якоби 0-5 порядков; $\gamma = 0,5$, $c = 1$, $\alpha = 1$, $\beta = 0$

[**4.9**] $\Phi_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\tau) = P_k^{(2,0)}(\tau,\gamma) + j\,\dfrac{\partial P_k^{(2,0)}(\tau,\gamma)}{\partial\tau}.$

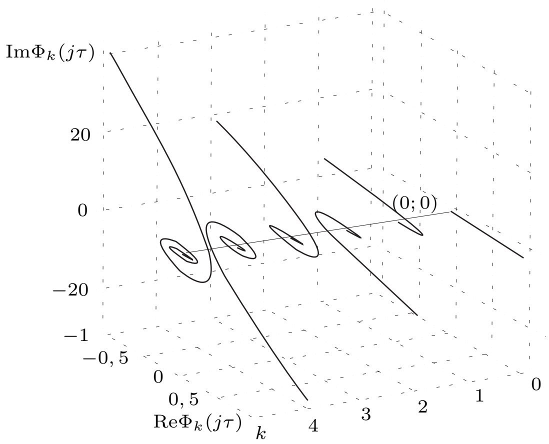
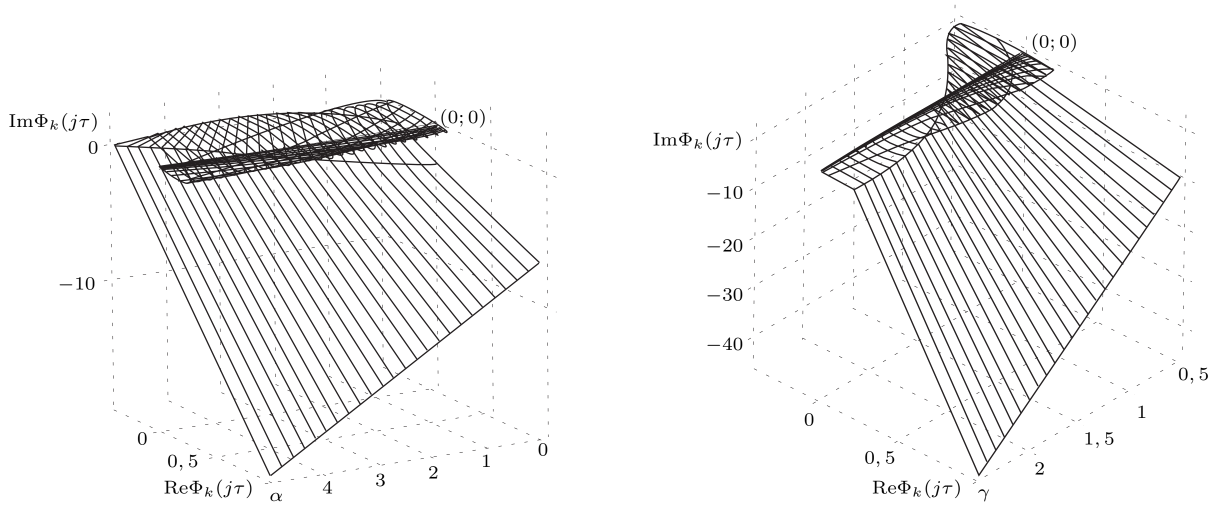


Рис. 4.9. Вид фазового представления ортогональных функций Якоби 0-5 порядков; $\gamma = 0,5$, $c = 2$, $\alpha = 2$, $\beta = 0$

[**4.10**] $\Phi_k^{\{P_k^{(\alpha,0)}(\tau,\gamma\}}(j\tau) = P_k^{(\alpha,0)}(\tau,\gamma) + j\,\dfrac{\partial P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}.$



а) **б)**

Рис. 4.10. Вид фазового представления ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,5$, $c = 2$, $\alpha \in [0;5]$, $\beta = 0$; б) $\gamma \in [0,5;2,5]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[4.11] $\Phi_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\tau) = P_k^{(0,1)}(\tau,\gamma) + j\dfrac{\partial P_k^{(0,1)}(\tau,\gamma)}{\partial\tau}.$

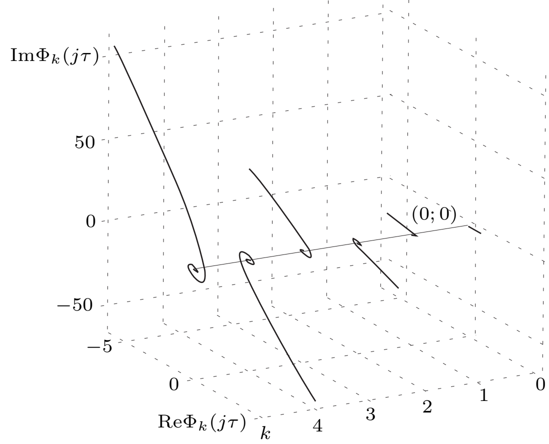


Рис. 4.11. Вид фазового представления ортогональных функций Якоби 0-5 порядков; $\gamma = 0,5$, $c = 2$, $\alpha = 0$, $\beta = 1$

[4.12] $\Phi_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\tau) = P_k^{(0,2)}(\tau,\gamma) + j\dfrac{\partial P_k^{(0,2)}(\tau,\gamma)}{\partial\tau}.$

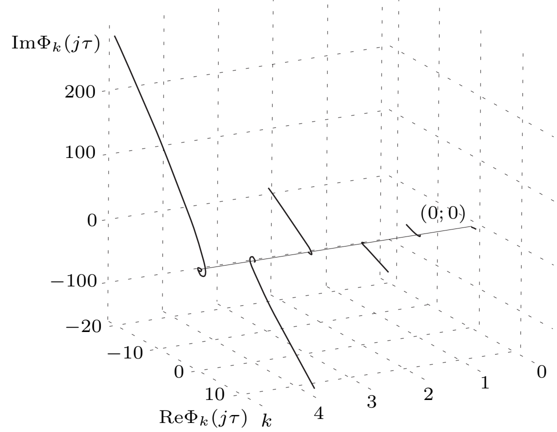


Рис. 4.12. Вид фазового представления ортогональных функций Якоби 0-5 порядков; $\gamma = 0,5$, $c = 2$, $\alpha = 0$, $\beta = 2$

[4.13] $\Phi_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\tau) = P_k^{(\alpha,0)}(\tau,\gamma) + j\dfrac{\partial P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}.$

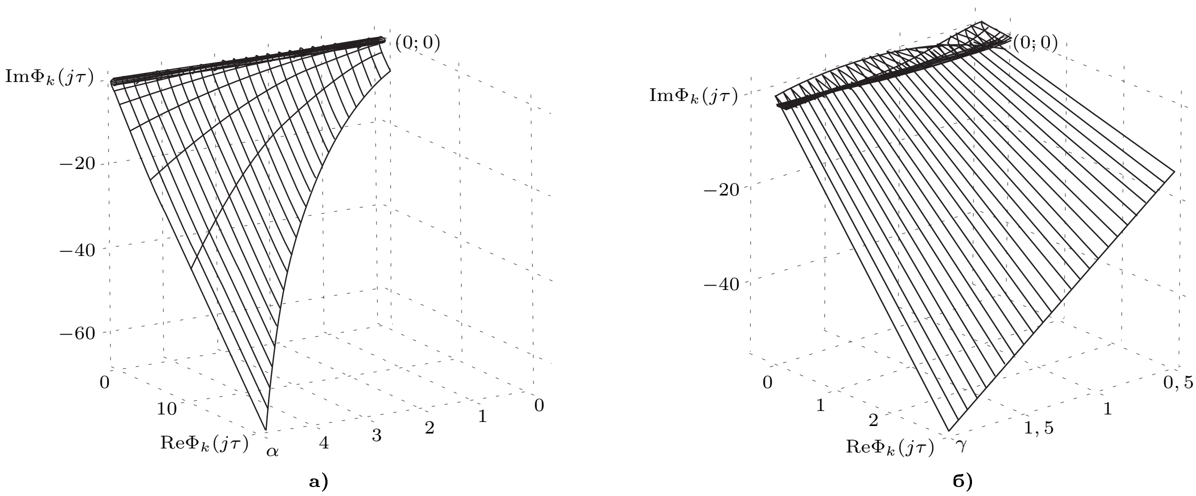


Рис. 4.13. Вид фазового представления ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,5$, $c = 2$, $\alpha = 0$, $\beta \in [0;5]$; б) $\gamma \in [0,5;2]$, $c = 2$, $\alpha = 0$, $\beta = 1$

Глава 5

Интегральные представления ортогональных функций

Определение.

Данное определение ортогональных функций введено как обратное преобразование Фурье

$$\psi_k(\tau,\gamma) = \int_0^\infty W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\exp(j\omega\tau)d\omega.$$

[5.1] $L_k(\tau,\gamma) =$

$$= \frac{2(-1)^k}{\pi}\int\limits_0^{\pi/2}\frac{\cos\bigl((2k+1)\phi\bigr)}{\cos\phi}\cos\Bigl(\frac{\tau\gamma}{2}\tan\phi\Bigr)d\phi.$$

[5.2] $L_k^{(1)}(\tau,\gamma) =$

$$= \frac{4(-1)^k(k+1)}{\pi\tau\gamma}\int\limits_0^{\pi/2}\cos\bigl((2k+2)\phi\bigr)\cos\Bigl(\frac{\tau\gamma}{2}\tan\phi\Bigr)d\phi.$$

[5.3] $L_k^{(2)}(\tau,\gamma) = \dfrac{8(-1)^k(k+1)(k+2)}{\pi(\tau\gamma)^2}\times$

$$\times\int\limits_0^{\pi/2}\cos\bigl((2k+3)\phi\bigr)\cos\phi\cos\Bigl(\frac{\tau\gamma}{2}\tan\phi\Bigr)d\phi.$$

[5.4] $L_k^{(\alpha)}(\tau,\gamma) = \dfrac{2^{\alpha+1}(-1)^k(k+\alpha)!}{\pi(\tau\gamma)^\alpha k!}\times$

$$\times\int\limits_0^{\pi/2}\cos\bigl((2k+\alpha+1)\phi\bigr)(\cos\phi)^{\alpha-1}\cos\Bigl(\frac{\tau\gamma}{2}\tan\phi\Bigr)d\phi.$$

[5.5] $P_k^{(-1/2,0)}(\tau,\gamma) = \dfrac{2}{\pi}\times$

$$\times\begin{cases}\displaystyle\int\limits_0^{\pi/2}\cos\bigl((4k+1)\gamma\tau/2\tan\phi\bigr)d\phi, & \text{если } k=0;\\ \displaystyle\int\limits_0^{\pi/2}\cos\Biggl(\phi+ \\ \displaystyle +2\sum_{s=0}^{k-1}\arctan\left(\frac{4k+1}{4s+1}\tan\phi\right)\Biggr)\times \\ \displaystyle\times\frac{\cos\bigl((4k+1)\gamma\tau/2\tan\phi\bigr)}{\cos\phi}d\phi, & \text{если } k>0.\end{cases}$$

[5.6] $Leg_k(\tau,\gamma) = \dfrac{2}{\pi}\times$

$$\times\begin{cases}\displaystyle\int\limits_0^{\pi/2}\cos\bigl((2k+1)\gamma\tau\tan\phi\bigr)d\phi, & \text{если } k=0;\\ \displaystyle\int\limits_0^{\pi/2}\cos\Biggl(\phi+ \\ \displaystyle +2\sum_{s=0}^{k-1}\arctan\left(\frac{2k+1}{2s+1}\tan\phi\right)\Biggr)\times \\ \displaystyle\times\frac{\cos\bigl((2k+1)\gamma\tau\tan\phi\bigr)}{\cos\phi}d\phi, & \text{если } k>0.\end{cases}$$

[5.7] $P_k^{(1/2,0)}(\tau,\gamma)=\frac{2}{\pi}\times$

$$\times\begin{cases}\displaystyle\int_0^{\pi/2}\cos\big((4k+3)\gamma\tau/2\tan\phi\big)d\phi, & \text{если } k=0;\\ \displaystyle\int_0^{\pi/2}\cos\Bigg(\phi+ 2\sum_{s=0}^{k-1}\arctan\left(\frac{4k+3}{4s+3}\tan\phi\right)\Bigg)\times\frac{\cos\big((4k+3)\gamma\tau/2\tan\phi\big)}{\cos\phi}d\phi, & \text{если } k>0.\end{cases}$$

[5.8] $P_k^{(1,0)}(\tau,\gamma)=\frac{2}{\pi}\times$

$$\times\begin{cases}\displaystyle\int_0^{\pi/2}\cos\big((k+1)\gamma\tau\tan\phi\big)d\phi, & \text{если } k=0;\\ \displaystyle\int_0^{\pi/2}\cos\Bigg(\phi+ 2\sum_{s=0}^{k-1}\arctan\left(\frac{k+1}{s+1}\tan\phi\right)\Bigg)\times\frac{\cos\big((k+1)\gamma\tau\tan\phi\big)}{\cos\phi}d\phi, & \text{если } k>0.\end{cases}$$

[5.9] $P_k^{(2,0)}(\tau,\gamma)=\frac{2}{\pi}\times$

$$\times\begin{cases}\displaystyle\int_0^{\pi/2}\cos\big((2k+3)\gamma\tau\tan\phi\big)d\phi, & \text{если } k=0;\\ \displaystyle\int_0^{\pi/2}\cos\Bigg(\phi+ 2\sum_{s=0}^{k-1}\arctan\left(\frac{2k+3}{2s+3}\tan\phi\right)\Bigg)\times\frac{\cos\big((2k+3)\gamma\tau\tan\phi\big)}{\cos\phi}d\phi, & \text{если } k>0.\end{cases}$$

[5.10] $P_k^{(\alpha,0)}(\tau,\gamma)=\frac{2}{\pi}\times$

$$\times\begin{cases}\displaystyle\int_0^{\pi/2}\cos\big((2k+\alpha+1)c\gamma\tau/2\tan\phi\big)d\phi, & \text{если } k=0;\\ \displaystyle\int_0^{\pi/2}\cos\Bigg(\phi+2\times \sum_{s=0}^{k-1}\arctan\left(\frac{2k+\alpha+1}{2s+\alpha+1}\tan\phi\right)\Bigg)\times\frac{\cos\big((2k+\alpha+1)c\gamma\tau/2\tan\phi\big)}{\cos\phi}d\phi, & \text{если } k>0.\end{cases}$$

[5.11] $P_k^{(0,1)}(\tau,\gamma)=\frac{4(k+1)}{\pi(2k+3)\big(1-\exp(-2\gamma\tau)\big)}\times$

$$\times\begin{cases}\displaystyle\int_0^{\pi/2}\cos\left(\arctan\left(\frac{2k+1}{2k+3}\tan\phi\right)\right)\times \cos\left(\phi+\arctan\left(\frac{2k+1}{2k+3}\tan\phi\right)\right)\times\frac{\cos\big((2k+1)\gamma\tau\tan\phi\big)}{\cos\phi}d\phi, & \text{если } k=0;\\ \displaystyle\int_0^{\pi/2}\cos\left(\arctan\left(\frac{2k+1}{2k+3}\tan\phi\right)\right)\times \cos\left(\phi+\arctan\left(\frac{2k+1}{2k+3}\tan\phi\right)+ 2\sum_{s=0}^{k-1}\arctan\left(\frac{2k+1}{2s+1}\tan\phi\right)\right)\times\frac{\cos\big((2k+1)\gamma\tau\tan\phi\big)}{\cos\phi}d\phi, & \text{если } k>0.\end{cases}$$

[5.12] $P_k^{(0,2)}(\tau,\gamma)=\frac{8(k+1)(k+2)}{(2k+3)(2k+5)\big(1-\exp(-2\gamma\tau)\big)^2}\times$

$$\times\frac{1}{\pi}\begin{cases}\displaystyle\int_0^{\pi/2}\cos\left(\arctan\left(\frac{2k+1}{2k+3}\tan\phi\right)\right)\times \cos\left(\arctan\left(\frac{2k+1}{2k+5}\tan\phi\right)\right)\times \cos\left(\phi+\arctan\left(\frac{2k+1}{2k+3}\tan\phi\right)+ \arctan\left(\frac{2k+1}{2k+5}\tan\phi\right)\right)\times \frac{\cos\big((2k+1)\gamma\tau\tan\phi\big)}{\cos\phi}d\phi, & \text{если } k=0;\\ \displaystyle\int_0^{\pi/2}\cos\left(\arctan\left(\frac{2k+1}{2k+3}\tan\phi\right)\right)\times \cos\left(\arctan\left(\frac{2k+1}{2k+5}\tan\phi\right)\right)\times \cos\left(\phi+\arctan\left(\frac{2k+1}{2k+3}\tan\phi\right) +\arctan\left(\frac{2k+1}{2k+5}\tan\phi\right)+ 2\sum_{s=0}^{k-1}\arctan\left(\frac{2k+1}{2s+1}\tan\phi\right)\right)\times \frac{\cos\big((2k+1)\gamma\tau\tan\phi\big)}{\cos\phi}d\phi, & \text{если } k>0.\end{cases}$$

[5.13] $P_k^{(0,\beta)}(\tau,\gamma) =$

$$= \frac{2^{\beta+1}(k+\beta)!(2k+1)}{\pi k!\big(1-\exp(-2c\gamma\tau/2)\big)^{\beta}\prod\limits_{p=0}^{\beta}(2k+2p+1)}\times$$

$$\times\begin{cases}\int\limits_0^{\pi/2}\prod\limits_{p=0}^{\beta}\cos\left(\arctan\left(\frac{2k+1}{2k+2p+1}\tan\phi\right)\right)\times \\ \times\cos\Bigg(\phi+ \\ +\sum\limits_{p=0}^{\beta}\arctan\left(\frac{2k+1}{2k+2p+1}\tan\phi\right)\Bigg)\times \\ \times\frac{\cos\big((2k+1)c\gamma\tau/2\tan\phi\big)}{(\cos\phi)^2}d\phi, & k=0; \\ \int\limits_0^{\pi/2}\prod\limits_{p=0}^{\beta}\cos\left(\arctan\left(\frac{2k+1}{2k+2p+1}\tan\phi\right)\right)\times \\ \times\cos\Bigg(\phi+ \\ +\sum\limits_{p=0}^{\beta}\arctan\left(\frac{2k+1}{2k+2p+1}\tan\phi\right)+ \\ +2\sum\limits_{s=0}^{k-1}\arctan\left(\frac{2k+1}{2s+1}\tan\phi\right)\Bigg)\times \\ \times\frac{\cos\big((2k+1)c\gamma\tau/2\tan\phi\big)}{(\cos\phi)^2}d\phi, & k>0.\end{cases}$$

Глава 6

Аналитические представления в частотной области

Определение.

Для представления ортогональных функций в частотной области имеет место ряд определений в зависимости от дальнейшего приложения. Наиболее распространено следующее определение, известное как преобразование Фурье ортогональных функций[9, 1]

$$W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega) = \int_0^\infty \psi_k(\tau,\gamma)\exp(-j\omega\tau)d\tau.$$

В отличие от вышеприведенной характеристики преобразование Фурье ортогональных фильтров является физически реализуемым [6]

$$V_k^{\{\psi_k(\tau,\gamma)\}}(j\omega) = \int_0^\infty \psi_k(\tau,\gamma)\exp(-j\omega\tau)\mu^{\{\psi_k(\tau,\gamma)\}}(\tau,\gamma)d\tau.$$

Значительно реже используется определение преобразования Фурье производных ортогональных функций $W_k^{\left\{\frac{\partial\psi_k(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)$.

6.1 Преобразование Фурье ортогональных функций

[6.1] $$W_k^{[1]\{L_k(\tau,\gamma)\}}(j\omega) = \frac{1}{j\omega+\gamma/2}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

[6.2] $$W_k^{[2]\{L_k(\tau,\gamma)\}}(j\omega) = \frac{1}{j\omega+\gamma/2}\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^k.$$

[6.3] $$W_k^{[3]\{L_k(\tau,\gamma)\}}(j\omega) = \frac{2}{\gamma}(-1)^k\cos\varphi\times \exp(-j(2k+1)\varphi), \quad \varphi = \arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{L_0(\tau,\gamma)\}}(j\omega) = \frac{1}{j\omega+\gamma/2};$$

$$W_1^{\{L_1(\tau,\gamma)\}}(j\omega) = \frac{j\omega-\gamma/2}{(j\omega+\gamma/2)^2};$$

$$W_2^{\{L_2(\tau,\gamma)\}}(j\omega) = \frac{(j\omega-\gamma/2)^2}{(j\omega+\gamma/2)^3};$$

$$W_3^{\{L_3(\tau,\gamma)\}}(j\omega) = \frac{(j\omega-\gamma/2)^3}{(j\omega+\gamma/2)^4};$$

$$W_4^{\{L_4(\tau,\gamma)\}}(j\omega) = \frac{(j\omega - \gamma/2)^4}{(j\omega + \gamma/2)^5};$$

$$W_5^{\{L_5(\tau,\gamma)\}}(j\omega) = \frac{(j\omega - \gamma/2)^5}{(j\omega + \gamma/2)^6}.$$

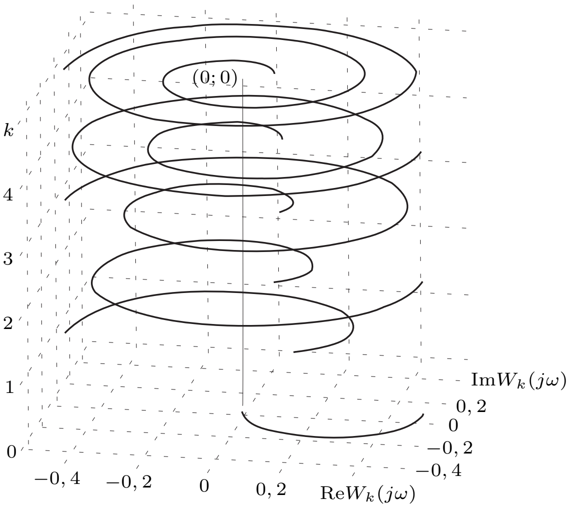


Рис. 6.1. Вид преобразования Фурье ортогональных функций Лагерра 0-5 порядков; $\gamma = 4$

[6.4] $$W_k^{[1]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{j\omega + \gamma/2}\sum_{s=0}^{k}\binom{k+1}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

[6.5] $$W_k^{[2]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma}\left(1 - \left(\frac{j\omega - \gamma/2}{j\omega + \gamma/2}\right)^{k+1}\right).$$

[6.6] $$W_k^{[3]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma}\Big(1+(-1)^k \exp(-j(2k+2)\varphi)\Big), \quad \varphi = \arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{L_0^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{j\omega + \gamma/2};$$

$$W_1^{\{L_1^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{2j\omega}{(j\omega + \gamma/2)^2};$$

$$W_2^{\{L_2^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^2/4 - 3\omega^2}{(j\omega + \gamma/2)^3};$$

$$W_3^{\{L_3^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^2 j\omega - 4j\omega^3}{(j\omega + \gamma/2)^4};$$

$$W_4^{\{L_4^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^4/16 - 5\gamma^2\omega^2/2 + 5\omega^4}{(j\omega + \gamma/2)^5};$$

$$W_5^{\{L_5^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{3\gamma^4 j\omega/8 - 5\gamma^2 j\omega^3 + 6j\omega^4}{(j\omega + \gamma/2)^6}.$$

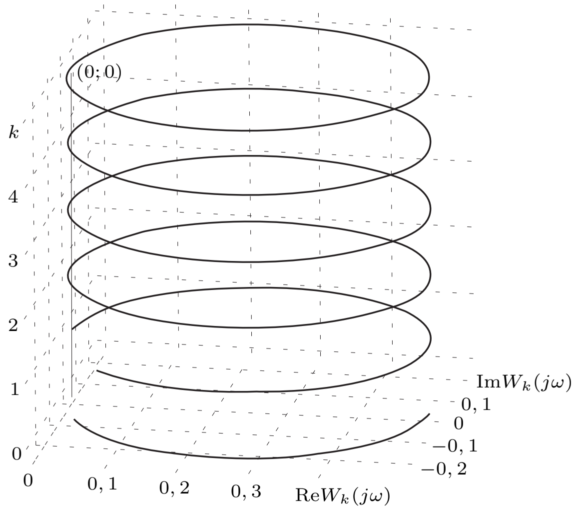


Рис. 6.2. Вид преобразования Фурье ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 4$, $\alpha = 1$

[6.7] $$W_k^{[1]\{L_k^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{j\omega + \gamma/2}\sum_{s=0}^{k}\binom{k+2}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

[6.8] $$W_k^{[2]\{L_k^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma^2}\times \left[\left(\left(\frac{j\omega - \gamma/2}{j\omega + \gamma/2}\right)^{k+1} - 1\right)\Big(j\omega - \gamma/2\Big) + \gamma(k+1)\right].$$

[6.9] $$W_k^{[3]\{L_k^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma}\times \left(\frac{\exp(-j\varphi) + (-1)^k \exp(-j(2k+3)\varphi)}{2\cos\varphi} + k + 1\right),$$
$$\varphi = \arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{L_0^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{j\omega + \gamma/2};$$

$$W_1^{\{L_1^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma/2 + 3j\omega}{(j\omega + \gamma/2)^2};$$

$$W_2^{\{L_2^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^2/2 + 2\gamma j\omega - 6\omega^2}{(j\omega + \gamma/2)^3};$$

$$W_3^{\{L_3^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^3/4 + 5\gamma^2 j\omega/2 - 5\gamma\omega^2 - 10j\omega^3}{(j\omega + \gamma/2)^4};$$

$$W_4^{\{L_4^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(j\omega + \gamma/2)^5}\big(3\gamma^4/16 + 3\gamma^3 j\omega/2 - 15\gamma^2\omega^2/2 - 10\gamma j\omega^3 + 15\omega^4\big);$$

$$W_5^{\{L_5^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(j\omega + \gamma/2)^6}\big(3\gamma^5/32 + 21\gamma^4 j\omega/16 - 21\gamma^3\omega^2/4 - 35\gamma^2 j\omega^3/2 + 35\gamma\omega^4/2 + 21j\omega^5\big).$$

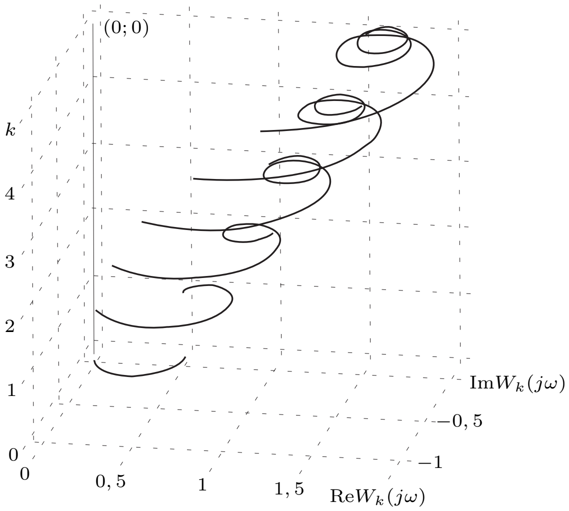


Рис. 6.3. Вид преобразования Фурье ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 4$, $\alpha = 2$

[6.10] $$W_k^{[1]\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{1}{j\omega + \gamma/2} \sum_{s=0}^{k} \binom{k+\alpha}{k-s} \left(\frac{-\gamma}{j\omega+\gamma/2} \right)^s.$$

[6.11] $$W_k^{[2]\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{\left(j\omega+\gamma/2\right)^{\alpha-1}}{(-\gamma)^\alpha} \left[\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2} \right)^{k+\alpha} - \sum_{p=0}^{\alpha-1} \binom{k+\alpha}{p} \left(-\frac{\gamma}{j\omega+\gamma/2} \right)^p \right], \qquad \alpha \in \mathbb{Z}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{L_0^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{1}{j\omega+\gamma/2};$$

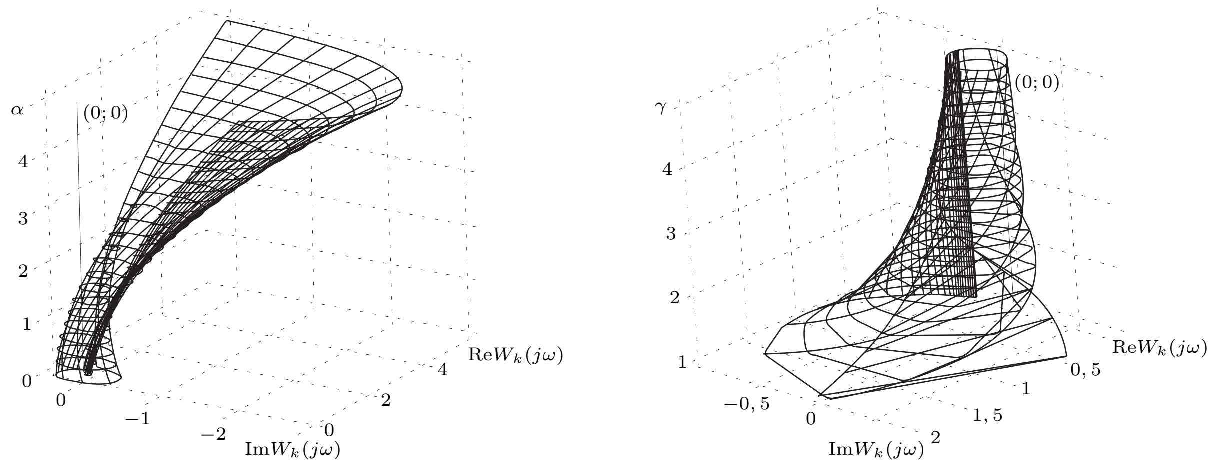
$$W_1^{\{L_1^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma(\alpha-1)/2 + j\omega(\alpha+1)}{\left(j\omega+\gamma/2\right)^2};$$

$$W_2^{\{L_2^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\left(j\omega+\gamma/2\right)^3} \big(\gamma^2(\alpha^2+\alpha-2)/8 + \gamma j\omega(\alpha^2 - \alpha + 2)/2 - \omega^2(\alpha^2+3\alpha+2)/2\big);$$

$$W_3^{\{L_3^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\left(j\omega+\gamma/2\right)^4} \big(-\gamma^3 + \gamma^2(j\omega+\gamma/2)(\alpha+3) - \gamma(j\omega+\gamma/2)^2(\alpha+2)(\alpha+3)/2 - (j\omega+\gamma/2)^3(\alpha+3)!/(6\alpha!)\big);$$

$$W_4^{\{L_4^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\left(j\omega+\gamma/2\right)^5} \Big(\gamma^4 - \gamma^3(j\omega+\gamma/2)(\alpha+4) + \gamma^2(j\omega+\gamma/2)^2(\alpha+3)(\alpha+4)/2 - \gamma(j\omega+\gamma/2)^3 \times (\alpha+4)!/\big(6(\alpha+1)!\big) + (j\omega+\gamma/2)^4(\alpha+4)!/(24\alpha!)\Big);$$

$$W_5^{\{L_5^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\left(j\omega+\gamma/2\right)^6} \Big(-\gamma^5 + \gamma^4(j\omega+\gamma/2)(\alpha+5) - \gamma^3(j\omega+\gamma/2)^2(\alpha+4)(\alpha+5)/2 + \gamma^2(j\omega+\gamma/2)^3 \times (\alpha+5)!/\big(6(\alpha+2)!\big) - \gamma(j\omega+\gamma/2)^4(\alpha+5)!/\big(24(\alpha+1)!\big) + (j\omega+\gamma/2)^5(\alpha+5)!/(120\alpha!)\Big).$$



а) б)

Рис. 6.4. Вид преобразования Фурье ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 4$, $\alpha \in [0; 5]$; б) $\gamma \in [1; 5]$, $\alpha = 1$

[6.12] $$W_k^{[1]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) = \sum_{s=0}^{k} \binom{k}{s} \binom{k+s-1/2}{s-1/2} (-1)^s \frac{1}{(4s+1)\gamma/2 + j\omega}.$$

[6.13] $$W_k^{[2]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)=$$
$$=\begin{cases}\dfrac{1}{\gamma/2+j\omega}, & \text{если } k=0;\\ \dfrac{1}{(4k+1)\gamma/2+j\omega}\times\\ \times\displaystyle\prod_{s=0}^{k-1}\dfrac{(4s+1)\gamma/2-j\omega}{(4s+1)\gamma/2+j\omega}, & \text{если } k>0.\end{cases}$$

[6.14] $$W_k^{[3]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)=$$
$$=\begin{cases}\dfrac{2}{\gamma}\cos\varphi_0\exp\bigl(-j\varphi_0\bigr), & \text{если } k=0;\\ \dfrac{2}{(4k+1)\gamma}\cos\varphi_k\times\\ \times\exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k>0,\end{cases}$$
$$\varphi_k=\arctan\frac{2\omega}{(4k+1)\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{P_0^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)=\frac{1}{\gamma/2+j\omega};$$
$$W_1^{\{P_1^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)=\frac{\gamma/2-j\omega}{(\gamma/2+j\omega)(5\gamma/2+j\omega)};$$
$$W_2^{\{P_2^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)=\frac{(\gamma/2-j\omega)(5\gamma/2-j\omega)}{(\gamma/2+j\omega)(5\gamma/2+j\omega)(9\gamma/2+j\omega)};$$
$$W_3^{\{P_3^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)=\frac{1}{(13\gamma/2+j\omega)}\frac{(\gamma/2-j\omega)}{(\gamma/2+j\omega)}\times$$
$$\times\frac{(5\gamma/2-j\omega)(9\gamma/2-j\omega)}{(5\gamma/2+j\omega)(9\gamma/2+j\omega)};$$
$$W_4^{\{P_4^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)=\frac{1}{(17\gamma/2+j\omega)}\frac{(\gamma/2-j\omega)}{(\gamma/2+j\omega)}\times$$
$$\times\frac{(5\gamma/2-j\omega)(9\gamma/2-j\omega)}{(5\gamma/2+j\omega)(9\gamma/2+j\omega)}\frac{(13\gamma/2-j\omega)}{(13\gamma/2+j\omega)};$$
$$W_5^{\{P_5^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)=\frac{1}{(21\gamma/2+j\omega)}\frac{(\gamma/2-j\omega)}{(\gamma/2+j\omega)}\times$$
$$\times\frac{(5\gamma/2-j\omega)(9\gamma/2-j\omega)}{(5\gamma/2+j\omega)(9\gamma/2+j\omega)}\frac{(13\gamma/2-j\omega)(17\gamma/2-j\omega)}{(13\gamma/2+j\omega)(17\gamma/2+j\omega)}.$$

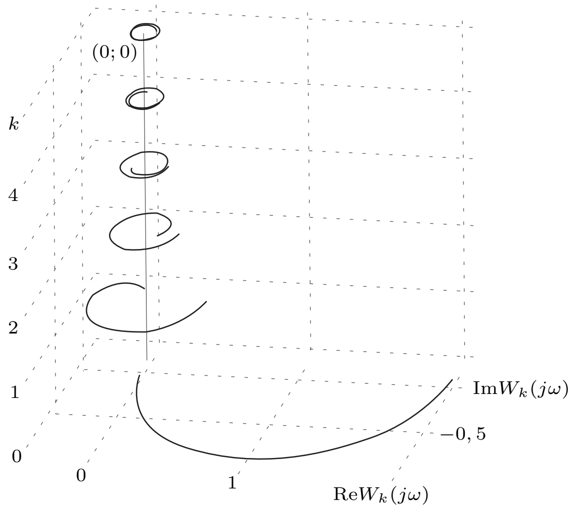


Рис. 6.5. Вид преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma=1$, $c=2$, $\alpha=-1/2$, $\beta=0$

[6.15]
$$W_k^{[1]\{Leg_k(\tau,\gamma)\}}(j\omega)=\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{(2s+1)\gamma+j\omega}.$$

[6.16] $$W_k^{[2]\{Leg_k(\tau,\gamma)\}}(j\omega)=$$
$$=\begin{cases}\dfrac{1}{\gamma+j\omega}, & \text{если } k=0;\\ \dfrac{1}{(2k+1)\gamma+j\omega}\displaystyle\prod_{s=0}^{k-1}\dfrac{(2s+1)\gamma-j\omega}{(2s+1)\gamma+j\omega}, & \text{если } k>0.\end{cases}$$

[6.17] $$W_k^{[3]\{Leg_k(\tau,\gamma)\}}(j\omega)=$$
$$=\begin{cases}\dfrac{1}{\gamma}\cos\varphi_0\exp\bigl(-j\varphi_0\bigr), & \text{если } k=0;\\ \dfrac{1}{(2k+1)\gamma}\cos\varphi_k\times\\ \times\exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k>0,\end{cases}$$
$$\varphi_k=\arctan\frac{\omega}{(2k+1)\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{Leg_0(\tau,\gamma)\}}(j\omega)=\frac{1}{\gamma+j\omega};$$
$$W_1^{\{Leg_1(\tau,\gamma)\}}(j\omega)=\frac{\gamma-j\omega}{(\gamma+j\omega)(3\gamma+j\omega)};$$
$$W_2^{\{Leg_2(\tau,\gamma)\}}(j\omega)=\frac{(\gamma-j\omega)(3\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)(5\gamma+j\omega)};$$
$$W_3^{\{Leg_3(\tau,\gamma)\}}(j\omega)=\frac{(\gamma-j\omega)(3\gamma-j\omega)(5\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)(5\gamma+j\omega)(7\gamma+j\omega)};$$
$$W_4^{\{Leg_4(\tau,\gamma)\}}(j\omega)=\frac{1}{(9\gamma+j\omega)}\frac{(\gamma-j\omega)(3\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)}\times$$
$$\times\frac{(5\gamma-j\omega)(7\gamma-j\omega)}{(5\gamma+j\omega)(7\gamma+j\omega)};$$

$$W_5^{\{Leg_5(\tau,\gamma)\}}(j\omega) = \frac{1}{(11\gamma + j\omega)} \frac{(\gamma - j\omega)(3\gamma - j\omega)}{(\gamma + j\omega)(3\gamma + j\omega)} \times$$
$$\times \frac{(5\gamma - j\omega)(7\gamma - j\omega)(9\gamma - j\omega)}{(5\gamma + j\omega)(7\gamma + j\omega)(9\gamma + j\omega)}.$$

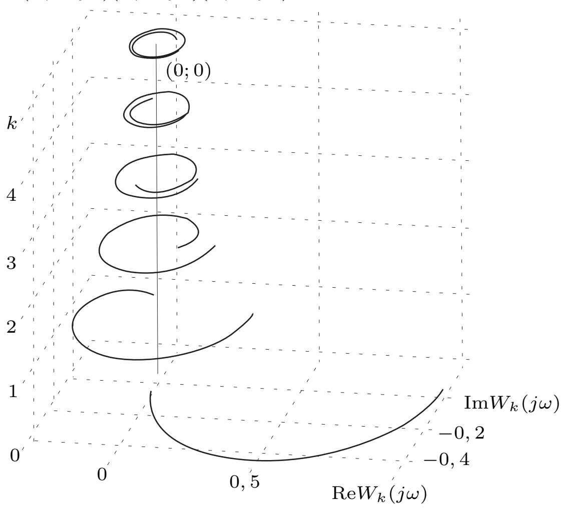


Рис. 6.6. Вид преобразования Фурье ортогональных функций Лежандра 0-5 порядков; $\gamma = 1$, $c = 2$

[6.18] $$W_k^{[1]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega) =$$
$$= \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+1/2}{s+1/2} (-1)^s \frac{1}{(4s+3)\gamma/2 + j\omega}.$$

[6.19] $$W_k^{[2]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega) =$$
$$= \begin{cases} \dfrac{1}{3\gamma/2 + j\omega}, & \text{если } k = 0; \\ \dfrac{1}{(4k+3)\gamma/2 + j\omega} \times \\ \times \displaystyle\prod_{s=0}^{k-1} \frac{(4s+3)\gamma/2 - j\omega}{(4s+3)\gamma/2 + j\omega}, & \text{если } k > 0. \end{cases}$$

[6.20] $$W_k^{[3]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega) =$$
$$= \begin{cases} \dfrac{2}{3\gamma} \cos\varphi_0 \exp(-j\varphi_0), & \text{если } k = 0; \\ \dfrac{2}{(4k+3)\gamma} \cos\varphi_k \times \\ \times \exp\left(-j\left(\varphi_k + 2\displaystyle\sum_{s=0}^{k-1} \varphi_s\right)\right), & \text{если } k > 0, \end{cases}$$
$$\varphi_k = \arctan \frac{2\omega}{(4k+3)\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{P_0^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{3\gamma/2 + j\omega};$$
$$W_1^{\{P_1^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{3\gamma/2 - j\omega}{(3\gamma/2 + j\omega)(7\gamma/2 + j\omega)};$$
$$W_2^{\{P_2^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{(3\gamma/2 - j\omega)(7\gamma/2 - j\omega)}{(3\gamma/2 + j\omega)(7\gamma/2 + j\omega)(11\gamma/2 + j\omega)};$$
$$W_3^{\{P_3^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(15\gamma/2 + j\omega)} \frac{(3\gamma/2 - j\omega)}{(3\gamma/2 + j\omega)} \times$$
$$\times \frac{(7\gamma/2 - j\omega)(11\gamma/2 - j\omega)}{(7\gamma/2 + j\omega)(11\gamma/2 + j\omega)};$$
$$W_4^{\{P_4^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(19\gamma/2 + j\omega)} \frac{(3\gamma/2 - j\omega)}{(3\gamma/2 + j\omega)} \times$$
$$\times \frac{(7\gamma/2 - j\omega)(11\gamma/2 - j\omega)}{(7\gamma/2 + j\omega)(11\gamma/2 + j\omega)} \frac{(15\gamma/2 - j\omega)}{(15\gamma/2 + j\omega)};$$
$$W_5^{\{P_5^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(23\gamma/2 + j\omega)} \frac{(3\gamma/2 - j\omega)}{(3\gamma/2 + j\omega)} \times$$
$$\times \frac{(7\gamma/2 - j\omega)(11\gamma/2 - j\omega)}{(7\gamma/2 + j\omega)(11\gamma/2 + j\omega)} \frac{(15\gamma/2 - j\omega)(19\gamma/2 - j\omega)}{(15\gamma/2 + j\omega)(19\gamma/2 + j\omega)}.$$

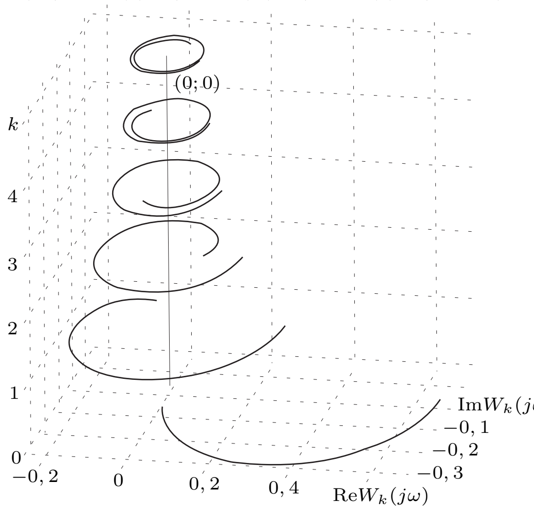


Рис. 6.7. Вид преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[6.21] $$W_k^{[1]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega) =$$
$$= \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+1}{s+1} (-1)^s \frac{1}{(s+1)\gamma + j\omega}.$$

[6.22] $$W_k^{[2]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega) =$$
$$= \begin{cases} \dfrac{1}{\gamma + j\omega}, & \text{если } k = 0; \\ \dfrac{1}{(k+1)\gamma + j\omega} \displaystyle\prod_{s=0}^{k-1} \frac{(s+1)\gamma - j\omega}{(s+1)\gamma + j\omega}, & \text{если } k > 0. \end{cases}$$

[6.23] $W_k^{[3]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega) =$

$$= \begin{cases} \dfrac{1}{\gamma}\cos\varphi_0 \exp\bigl(-j\varphi_0\bigr), & \text{если } k=0;\\ \dfrac{1}{(k+1)\gamma}\cos\varphi_k \times \\ \times \exp\left(-j\left(\varphi_k + 2\sum\limits_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k>0, \end{cases}$$

$$\varphi_k = \operatorname{arctan}\frac{\omega}{(k+1)\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{P_0^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega};$$

$$W_1^{\{P_1^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma - j\omega}{(\gamma + j\omega)(2\gamma + j\omega)};$$

$$W_2^{\{P_2^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{(\gamma - j\omega)(2\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)(3\gamma + j\omega)};$$

$$W_3^{\{P_3^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{(\gamma - j\omega)(2\gamma - j\omega)(3\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)(3\gamma + j\omega)(4\gamma + j\omega)};$$

$$W_4^{\{P_4^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(5\gamma + j\omega)}\,\frac{(\gamma - j\omega)(2\gamma - j\omega)(3\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)(3\gamma + j\omega)} \times$$
$$\times \frac{(4\gamma - j\omega)}{(4\gamma + j\omega)};$$

$$W_5^{\{P_5^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(6\gamma + j\omega)}\,\frac{(\gamma - j\omega)}{(\gamma + j\omega)}\,\frac{(2\gamma - j\omega)(3\gamma - j\omega)}{(2\gamma + j\omega)(3\gamma + j\omega)} \times$$
$$\times \frac{(4\gamma - j\omega)(5\gamma - j\omega)}{(4\gamma + j\omega)(5\gamma + j\omega)}.$$

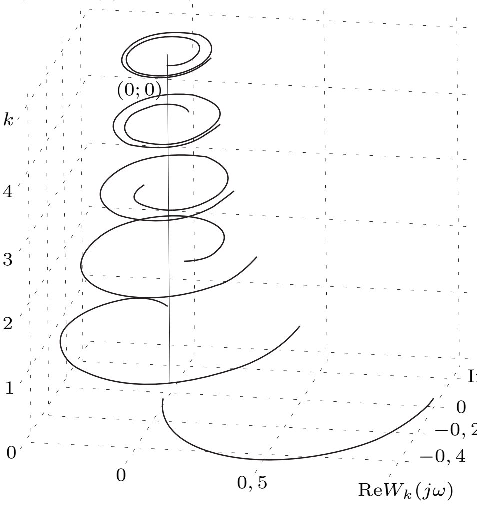


Рис. 6.8. Вид преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 1$, $\alpha = 1$, $\beta = 0$

[6.24] $W_k^{[1]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega) =$

$$= \sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s\frac{1}{(2s+3)\gamma + j\omega}.$$

[6.25] $W_k^{[2]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega) =$

$$= \begin{cases} \dfrac{1}{3\gamma + j\omega}, & \text{если } k=0;\\ \dfrac{1}{(2k+3)\gamma + j\omega}\prod\limits_{s=0}^{k-1}\dfrac{(2s+3)\gamma - j\omega}{(2s+3)\gamma + j\omega}, & \text{если } k>0. \end{cases}$$

[6.26] $W_k^{[3]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega) =$

$$= \begin{cases} \dfrac{1}{3\gamma}\cos\varphi_0 \exp\bigl(-j\varphi_0\bigr), & \text{если } k=0;\\ \dfrac{1}{(2k+3)\gamma}\cos\varphi_k \times \\ \exp\left(-j\left(\varphi_k + 2\sum\limits_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k>0, \end{cases}$$

$$\varphi_k = \operatorname{arctan}\frac{\omega}{(2k+3)\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{P_0^{(2,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{3\gamma + j\omega};$$

$$W_1^{\{P_1^{(2,0)}(\tau,\gamma)\}}(j\omega) = \frac{3\gamma - j\omega}{(3\gamma + j\omega)(5\gamma + j\omega)};$$

$$W_2^{\{P_2^{(2,0)}(\tau,\gamma)\}}(j\omega) = \frac{(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)(7\gamma + j\omega)};$$

$$W_3^{\{P_3^{(2,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(9\gamma + j\omega)}\,\frac{(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)}\,\frac{(7\gamma - j\omega)}{(7\gamma + j\omega)};$$

$$W_4^{\{P_4^{(2,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(11\gamma + j\omega)}\,\frac{(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)} \times$$
$$\times \frac{(7\gamma - j\omega)(9\gamma - j\omega)}{(7\gamma + j\omega)(9\gamma + j\omega)};$$

$$W_5^{\{P_5^{(2,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(13\gamma + j\omega)}\,\frac{(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)} \times$$
$$\times \frac{(7\gamma - j\omega)(9\gamma - j\omega)(11\gamma - j\omega)}{(7\gamma + j\omega)(9\gamma + j\omega)(11\gamma + j\omega)}.$$

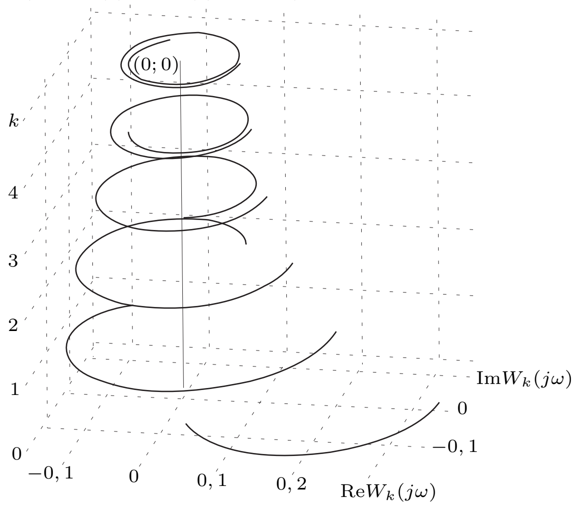


Рис. 6.9. Вид преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 2$, $\beta = 0$

[6.27] $$W_k^{[1]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s\frac{1}{(2s+\alpha+1)c\gamma/2+j\omega}.$$

[6.28] $$W_k^{[2]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \begin{cases}\dfrac{1}{(\alpha+1)c\gamma/2+j\omega}, & \text{если } k=0;\\ \dfrac{1}{(2k+\alpha+1)c\gamma/2+j\omega}\times\\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+\alpha+1)c\gamma/2-j\omega}{(2s+\alpha+1)c\gamma/2+j\omega}, & \text{если } k>0.\end{cases}$$

[6.29] $$W_k^{[3]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \begin{cases}\dfrac{2}{(\alpha+1)c\gamma}\cos\varphi_0\exp\bigl(-j\varphi_0\bigr), & \text{если } k=0;\\ \dfrac{2}{(2k+\alpha+1)c\gamma}\cos\varphi_k\times\\ \times\exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k>0,\end{cases}$$
$$\varphi_k = \arctan\frac{2\omega}{(2k+\alpha+1)c\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{P_0^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(\alpha+1)c\gamma/2+j\omega};$$

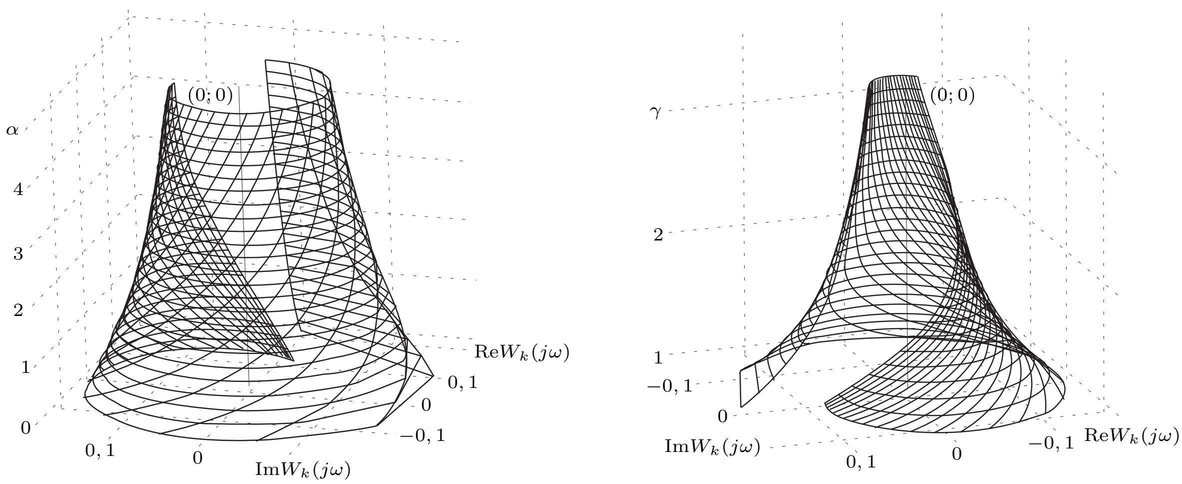
$$W_1^{\{P_1^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \frac{(\alpha+1)c\gamma/2-j\omega}{\bigl((\alpha+1)c\gamma/2+j\omega\bigr)\bigl((\alpha+3)c\gamma/2+j\omega\bigr)};$$

$$W_2^{\{P_2^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\bigl((\alpha+5)c\gamma/2+j\omega\bigr)}\frac{\bigl((\alpha+1)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2+j\omega\bigr)}\times\frac{\bigl((\alpha+3)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+3)c\gamma/2+j\omega\bigr)};$$

$$W_3^{\{P_3^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\bigl((\alpha+7)c\gamma/2+j\omega\bigr)}\frac{\bigl((\alpha+1)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2+j\omega\bigr)}\times\frac{\bigl((\alpha+3)c\gamma/2-j\omega\bigr)\bigl((\alpha+5)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+3)c\gamma/2+j\omega\bigr)\bigl((\alpha+5)c\gamma/2+j\omega\bigr)};$$

$$W_4^{\{P_4^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\bigl((\alpha+9)c\gamma/2+j\omega\bigr)}\frac{\bigl((\alpha+1)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+7)c\gamma/2+j\omega\bigr)}\times\frac{\bigl((\alpha+3)c\gamma/2-j\omega\bigr)\bigl((\alpha+5)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+3)c\gamma/2+j\omega\bigr)\bigl((\alpha+5)c\gamma/2+j\omega\bigr)}\frac{\bigl((\alpha+7)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+7)c\gamma/2+j\omega\bigr)};$$

$$W_5^{\{P_5^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\bigl((\alpha+11)c\gamma/2+j\omega\bigr)}\frac{\bigl((\alpha+1)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2+j\omega\bigr)}\times\frac{\bigl((\alpha+3)c\gamma/2-j\omega\bigr)\bigl((\alpha+5)c\gamma/2-j\omega\bigr)\bigl((\alpha+7)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+3)c\gamma/2+j\omega\bigr)\bigl((\alpha+5)c\gamma/2+j\omega\bigr)\bigl((\alpha+7)c\gamma/2+j\omega\bigr)}\times\frac{\bigl((\alpha+9)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+9)c\gamma/2+j\omega\bigr)}.$$



а) б)

Рис. 6.10. Вид преобразования Фурье ортогональных функций Якоби 2-ого порядка: а) $\gamma = 1$, $c = 2$, $\alpha \in [0; 5]$, $\beta = 0$; б) $\gamma \in [1; 3, 5]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[6.30] $$W_k^{[1]\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega) = \sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s\frac{1}{(2s+1)\gamma+j\omega}.$$

[6.31] $$W_k^{[2]\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega) = \sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}\times(-1)^s\frac{\cos\varphi_s\exp\bigl(-j\varphi_s\bigr)}{(2s+1)\gamma},\quad \varphi_k = \arctan\frac{\omega}{(2k+1)\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{P_0^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma+j\omega};$$

$$W_1^{\{P_1^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{3}{3\gamma + j\omega};$$

$$W_2^{\{P_2^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{8}{3\gamma + j\omega} + \frac{10}{5\gamma + j\omega};$$

$$W_3^{\{P_3^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{15}{3\gamma + j\omega} + \frac{45}{5\gamma + j\omega} - \frac{35}{7\gamma + j\omega};$$

$$W_4^{\{P_4^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{24}{3\gamma + j\omega} + \frac{126}{5\gamma + j\omega} - \frac{224}{7\gamma + j\omega} + \frac{126}{9\gamma + j\omega};$$

$$W_5^{\{P_5^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{35}{3\gamma + j\omega} + \frac{280}{5\gamma + j\omega} - \frac{840}{7\gamma + j\omega} + \frac{1050}{9\gamma + j\omega} - \frac{462}{11\gamma + j\omega}.$$

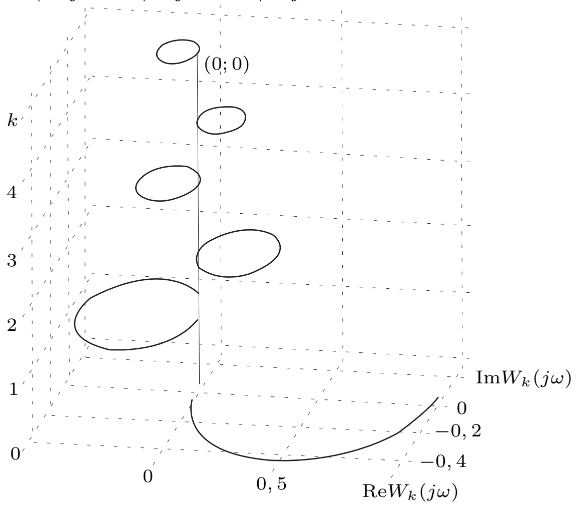


Рис. 6.11. Вид преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta = 1$

[6.32]
$$W_k^{[1]\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega) = \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+2}{s} (-1)^s \frac{1}{(2s+1)\gamma + j\omega}.$$

[6.33]
$$W_k^{[2]\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega) = \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+2}{s} \times (-1)^s \frac{\cos\varphi_s \exp(-j\varphi_s)}{(2s+1)\gamma}, \quad \varphi_k = \arctan\frac{\omega}{(2k+1)\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{P_0^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega};$$

$$W_1^{\{P_1^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{4}{3\gamma + j\omega};$$

$$W_2^{\{P_2^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{10}{3\gamma + j\omega} + \frac{15}{5\gamma + j\omega};$$

$$W_3^{\{P_3^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{18}{3\gamma + j\omega} + \frac{63}{5\gamma + j\omega} - \frac{56}{7\gamma + j\omega};$$

$$W_4^{\{P_4^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{28}{3\gamma + j\omega} + \frac{168}{5\gamma + j\omega} - \frac{336}{7\gamma + j\omega} + \frac{210}{9\gamma + j\omega};$$

$$W_5^{\{P_5^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma + j\omega} - \frac{40}{3\gamma + j\omega} + \frac{360}{5\gamma + j\omega} - \frac{1200}{7\gamma + j\omega} + \frac{1650}{9\gamma + j\omega} - \frac{792}{11\gamma + j\omega}.$$

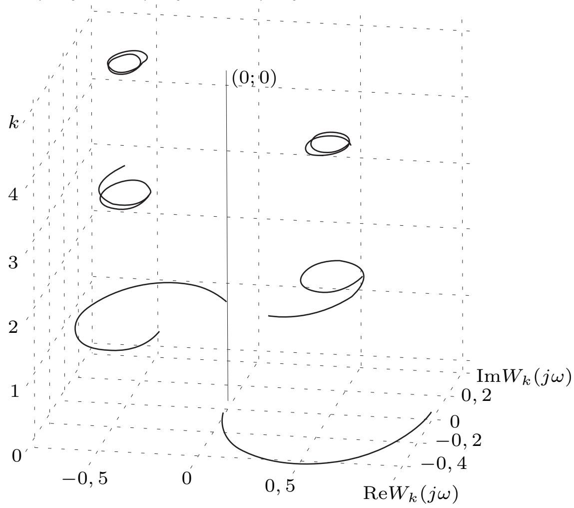


Рис. 6.12. Вид преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta = 2$

[6.34]
$$W_k^{[1]\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega) = \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+\beta}{s} (-1)^s \frac{1}{(2s+1)c\gamma/2 + j\omega}.$$

[6.35]
$$W_k^{[2]\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega) = \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+\beta}{s} \times (-1)^s \frac{\cos\varphi_s \exp(-j\varphi_s)}{(2s+1)c\gamma/2}, \quad \varphi_k = \arctan\frac{2\omega}{(2k+1)c\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$$W_0^{\{P_0^{(0,\beta)}(\tau,\gamma)\}}(j\omega) = \frac{1}{c\gamma/2 + j\omega};$$

$$W_1^{\{P_1^{(0,\beta)}(\tau,\gamma)\}}(j\omega) = \frac{1}{c\gamma/2 + j\omega} - \frac{\beta+2}{3c\gamma/2 + j\omega};$$

$$W_2^{\{P_2^{(0,\beta)}(\tau,\gamma)\}}(j\omega) = \frac{1}{c\gamma/2 + j\omega} - \frac{2(\beta+3)}{3c\gamma/2 + j\omega} + \frac{(\beta+3)(\beta+4)/2}{5c\gamma/2 + j\omega};$$

$$W_3^{\{P_3^{(0,\beta)}(\tau,\gamma)\}}(j\omega) = \frac{1}{c\gamma/2 + j\omega} - \frac{3(\beta+4)}{3c\gamma/2 + j\omega} + \frac{3(\beta+4)(\beta+5)/2}{5c\gamma/2 + j\omega} - \frac{(\beta+4)(\beta+5)(\beta+6)/6}{7c\gamma/2 + j\omega};$$

$$W_4^{\{P_4^{(0,\beta)}(\tau,\gamma)\}}(j\omega) = \frac{1}{c\gamma/2 + j\omega} - \frac{4(\beta+5)}{3c\gamma/2 + j\omega} + \frac{3(\beta+5)(\beta+6)}{5c\gamma/2 + j\omega} - \frac{2(\beta+5)(\beta+6)(\beta+7)/3}{7c\gamma/2 + j\omega} +$$

$$+\frac{(\beta+8)!}{24(\beta+4)!(9c\gamma/2+j\omega)};$$

$$W_5^{\{P_5^{(0,\beta)}(\tau,\gamma)\}}(j\omega)=\frac{1}{c\gamma/2+j\omega}-\frac{5(\beta+6)}{3c\gamma/2+j\omega}+$$

$$+\frac{5(\beta+6)(\beta+7)}{5c\gamma/2+j\omega}-\frac{5(\beta+6)(\beta+7)(\beta+8)/3}{7c\gamma/2+j\omega}+$$

$$+\frac{5(\beta+9)!}{24(\beta+5)!(9c\gamma/2+j\omega)}-\frac{(\beta+10)!}{120(\beta+5)!(11c\gamma/2+j\omega)}.$$

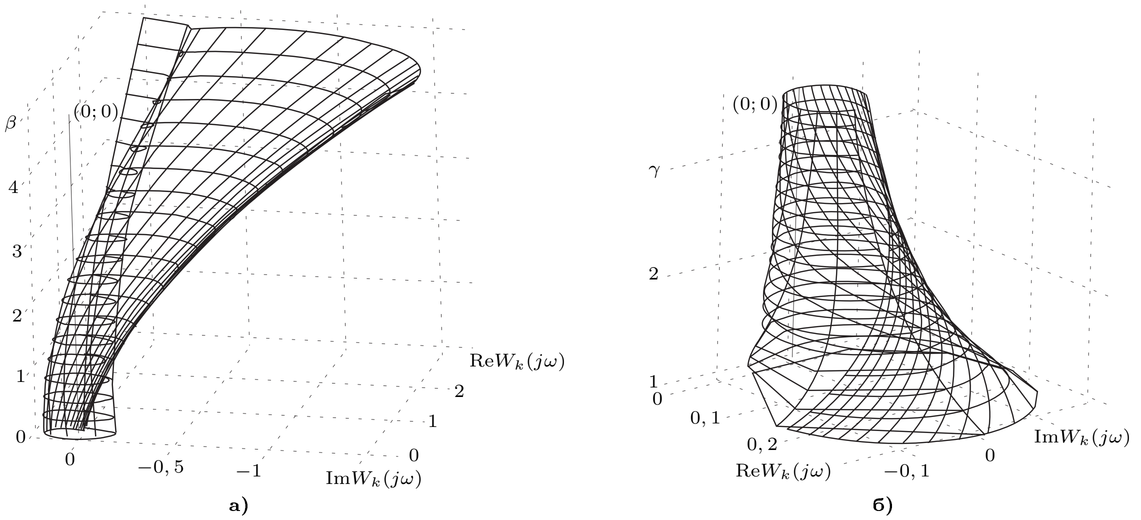


Рис. 6.13. Вид преобразования Фурье ортогональных функций Якоби 2-ого порядка: а) $\gamma=1$, $c=2$, $\alpha=0$, $\beta\in[0;5]$; б) $\gamma\in[1;3,5]$, $c=2$, $\alpha=0$, $\beta=1$

6.2 Преобразование Фурье ортогональных фильтров

[6.36] $V_k^{[1]\{L_k(\tau,\gamma)\}}(j\omega)=$

$$=\frac{\gamma}{j\omega+\gamma/2}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

[6.37] $V_k^{[2]\{L_k(\tau,\gamma)\}}(j\omega)=\dfrac{\gamma}{j\omega+\gamma/2}\left(\dfrac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^k.$

[6.38] $V_k^{[3]\{L_k(\tau,\gamma)\}}(j\omega)=2(-1)^k\cos\varphi\times$

$$\times\exp(-j(2k+1)\varphi),\quad \varphi=\arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$$V_0^{\{L_0(\tau,\gamma)\}}(j\omega)=\frac{\gamma}{j\omega+\gamma/2};$$

$$V_1^{\{L_1(\tau,\gamma)\}}(j\omega)=\frac{\gamma(j\omega-\gamma/2)}{(j\omega+\gamma/2)^2};$$

$$V_2^{\{L_2(\tau,\gamma)\}}(j\omega)=\frac{\gamma(j\omega-\gamma/2)^2}{(j\omega+\gamma/2)^3};$$

$$V_3^{\{L_3(\tau,\gamma)\}}(j\omega)=\frac{\gamma(j\omega-\gamma/2)^3}{(j\omega+\gamma/2)^4};$$

$$V_4^{\{L_4(\tau,\gamma)\}}(j\omega)=\frac{\gamma(j\omega-\gamma/2)^4}{(j\omega+\gamma/2)^5};$$

$$V_5^{\{L_5(\tau,\gamma)\}}(j\omega)=\frac{\gamma(j\omega-\gamma/2)^5}{(j\omega+\gamma/2)^6}.$$

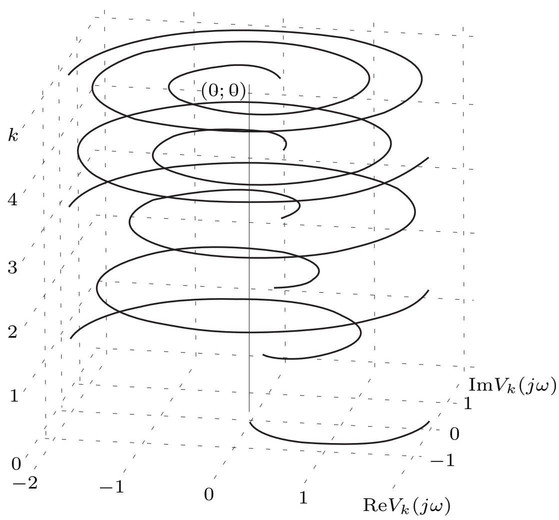


Рис. 6.14. Вид преобразования Фурье ортогональных фильтров Лагерра 0-5 порядков; $\gamma=4$

[6.39] $V_k^{[1]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)=$

$$=\frac{\gamma^2}{(j\omega+\gamma/2)^2}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

[6.40] $V_k^{[2]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)=\dfrac{\gamma^2}{(j\omega+\gamma/2)^2}\left(\dfrac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^k.$

[6.41] $$V_k^{[3]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega) = 4(-1)^k(\cos\varphi)^2\times \times\exp(-j(2k+2)\varphi),\quad \varphi=\arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$$V_0^{\{L_0^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^2}{(j\omega+\gamma/2)^2};$$

$$V_1^{\{L_1^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^2(j\omega-\gamma/2)}{(j\omega+\gamma/2)^3};$$

$$V_2^{\{L_2^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^2(j\omega-\gamma/2)^2}{(j\omega+\gamma/2)^4};$$

$$V_3^{\{L_3^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^2(j\omega-\gamma/2)^3}{(j\omega+\gamma/2)^5};$$

$$V_4^{\{L_4^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^2(j\omega-\gamma/2)^4}{(j\omega+\gamma/2)^6};$$

$$V_5^{\{L_5^{(1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^2(j\omega-\gamma/2)^5}{(j\omega+\gamma/2)^7}.$$

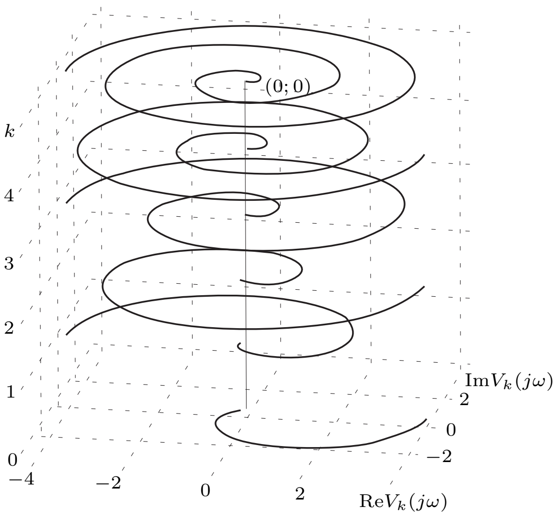


Рис. 6.15. Вид преобразования Фурье ортогональных фильтров Сонина-Лагерра 0-5 порядков; $\gamma=4$, $\alpha=1$

[6.42] $$V_k^{[1]\{L_k^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^3}{(j\omega+\gamma/2)^3}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

[6.43] $$V_k^{[2]\{L_k^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^3}{(j\omega+\gamma/2)^3}\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^k.$$

[6.44] $$V_k^{[3]\{L_k^{(2)}(\tau,\gamma)\}}(j\omega) = 8(-1)^k(\cos\varphi)^3\times \times\exp(-j(2k+3)\varphi),\quad \varphi=\arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$$V_0^{\{L_0^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^3}{(j\omega+\gamma/2)^3};$$

$$V_1^{\{L_1^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^3(j\omega-\gamma/2)}{(j\omega+\gamma/2)^4};$$

$$V_2^{\{L_2^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^3(j\omega-\gamma/2)^2}{(j\omega+\gamma/2)^5};$$

$$V_3^{\{L_3^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^3(j\omega-\gamma/2)^3}{(j\omega+\gamma/2)^6};$$

$$V_4^{\{L_4^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^3(j\omega-\gamma/2)^4}{(j\omega+\gamma/2)^7};$$

$$V_5^{\{L_5^{(2)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^3(j\omega-\gamma/2)^5}{(j\omega+\gamma/2)^8}.$$

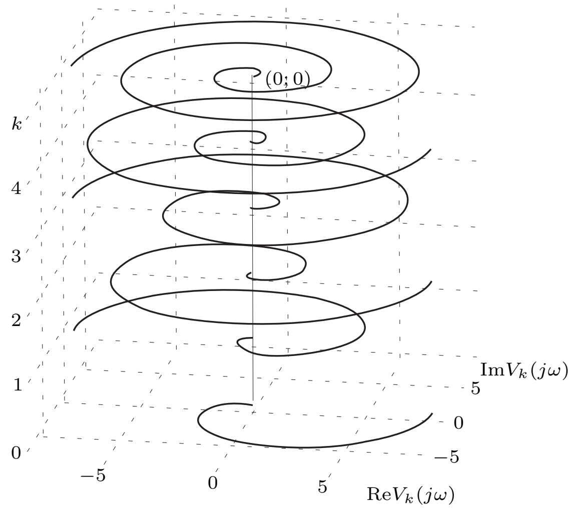


Рис. 6.16. Вид преобразования Фурье ортогональных фильтров Сонина-Лагерра 0-5 порядков; $\gamma=4$, $\alpha=2$

[6.45] $$V_k^{[1]\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^{\alpha+1}}{(j\omega+\gamma/2)^{\alpha+1}}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

[6.46] $$V_k^{[2]\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \left(\frac{\gamma}{j\omega+\gamma/2}\right)^{\alpha+1}\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^k.$$

[6.47] $$V_k^{[3]\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega) = (-1)^k(2\cos\varphi)^{\alpha+1}\times \times\exp(-j(2k+\alpha+1)\varphi),\quad \varphi=\arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$$V_0^{\{L_0^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^{\alpha+1}}{(j\omega+\gamma/2)^{\alpha+1}};$$

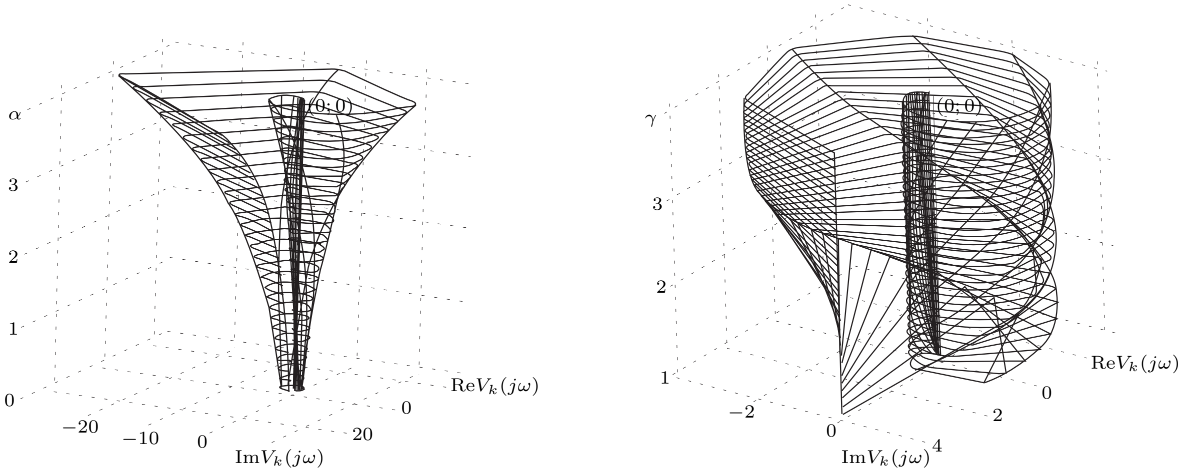
$$V_1^{\{L_1^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^{\alpha+1}(j\omega-\gamma/2)}{(j\omega+\gamma/2)^{\alpha+2}};$$

$$V_2^{\{L_2^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^{\alpha+1}(j\omega-\gamma/2)^2}{(j\omega+\gamma/2)^{\alpha+3}};$$

$$V_3^{\{L_3^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^{\alpha+1}\big(j\omega-\gamma/2\big)^3}{\big(j\omega+\gamma/2\big)^{\alpha+4}};$$

$$V_4^{\{L_4^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^{\alpha+1}\big(j\omega-\gamma/2\big)^4}{\big(j\omega+\gamma/2\big)^{\alpha+5}};$$

$$V_5^{\{L_5^{(\alpha)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma^{\alpha+1}\big(j\omega-\gamma/2\big)^5}{\big(j\omega+\gamma/2\big)^{\alpha+6}}.$$



а) б)

Рис. 6.17. Вид преобразования Фурье ортогональных фильтров Сонина-Лагерра 2-ого порядка: а) $\gamma = 4$, $\alpha \in [0;4]$; б) $\gamma \in [1;4]$, $\alpha = 1$

[6.48] $$V_k^{[1]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) = (4k+1)\gamma\times$$
$$\times\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\frac{1}{(4s+1)\gamma/2+j\omega}.$$

[6.49] $$V_k^{[2]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) =$$
$$=\begin{cases}\dfrac{\gamma}{\gamma/2+j\omega}, & \text{если } k=0;\\ \dfrac{(4k+1)\gamma}{(4k+1)\gamma/2+j\omega}\times & \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(4s+1)\gamma/2-j\omega}{(4s+1)\gamma/2+j\omega}, & \text{если } k>0.\end{cases}$$

[6.50] $$V_k^{[3]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) =$$
$$=\begin{cases}2\cos\varphi_0\exp\big(-j\varphi_0\big), & \text{если } k=0;\\ 2\cos\varphi_k\times & \\ \exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k>0,\end{cases}$$
$$\varphi_k = \arctan\frac{2\omega}{(4k+1)\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$$V_0^{\{P_0^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma}{\gamma/2+j\omega};$$

$$V_1^{\{P_1^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{5\gamma(\gamma/2-j\omega)}{(\gamma/2+j\omega)(5\gamma/2+j\omega)};$$

$$V_2^{\{P_2^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{9\gamma(\gamma/2-j\omega)(5\gamma/2-j\omega)}{(\gamma/2+j\omega)(5\gamma/2+j\omega)(9\gamma/2+j\omega)};$$

$$V_3^{\{P_3^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{13\gamma}{(13\gamma/2+j\omega)}\frac{(\gamma/2-j\omega)}{(\gamma/2+j\omega)}\times$$
$$\times\frac{(5\gamma/2-j\omega)(9\gamma/2-j\omega)}{(5\gamma/2+j\omega)(9\gamma/2+j\omega)};$$

$$V_4^{\{P_4^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{17\gamma}{(17\gamma/2+j\omega)}\frac{(\gamma/2-j\omega)}{(\gamma/2+j\omega)}\times$$
$$\times\frac{(5\gamma/2-j\omega)(9\gamma/2-j\omega)}{(5\gamma/2+j\omega)(9\gamma/2+j\omega)}\frac{(13\gamma/2-j\omega)}{(13\gamma/2+j\omega)};$$

$$V_5^{\{P_5^{(-1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{21\gamma}{(21\gamma/2+j\omega)}\frac{(\gamma/2-j\omega)}{(\gamma/2+j\omega)}\times$$
$$\times\frac{(5\gamma/2-j\omega)(9\gamma/2-j\omega)}{(5\gamma/2+j\omega)(9\gamma/2+j\omega)}\frac{(13\gamma/2-j\omega)(17\gamma/2-j\omega)}{(13\gamma/2+j\omega)(17\gamma/2+j\omega)}.$$

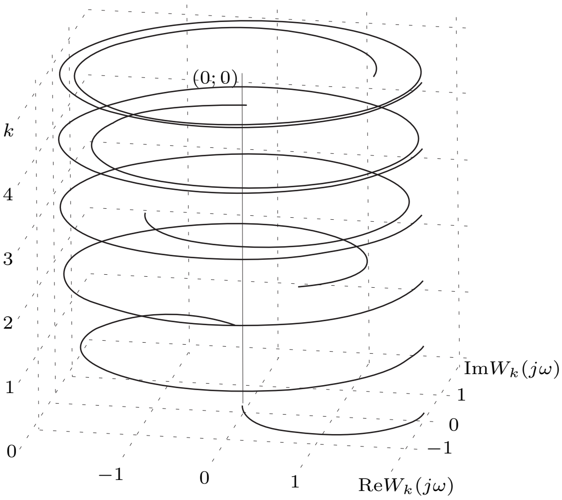


Рис. 6.18. Вид преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

[6.51] $$V_k^{[1]\{Leg_k(\tau,\gamma)\}}(j\omega) = 2(2k+1)\gamma \times \sum_{s=0}^{k} \binom{k}{s}\binom{k+s}{s}(-1)^s \frac{1}{(2s+1)\gamma + j\omega}.$$

[6.52] $$V_k^{[2]\{Leg_k(\tau,\gamma)\}}(j\omega) = \begin{cases} \dfrac{2\gamma}{\gamma + j\omega}, & \text{если } k = 0; \\ \dfrac{2(2k+1)\gamma}{(2k+1)\gamma + j\omega} \prod\limits_{s=0}^{k-1} \dfrac{(2s+1)\gamma - j\omega}{(2s+1)\gamma + j\omega}, & \text{если } k > 0. \end{cases}$$

[6.53] $$V_k^{[3]\{Leg_k(\tau,\gamma)\}}(j\omega) = \begin{cases} 2\cos\varphi_0 \exp\big(-j\varphi_0\big), & \text{если } k = 0; \\ 2\cos\varphi_k \times \\ \times \exp\left(-j\left(\varphi_k + 2\sum\limits_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k > 0, \end{cases}$$
$$\varphi_k = \arctan\frac{\omega}{(2k+1)\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$V_0^{\{Leg_0(\tau,\gamma)\}}(j\omega) = \dfrac{2\gamma}{\gamma + j\omega}$;

$V_1^{\{Leg_1(\tau,\gamma)\}}(j\omega) = \dfrac{6\gamma(\gamma - j\omega)}{(\gamma + j\omega)(3\gamma + j\omega)}$;

$V_2^{\{Leg_2(\tau,\gamma)\}}(j\omega) = \dfrac{10\gamma(\gamma - j\omega)(3\gamma - j\omega)}{(\gamma + j\omega)(3\gamma + j\omega)(5\gamma + j\omega)}$;

$V_3^{\{Leg_3(\tau,\gamma)\}}(j\omega) = \dfrac{14\gamma(\gamma - j\omega)(3\gamma - j\omega)(5\gamma - j\omega)}{(\gamma + j\omega)(3\gamma + j\omega)(5\gamma + j\omega)(7\gamma + j\omega)}$;

$V_4^{\{Leg_4(\tau,\gamma)\}}(j\omega) = \dfrac{18\gamma}{(9\gamma + j\omega)} \dfrac{(\gamma - j\omega)(3\gamma - j\omega)(5\gamma - j\omega)}{(\gamma + j\omega)(3\gamma + j\omega)(5\gamma + j\omega)} \times$
$\times \dfrac{(7\gamma - j\omega)}{(7\gamma + j\omega)}$;

$V_5^{\{Leg_5(\tau,\gamma)\}}(j\omega) = \dfrac{22\gamma}{(11\gamma + j\omega)} \dfrac{(\gamma - j\omega)}{(\gamma + j\omega)} \dfrac{(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)} \times$
$\times \dfrac{(7\gamma - j\omega)(9\gamma - j\omega)}{(7\gamma + j\omega)(9\gamma + j\omega)}$.

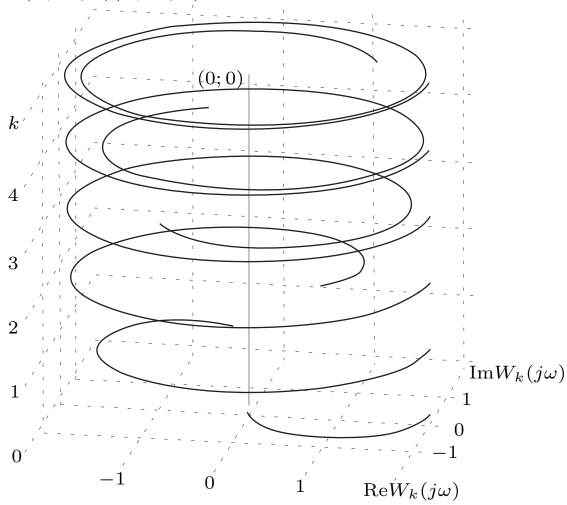


Рис. 6.19. Вид преобразования Фурье ортогональных фильтров Лежандра 0-5 порядков; $\gamma = 1$, $c = 2$

[6.54] $$V_k^{[1]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = (4k+3)\gamma \times \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s \frac{1}{(4s+3)\gamma/2 + j\omega}.$$

[6.55] $$V_k^{[2]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \begin{cases} \dfrac{3\gamma}{3\gamma/2 + j\omega}, & \text{если } k = 0; \\ \dfrac{(4k+3)\gamma}{(4k+3)\gamma/2 + j\omega} \times \\ \times \prod\limits_{s=0}^{k-1} \dfrac{(4s+3)\gamma/2 - j\omega}{(4s+3)\gamma/2 + j\omega}, & \text{если } k > 0. \end{cases}$$

[6.56] $$V_k^{[3]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \begin{cases} 2\cos\varphi_0 \exp\big(-j\varphi_0\big), & \text{если } k = 0; \\ 2\cos\varphi_k \times \\ \times \exp\left(-j\left(\varphi_k + 2\sum\limits_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k > 0, \end{cases}$$
$$\varphi_k = \arctan\frac{2\omega}{(4k+3)\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$V_0^{\{P_0^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{3\gamma}{3\gamma/2 + j\omega}$;

$V_1^{\{P_1^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{7\gamma(3\gamma/2 - j\omega)}{(3\gamma/2 + j\omega)(7\gamma/2 + j\omega)}$;

$V_2^{\{P_2^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{11\gamma(3\gamma/2 - j\omega)(7\gamma/2 - j\omega)}{(3\gamma/2 + j\omega)(7\gamma/2 + j\omega)(11\gamma/2 + j\omega)}$;

$V_3^{\{P_3^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{15\gamma}{(15\gamma/2 + j\omega)} \dfrac{(3\gamma/2 - j\omega)}{(3\gamma/2 + j\omega)} \times$

$$\times \frac{(7\gamma/2 - j\omega)(11\gamma/2 - j\omega)}{(7\gamma/2 + j\omega)(11\gamma/2 + j\omega)};$$

$$V_4^{\{P_4^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{19\gamma}{(19\gamma/2 + j\omega)} \frac{(3\gamma/2 - j\omega)}{(3\gamma/2 + j\omega)} \times$$
$$\times \frac{(7\gamma/2 - j\omega)(11\gamma/2 - j\omega)}{(7\gamma/2 + j\omega)(11\gamma/2 + j\omega)} \frac{(15\gamma/2 - j\omega)}{(15\gamma/2 + j\omega)};$$

$$V_5^{\{P_5^{(1/2,0)}(\tau,\gamma)\}}(j\omega) = \frac{23\gamma}{(23\gamma/2 + j\omega)} \frac{(3\gamma/2 - j\omega)}{(3\gamma/2 + j\omega)} \times$$
$$\times \frac{(7\gamma/2 - j\omega)(11\gamma/2 - j\omega)}{(7\gamma/2 + j\omega)(11\gamma/2 + j\omega)} \frac{(15\gamma/2 - j\omega)(19\gamma/2 - j\omega)}{(15\gamma/2 + j\omega)(19\gamma/2 + j\omega)}.$$

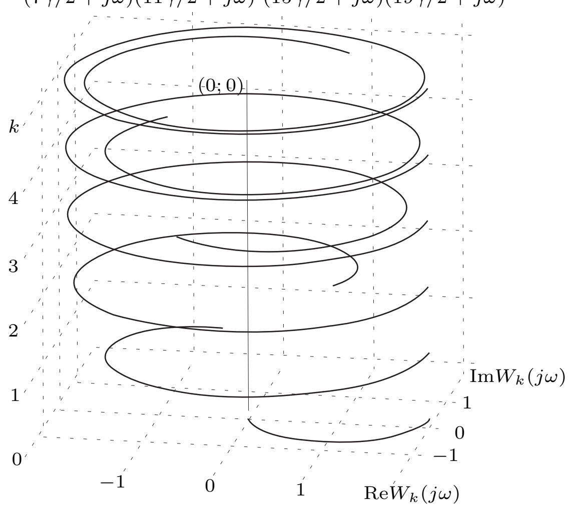


Рис. 6.20. Вид преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[6.57] $$V_k^{[1]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega) = 2(k+1)\gamma \times$$
$$\times \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+1}{s+1} (-1)^s \frac{1}{(s+1)\gamma + j\omega}.$$

[6.58] $$V_k^{[2]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega) =$$
$$= \begin{cases} \dfrac{2\gamma}{\gamma + j\omega}, & \text{если } k = 0; \\ \dfrac{2(k+1)\gamma}{(k+1)\gamma + j\omega} \displaystyle\prod_{s=0}^{k-1} \frac{(s+1)\gamma - j\omega}{(s+1)\gamma + j\omega}, & \text{если } k > 0. \end{cases}$$

[6.59] $$V_k^{[3]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega) =$$
$$= \begin{cases} 2\cos\varphi_0 \exp(-j\varphi_0), & \text{если } k = 0; \\ 2\cos\varphi_k \times \\ \times \exp\left(-j\left(\varphi_k + 2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k > 0, \end{cases}$$
$$\varphi_k = \arctan\frac{\omega}{(k+1)\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$$V_0^{\{P_0^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{2\gamma}{\gamma + j\omega};$$

$$V_1^{\{P_1^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{4\gamma(\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)};$$

$$V_2^{\{P_2^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{6\gamma(\gamma - j\omega)(2\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)(3\gamma + j\omega)};$$

$$V_3^{\{P_3^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{8\gamma(\gamma - j\omega)(2\gamma - j\omega)(3\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)(3\gamma + j\omega)(4\gamma + j\omega)};$$

$$V_4^{\{P_4^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{10\gamma}{(5\gamma + j\omega)} \frac{(\gamma - j\omega)(2\gamma - j\omega)(3\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)(3\gamma + j\omega)} \times$$
$$\times \frac{(4\gamma - j\omega)}{(4\gamma + j\omega)};$$

$$V_5^{\{P_5^{(1,0)}(\tau,\gamma)\}}(j\omega) = \frac{12\gamma}{(6\gamma + j\omega)} \frac{(\gamma - j\omega)}{(\gamma + j\omega)} \frac{(2\gamma - j\omega)(3\gamma - j\omega)}{(2\gamma + j\omega)(3\gamma + j\omega)} \times$$
$$\times \frac{(4\gamma - j\omega)(5\gamma - j\omega)}{(4\gamma + j\omega)(5\gamma + j\omega)}.$$

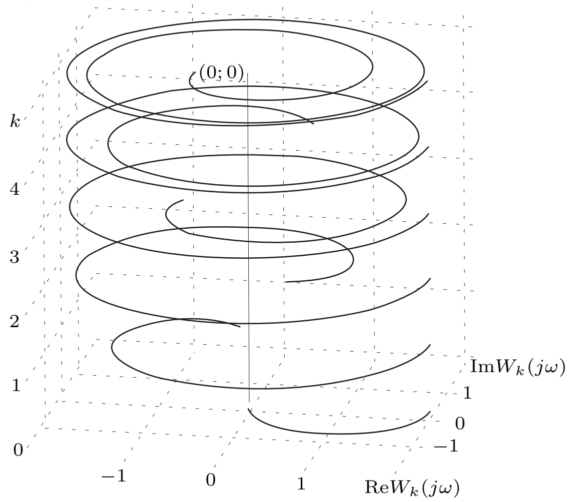


Рис. 6.21. Вид преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 1$, $\alpha = 1$, $\beta = 0$

[6.60] $$V_k^{[1]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega) = 2(2k+3)\gamma \times$$
$$\times \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+2}{s+2} (-1)^s \frac{1}{(2s+3)\gamma + j\omega}.$$

[6.61] $$V_k^{[2]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega) =$$
$$= \begin{cases} \dfrac{6\gamma}{3\gamma + j\omega}, & \text{если } k = 0; \\ \dfrac{2(2k+3)\gamma}{(2k+3)\gamma + j\omega} \displaystyle\prod_{s=0}^{k-1} \frac{(2s+3)\gamma - j\omega}{(2s+3)\gamma + j\omega}, & \text{если } k > 0. \end{cases}$$

[6.62] $$V_k^{[3]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega) =$$
$$= \begin{cases} 2\cos\varphi_0 \exp(-j\varphi_0), & \text{если } k = 0; \\ 2\cos\varphi_k \times \\ \times \exp\left(-j\left(\varphi_k + 2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right), & \text{если } k > 0, \end{cases}$$
$$\varphi_k = \arctan\frac{\omega}{(2k+3)\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$V_0^{\{P_0^{(2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{6\gamma}{3\gamma + j\omega};$

$V_1^{\{P_1^{(2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{10\gamma(3\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)};$

$V_2^{\{P_2^{(2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{14\gamma(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)(7\gamma + j\omega)};$

$V_3^{\{P_3^{(2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{18\gamma}{(9\gamma + j\omega)}\dfrac{(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)} \times$
$\times \dfrac{(7\gamma - j\omega)}{(7\gamma + j\omega)};$

$V_4^{\{P_4^{(2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{22\gamma}{(11\gamma + j\omega)}\dfrac{(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)} \times$
$\times \dfrac{(7\gamma - j\omega)(9\gamma - j\omega)}{(7\gamma + j\omega)(9\gamma + j\omega)};$

$V_5^{\{P_5^{(2,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{26\gamma}{(13\gamma + j\omega)}\dfrac{(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)} \times$
$\times \dfrac{(7\gamma - j\omega)(9\gamma - j\omega)(11\gamma - j\omega)}{(7\gamma + j\omega)(9\gamma + j\omega)(11\gamma + j\omega)}.$

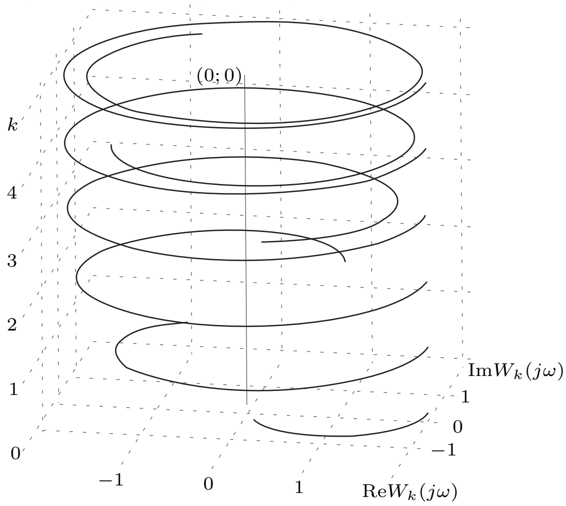


Рис. 6.22. Вид преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 2$, $\beta = 0$

[6.63] $V_k^{[1]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = (2k + \alpha + 1)c\gamma \times$
$$\times \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s \frac{1}{(2s+\alpha+1)c\gamma/2 + j\omega}.$$

[6.64] $V_k^{[2]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) =$
$$= \begin{cases} \dfrac{(\alpha+1)c\gamma}{(\alpha+1)c\gamma/2 + j\omega}, & \text{если } k = 0; \\ \dfrac{(2k+\alpha+1)c\gamma}{(2k+\alpha+1)c\gamma/2 + j\omega} \times \\ \times \displaystyle\prod_{s=0}^{k-1} \dfrac{(2s+\alpha+1)c\gamma/2 - j\omega}{(2s+\alpha+1)c\gamma/2 + j\omega}, & \text{если } k > 0. \end{cases}$$

[6.65] $V_k^{[3]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) =$
$$= \begin{cases} 2\cos\varphi_0 \exp\bigl(-j\varphi_0\bigr), & \text{если } k = 0; \\ 2\cos\varphi_k \times \\ \times \exp\Biggl(-j\biggl(\varphi_k + 2\displaystyle\sum_{s=0}^{k-1}\varphi_s\biggr)\Biggr), & \text{если } k > 0, \end{cases}$$
$$\varphi_k = \arctan\frac{2\omega}{(2k+\alpha+1)c\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$V_0^{\{P_0^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{(\alpha+1)c\gamma}{(\alpha+1)c\gamma/2 + j\omega};$

$V_1^{\{P_1^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{(\alpha+3)c\gamma\bigl((\alpha+1)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2 + j\omega\bigr)\bigl((\alpha+3)c\gamma/2 + j\omega\bigr)};$

$V_2^{\{P_2^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{(\alpha+5)c\gamma}{((\alpha+5)c\gamma/2 + j\omega)}\dfrac{\bigl((\alpha+1)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2 + j\omega\bigr)} \times$
$\times \dfrac{\bigl((\alpha+3)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+3)c\gamma/2 + j\omega\bigr)};$

$V_3^{\{P_3^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{(\alpha+7)c\gamma}{((\alpha+7)c\gamma/2 + j\omega)}\dfrac{\bigl((\alpha+1)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2 + j\omega\bigr)} \times$
$\times \dfrac{\bigl((\alpha+3)c\gamma/2 - j\omega\bigr)\bigl((\alpha+5)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+3)c\gamma/2 + j\omega\bigr)\bigl((\alpha+5)c\gamma/2 + j\omega\bigr)};$

$V_4^{\{P_4^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{(\alpha+9)c\gamma}{((\alpha+9)c\gamma/2 + j\omega)}\dfrac{\bigl((\alpha+1)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+7)c\gamma/2 + j\omega\bigr)} \times$
$\times \dfrac{\bigl((\alpha+3)c\gamma/2 - j\omega\bigr)\bigl((\alpha+5)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+3)c\gamma/2 + j\omega\bigr)\bigl((\alpha+5)c\gamma/2 + j\omega\bigr)}\dfrac{\bigl((\alpha+7)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+7)c\gamma/2 + j\omega\bigr)};$

$V_5^{\{P_5^{(\alpha,0)}(\tau,\gamma)\}}(j\omega) = \dfrac{(\alpha+11)c\gamma}{\bigl((\alpha+11)c\gamma/2 + j\omega\bigr)} \times$
$\times \dfrac{\bigl((\alpha+1)c\gamma/2 - j\omega\bigr)\bigl((\alpha+3)c\gamma/2 - j\omega\bigr)\bigl((\alpha+5)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+3)c\gamma/2 + j\omega\bigr)\bigl((\alpha+3)c\gamma/2 + j\omega\bigr)\bigl((\alpha+5)c\gamma/2 + j\omega\bigr)} \times$
$\times \dfrac{\bigl((\alpha+7)c\gamma/2 - j\omega\bigr)\bigl((\alpha+9)c\gamma/2 - j\omega\bigr)}{\bigl((\alpha+7)c\gamma/2 + j\omega\bigr)\bigl((\alpha+9)c\gamma/2 + j\omega\bigr)}.$

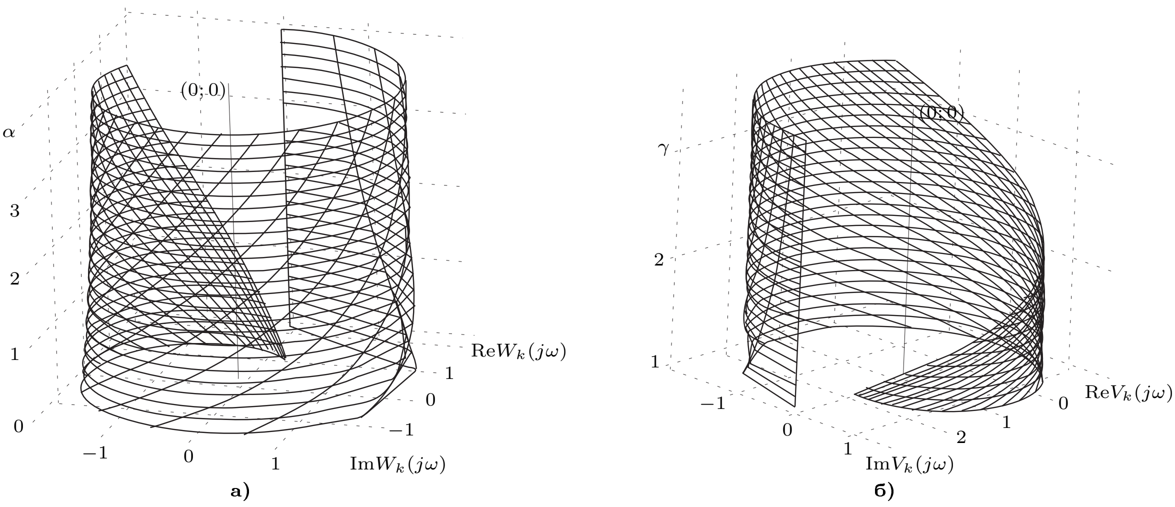


Рис. 6.23. Вид преобразования Фурье ортогональных фильтров Якоби 2-ого порядка: а) $\gamma = 1$, $c = 2$, $\alpha \in [0; 4]$, $\beta = 0$; б) $\gamma \in [1; 3,5]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[6.66]
$$V_k^{[1]\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{8\gamma^2(k+1)^2}{(2k+3)\gamma + j\omega}\times$$
$$\times\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{(2s+1)\gamma + j\omega}.$$

[6.67]
$$V_k^{[2]\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega) =$$
$$= \begin{cases} \dfrac{8\gamma^2}{(\gamma + j\omega)(3\gamma + j\omega)}, & \text{если } k = 0; \\ \dfrac{8(k+1)^2\gamma^2}{\big((2k+1)\gamma + j\omega\big)\big((2k+3)\gamma + j\omega\big)}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+1)\gamma - j\omega}{(2s+1)\gamma + j\omega}, & \text{если } k > 0. \end{cases}$$

[6.68]
$$V_k^{[3]\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega) =$$
$$= \begin{cases} 8(k+1)^2\cos\varphi_0^{[0]}\cos\varphi_0^{[1]}/3\times \\ \times\exp\big(-j(\varphi_0^{[0]} + \varphi_0^{[1]})\big), & \text{если } k = 0; \\ \dfrac{8(k+1)^2\cos\varphi_k^{[0]}\cos\varphi_k^{[1]}}{(2k+1)(2k+3)}\times \\ \times\exp\Bigg(-j\Bigg(\varphi_k^{[0]} + \varphi_k^{[1]} + \\ +2\displaystyle\sum_{s=0}^{k-1}\varphi_s^{[0]}\Bigg)\Bigg), & \text{если } k > 0, \end{cases}$$
$$\varphi_k^{[0]} = \arctan\frac{\omega}{(2k+1)\gamma};\quad \varphi_k^{[1]} = \arctan\frac{\omega}{(2k+3)\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$$V_0^{\{P_0^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{8\gamma^2}{(\gamma + j\omega)(3\gamma + j\omega)};$$
$$V_1^{\{P_1^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{32\gamma^2}{(3\gamma + j\omega)(5\gamma + j\omega)}\frac{(\gamma - j\omega)}{(\gamma + j\omega)};$$
$$V_2^{\{P_2^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{72\gamma^2}{(5\gamma + j\omega)(7\gamma + j\omega)}\frac{(\gamma - j\omega)}{(\gamma + j\omega)}\frac{(3\gamma - j\omega)}{(3\gamma + j\omega)};$$
$$V_3^{\{P_3^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{128\gamma^2}{(7\gamma + j\omega)(9\gamma + j\omega)}\frac{(\gamma - j\omega)}{(\gamma + j\omega)}\times$$
$$\times\frac{(3\gamma - j\omega)(5\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)};$$
$$V_4^{\{P_4^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{200\gamma^2}{(9\gamma + j\omega)(11\gamma + j\omega)}\frac{(\gamma - j\omega)}{(\gamma + j\omega)}\times$$
$$\times\frac{(3\gamma - j\omega)(5\gamma - j\omega)(7\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)(7\gamma + j\omega)};$$
$$V_5^{\{P_5^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{288\gamma^2}{(11\gamma + j\omega)(13\gamma + j\omega)}\frac{(\gamma - j\omega)}{(\gamma + j\omega)}\times$$
$$\times\frac{(3\gamma - j\omega)(5\gamma - j\omega)(7\gamma - j\omega)(9\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)(7\gamma + j\omega)(9\gamma + j\omega)}.$$

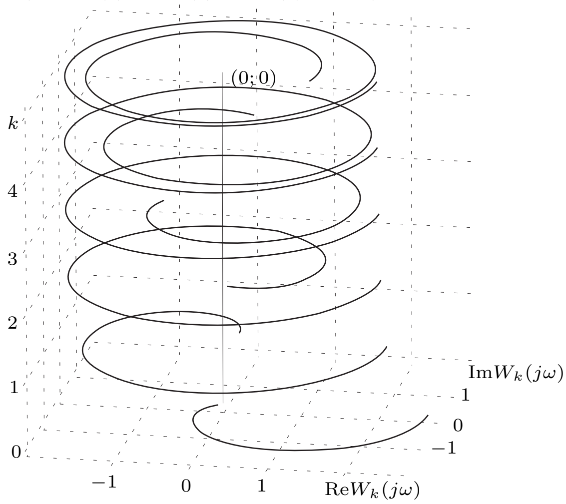


Рис. 6.24. Вид преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta = 1$

[6.69] $V_k^{[1]\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega) =$

$$= \frac{8(2k+3)(k+1)(k+2)\gamma^3}{\big((2k+3)\gamma+j\omega\big)\big((2k+5)\gamma+j\omega\big)} \times \sum_{s=0}^{k} \binom{k}{s}\binom{k+s}{s}(-1)^s \frac{1}{(2s+1)\gamma+j\omega}.$$

[6.70] $V_k^{[2]\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega) =$

$$= \begin{cases} \dfrac{48\gamma^3}{(\gamma+j\omega)(3\gamma+j\omega)(5\gamma+j\omega)}, & \text{если } k=0;\\ \dfrac{8(2k+3)\gamma^3}{\big((2k+1)\gamma+j\omega\big)\big((2k+3)\gamma+j\omega\big)}\times \\ \times\dfrac{(k+1)(k+2)}{\big((2k+5)\gamma+j\omega\big)}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+1)\gamma-j\omega}{(2s+1)\gamma+j\omega}, & \text{если } k>0. \end{cases}$$

[6.71] $V_k^{[3]\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega) =$

$$= \begin{cases} 16\cos\varphi_0^{[0]}\cos\varphi_0^{[1]}\cos\varphi_0^{[2]}/5\times \\ \times\exp\big(-j(\varphi_0^{[0]}+\varphi_0^{[1]}+\varphi_0^{[2]})\big), & \text{если } k=0;\\ \dfrac{8(k+1)(k+2)\cos\varphi_k^{[0]}\cos\varphi_k^{[1]}}{(2k+1)(2k+5)}\times \\ \times\cos\varphi_k^{[2]}\exp\Bigg(-j\bigg(\varphi_k^{[0]}+\varphi_k^{[1]}+ \\ +\varphi_k^{[2]}+2\displaystyle\sum_{s=0}^{k-1}\varphi_s^{[0]}\bigg)\Bigg), & \text{если } k>0, \end{cases}$$

$$\varphi_k^{[0]} = \arctan\frac{\omega}{(2k+1)\gamma};\quad \varphi_k^{[1]} = \arctan\frac{\omega}{(2k+3)\gamma};\quad \varphi_k^{[2]} = \arctan\frac{\omega}{(2k+5)\gamma}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$$V_0^{\{P_0^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{48\gamma^3}{(\gamma+j\omega)(3\gamma+j\omega)(5\gamma+j\omega)};$$

$$V_1^{\{P_1^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{240\gamma^3}{(3\gamma+j\omega)(5\gamma+j\omega)(7\gamma+j\omega)}\frac{(\gamma-j\omega)}{(\gamma+j\omega)};$$

$$V_2^{\{P_2^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{672\gamma^3}{(5\gamma+j\omega)(7\gamma+j\omega)(9\gamma+j\omega)}\frac{(\gamma-j\omega)}{(\gamma+j\omega)}\times \frac{(3\gamma-j\omega)}{(3\gamma+j\omega)};$$

$$V_3^{\{P_3^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{1440\gamma^3}{(7\gamma+j\omega)(9\gamma+j\omega)(11\gamma+j\omega)}\times \frac{(\gamma-j\omega)(3\gamma-j\omega)(5\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)(5\gamma+j\omega)};$$

$$V_4^{\{P_4^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{2640\gamma^3}{(9\gamma+j\omega)(11\gamma+j\omega)(13\gamma+j\omega)}\times \frac{(\gamma-j\omega)(3\gamma-j\omega)(5\gamma-j\omega)(7\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)(5\gamma+j\omega)(7\gamma+j\omega)};$$

$$V_5^{\{P_5^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{4368\gamma^3}{(11\gamma+j\omega)(13\gamma+j\omega)(15\gamma+j\omega)}\times \frac{(\gamma-j\omega)(3\gamma-j\omega)(5\gamma-j\omega)(7\gamma-j\omega)(9\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)(5\gamma+j\omega)(7\gamma+j\omega)(9\gamma+j\omega)}.$$

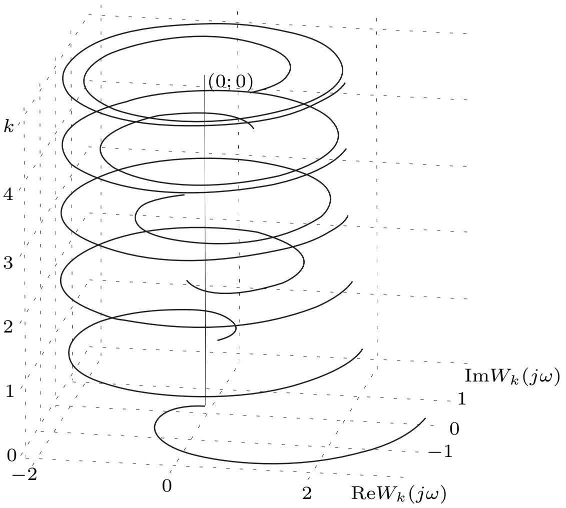


Рис. 6.25. Вид преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma=1$, $c=2$, $\alpha=0$, $\beta=2$

[6.72] $V_k^{[1]\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega) =$

$$= \frac{(c\gamma)^{\beta+1}(2k+\beta+1)(k+\beta)!}{k!\displaystyle\prod_{p=1}^{\beta}\big((2k+2p+1)c\gamma/2+j\omega\big)}\times \sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{(2s+1)c\gamma/2+j\omega}.$$

[6.73] $V_k^{[2]\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega) =$

$$= \begin{cases} \dfrac{(c\gamma)^{\beta+1}(\beta+1)!}{\prod_{p=0}^{\beta}\big((2p+1)c\gamma/2+j\omega\big)}, & \text{если } k=0;\\ \dfrac{(c\gamma)^{\beta+1}(2k+\beta+1)(k+\beta)!}{k!\displaystyle\prod_{p=0}^{\beta}\big((2k+2p+1)c\gamma/2+j\omega\big)}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+1)c\gamma/2-j\omega}{(2s+1)c\gamma/2+j\omega}, & \text{если } k>0. \end{cases}$$

[6.74] $V_k^{[3]\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega) =$

$$= \begin{cases} 2^{\beta+1}(\beta+1)!\prod_{p=0}^{\beta}\dfrac{\cos\varphi_0^{[p]}}{2p+1}\times \\ \times\exp\Big(-j\sum_{p=0}^{\beta}\varphi_0^{[p]}\Big), & \text{если } k=0;\\ 2^{\beta+1}(2k+\beta+1)(k+\beta)!/k!\times \\ \times\displaystyle\prod_{p=0}^{\beta}\frac{\cos\varphi_k^{[p]}}{2k+2p+1}\times \\ \times\exp\Bigg(-j\bigg(\displaystyle\sum_{p=0}^{\beta}\varphi_k^{[p]}+2\sum_{s=0}^{k-1}\varphi_s^{[0]}\bigg)\Bigg), & \text{если } k>0, \end{cases}$$

$$\varphi_k^{[p]} = \arctan\frac{2\omega}{(2k+2p+1)c\gamma};\quad \beta\in\mathbb{Z}.$$

Частные случаи для преобразования Фурье фильтров 0-5 порядков:

$$V_0^{\{P_0^{(0,\beta)}(\tau,\gamma)\}}(j\omega)=\frac{(c\gamma)^{\beta+1}(\beta+1)!}{\prod\limits_{p=0}^{\beta}\left((2p+1)c\gamma/2+j\omega\right)};$$

$$V_1^{\{P_1^{(0,\beta)}(\tau,\gamma)\}}(j\omega)=\frac{(c\gamma)^{\beta+1}(\beta+3)(\beta+1)!}{\prod\limits_{p=0}^{\beta}\left((2p+3)c\gamma/2+j\omega\right)}\,\frac{(c\gamma/2-j\omega)}{(c\gamma/2+j\omega)};$$

$$V_2^{\{P_2^{(0,\beta)}(\tau,\gamma)\}}(j\omega)=\frac{(c\gamma)^{\beta+1}(\beta+5)(\beta+2)!}{2\prod\limits_{p=0}^{\beta}\left((2p+5)c\gamma/2+j\omega\right)}\,\frac{(c\gamma/2-j\omega)}{(c\gamma/2+j\omega)}\times$$
$$\times\frac{(3c\gamma/2-j\omega)}{(3c\gamma/2+j\omega)};$$

$$V_3^{\{P_3^{(0,\beta)}(\tau,\gamma)\}}(j\omega)=\frac{(c\gamma)^{\beta+1}(\beta+7)(\beta+3)!}{6\prod\limits_{p=0}^{\beta}\left((2p+7)c\gamma/2+j\omega\right)}\,\frac{(c\gamma/2-j\omega)}{(c\gamma/2+j\omega)}\times$$
$$\times\frac{(3c\gamma/2-j\omega)(5c\gamma/2-j\omega)}{(3c\gamma/2+j\omega)(5c\gamma/2+j\omega)};$$

$$V_4^{\{P_4^{(0,\beta)}(\tau,\gamma)\}}(j\omega)=\frac{(c\gamma)^{\beta+1}(\beta+9)(\beta+4)!}{24\prod\limits_{p=0}^{\beta}\left((2p+9)c\gamma/2+j\omega\right)}\,\frac{(c\gamma/2-j\omega)}{(c\gamma/2+j\omega)}\times$$
$$\times\frac{(3c\gamma/2-j\omega)(5c\gamma/2-j\omega)(7c\gamma/2-j\omega)}{(3c\gamma/2+j\omega)(5c\gamma/2+j\omega)(7c\gamma/2+j\omega)};$$

$$V_5^{\{P_5^{(0,\beta)}(\tau,\gamma)\}}(j\omega)=\frac{(c\gamma)^{\beta+1}(\beta+11)(\beta+5)!}{120\prod\limits_{p=0}^{\beta}\left((2p+11)c\gamma/2+j\omega\right)}\,\frac{(c\gamma/2-j\omega)}{(c\gamma/2+j\omega)}\times$$
$$\times\frac{(3c\gamma/2-j\omega)(5c\gamma/2-j\omega)(7c\gamma/2-j\omega)(9c\gamma/2-j\omega)}{(3c\gamma/2+j\omega)(5c\gamma/2+j\omega)(7c\gamma/2+j\omega)(9c\gamma/2+j\omega)}.$$

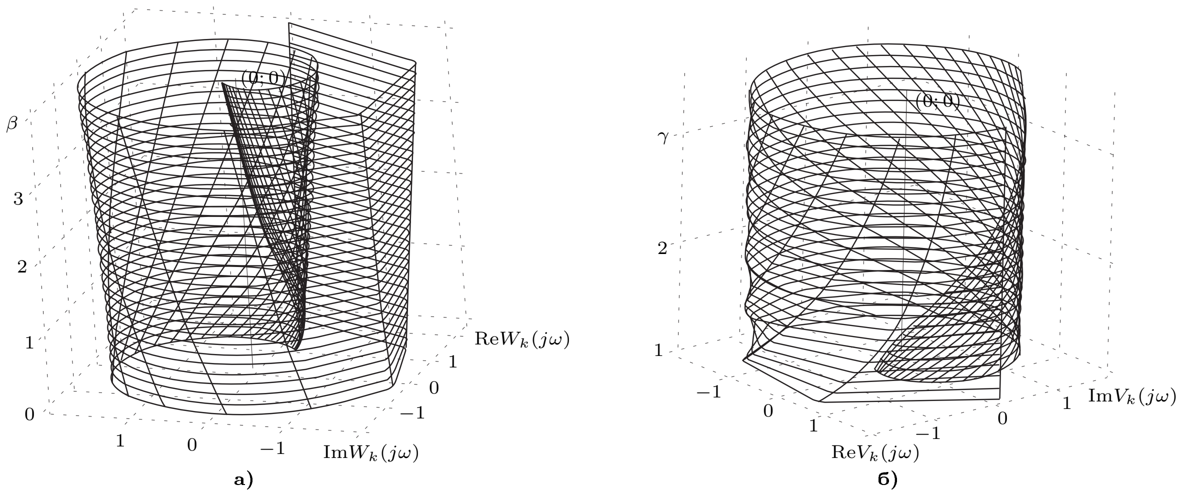


Рис. 6.26. Вид преобразования Фурье ортогональных фильтров Якоби 2-ого порядка: а) $\gamma=1$, $c=2$, $\alpha=0$, $\beta\in[0;5]$; б) $\gamma\in[1;3,5]$, $c=2$, $\alpha=0$, $\beta=1$

6.3 Преобразование Фурье производных ортогональных функций

[6.75] $$W_k^{[1]\left\{\frac{\partial L_k(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega}{j\omega+\gamma/2}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s-1.$$

[6.76] $$W_k^{[2]\left\{\frac{\partial L_k(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega}{j\omega+\gamma/2}\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^k-1.$$

[6.77] $$W_k^{[3]\left\{\frac{\partial L_k(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=(-1)^k j\sin\varphi\times$$
$$\times\exp(-j(2k+1)\varphi)-1,\quad \varphi=\arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$$W_0^{\left\{\frac{\partial L_0(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=-\frac{\gamma/2}{j\omega+\gamma/2};$$

$$W_1^{\left\{\frac{\partial L_1(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega(j\omega-\gamma/2)}{(j\omega+\gamma/2)^2}-1;$$

$$W_2^{\left\{\frac{\partial L_2(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega(j\omega-\gamma/2)^2}{(j\omega+\gamma/2)^3}-1;$$

$$W_3^{\left\{\frac{\partial L_3(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega(j\omega-\gamma/2)^3}{(j\omega+\gamma/2)^4}-1;$$

$$W_4^{\left\{\frac{\partial L_4(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega(j\omega-\gamma/2)^4}{(j\omega+\gamma/2)^5}-1;$$

$$W_5^{\left\{\frac{\partial L_5(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega(j\omega-\gamma/2)^5}{(j\omega+\gamma/2)^6}-1.$$

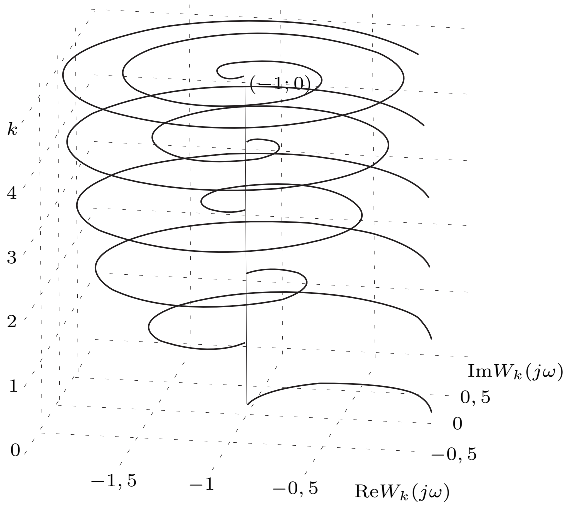


Рис. 6.27. Вид преобразования Фурье производных ортогональных функций Лагерра 0-5 порядков; $\gamma = 1$

[6.78] $$W_k^{[1]\left\{\frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{j\omega+\gamma/2}\sum_{s=0}^{k}\binom{k+1}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^{s} - (k+1).$$

[6.79] $$W_k^{[2]\left\{\frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{\gamma}\left(1-\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^{k+1}\right) - (k+1).$$

[6.80] $$W_k^{[3]\left\{\frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\tan(\varphi)}{2}\times \left(1+(-1)^k\exp(-j(2k+2)\varphi)\right)-(k+1), \quad \varphi = \arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$$W_0^{\left\{\frac{\partial L_0^{(1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = -\frac{\gamma/2}{j\omega+\gamma/2};$$

$$W_1^{\left\{\frac{\partial L_1^{(1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = -\frac{2\omega^2}{(j\omega+\gamma/2)^2} - 2;$$

$$W_2^{\left\{\frac{\partial L_2^{(1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma^2/4-3\omega^2)}{(j\omega+\gamma/2)^3} - 3;$$

$$W_3^{\left\{\frac{\partial L_3^{(1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma^2 j\omega-4j\omega^3)}{(j\omega+\gamma/2)^4} - 4;$$

$$W_4^{\left\{\frac{\partial L_4^{(1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma^4/16-5\gamma^2\omega^2/2+5\omega^4)}{(j\omega+\gamma/2)^5} - 5;$$

$$W_5^{\left\{\frac{\partial L_5^{(1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(3\gamma^4 j\omega/8-5\gamma^2 j\omega^3+6j\omega^4)}{(j\omega+\gamma/2)^6} - 6.$$

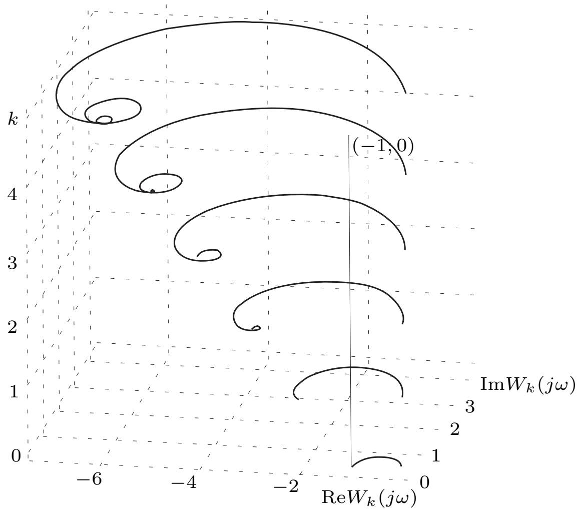


Рис. 6.28. Вид преобразования Фурье производных ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 1$, $\alpha = 1$

[6.81] $$W_k^{[1]\left\{\frac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{j\omega+\gamma/2}\sum_{s=0}^{k}\binom{k+2}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^{s} - \frac{(k+1)(k+2)}{2}.$$

[6.82] $$W_k^{[2]\left\{\frac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{\gamma^2}\times \left[\left(\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^{k+1}-1\right)\left(j\omega-\gamma/2\right)+\gamma(k+1)\right] - \frac{(k+1)(k+2)}{2}.$$

[6.83] $$W_k^{[3]\left\{\frac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\tan(\varphi)}{2}\times \left(\frac{(-1)^k\exp(-j(2k+3)\varphi)-\exp(-j\varphi)}{2\cos\varphi}+k+1\right) - \frac{(k+1)(k+2)}{2}, \quad \varphi = \arctan\frac{2\omega}{\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$$W_0^{\left\{\frac{\partial L_0^{(2)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = -\frac{\gamma/2}{j\omega+\gamma/2};$$

$$W_1^{\left\{\frac{\partial L_1^{(2)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma/2+3j\omega)}{(j\omega+\gamma/2)^2} - 3;$$

$$W_2^{\left\{\frac{\partial L_2^{(2)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma^2/2+2\gamma j\omega-6\omega^2)}{(j\omega+\gamma/2)^3} - 6;$$

$$W_3^{\left\{\frac{\partial L_3^{(2)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma^3/4+5\gamma^2 j\omega/2-5\gamma\omega^2-10j\omega^3)}{(j\omega+\gamma/2)^4} - 10;$$

$$W_4^{\left\{\frac{\partial L_4^{(2)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(j\omega+\gamma/2)^5}\big(3\gamma^4/16+3\gamma^3 j\omega/2 -$$
$$- 15\gamma^2\omega^2/2 - 10\gamma j\omega^3 + 15\omega^4\big) - 15;$$
$$W_5^{\left\{\frac{\partial L_5^{(2)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(j\omega+\gamma/2)^6}\big(3\gamma^5/32+21\gamma^4 j\omega/16 -$$
$$- 21\gamma^3\omega^2/4 - 35\gamma^2 j\omega^3/2 + 35\gamma\omega^4/2 + 21 j\omega^5\big) - 21.$$

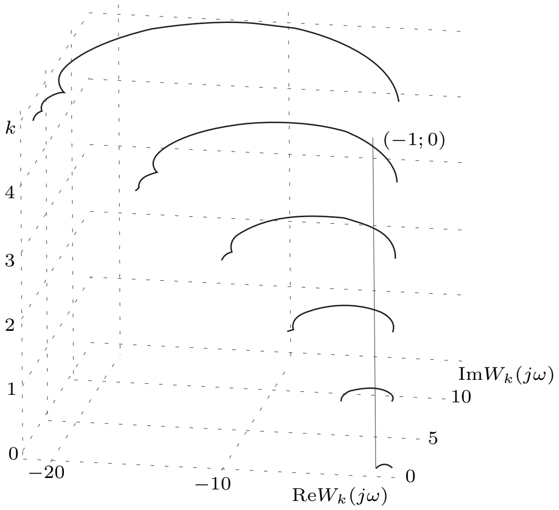


Рис. 6.29. Вид преобразования Фурье производных ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma = 1$, $\alpha = 2$

[6.84] $$W_k^{[1]\left\{\frac{\partial L_k^{(\alpha)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{j\omega+\gamma/2}\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s - \binom{k+\alpha}{k}.$$

[6.85] $$W_k^{[2]\left\{\frac{\partial L_k^{(\alpha)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega\Big(j\omega-\gamma/2\Big)^{\alpha-1}}{(-\gamma)^\alpha}\left[\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^{k+\alpha} - \sum_{p=0}^{\alpha-1}\binom{k+\alpha}{p}\left(-\frac{\gamma}{j\omega+\gamma/2}\right)^p\right] - \binom{k+\alpha}{k}, \qquad \alpha\in\mathbb{Z}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$$W_0^{\left\{\frac{\partial L_0^{(\alpha)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = -\frac{\gamma/2}{j\omega+\gamma/2};$$

$$W_1^{\left\{\frac{\partial L_1^{(\alpha)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega\big(\gamma(\alpha-1)/2+j\omega(\alpha+1)\big)}{(j\omega+\gamma/2)^2} - \alpha - 1;$$

$$W_2^{\left\{\frac{\partial L_2^{(\alpha)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(j\omega+\gamma/2)^3}\big(\gamma^2(\alpha^2+\alpha-2)/8+\gamma j\omega(\alpha^2-$$
$$-\alpha+2)/2 - \omega^2(\alpha^2+3\alpha+2)/2\big) - (\alpha+1)(\alpha+2)/2;$$

$$W_3^{\left\{\frac{\partial L_3^{(\alpha)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(j\omega+\gamma/2)^4}\big(-\gamma^3+\gamma^2(j\omega+\gamma/2)(\alpha+3)-$$
$$-\gamma(j\omega+\gamma/2)^2(\alpha+2)(\alpha+3)/2 - (j\omega+\gamma/2)^3(\alpha+3)!/(6\alpha!)\big) -$$
$$-(\alpha+1)(\alpha+2)(\alpha+3)/6;$$

$$W_4^{\left\{\frac{\partial L_4^{(\alpha)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(j\omega+\gamma/2)^5}\Big(\gamma^4-\gamma^3(j\omega+\gamma/2)(\alpha+4)+$$
$$+\gamma^2(j\omega+\gamma/2)^2(\alpha+3)(\alpha+4)/2 - \gamma(j\omega+\gamma/2)^3(\alpha+4)!/$$
$$/\big(6(\alpha+1)!+(j\omega+\gamma/2)^4(\alpha+4)!/(24\alpha!)\Big) - (\alpha+1)(\alpha+2)(\alpha+3)\times$$
$$\times(\alpha+4)/24;$$

$$W_5^{\left\{\frac{\partial L_5^{(\alpha)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(j\omega+\gamma/2)^6}\Big(-\gamma^5+\gamma^4(j\omega+\gamma/2)(\alpha+5)-$$
$$-\gamma^3(j\omega+\gamma/2)^2(\alpha+4)(\alpha+5)/2+\gamma^2(j\omega+\gamma/2)^3(\alpha+5)!/$$
$$/\big(6(\alpha+2)! - \gamma(j\omega+\gamma/2)^4(\alpha+5)!/\big(24(\alpha+1)!+(j\omega+\gamma/2)^5\times$$
$$\times(\alpha+5)!/(120\alpha!)\Big) - (\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4)(\alpha+5)/120.$$

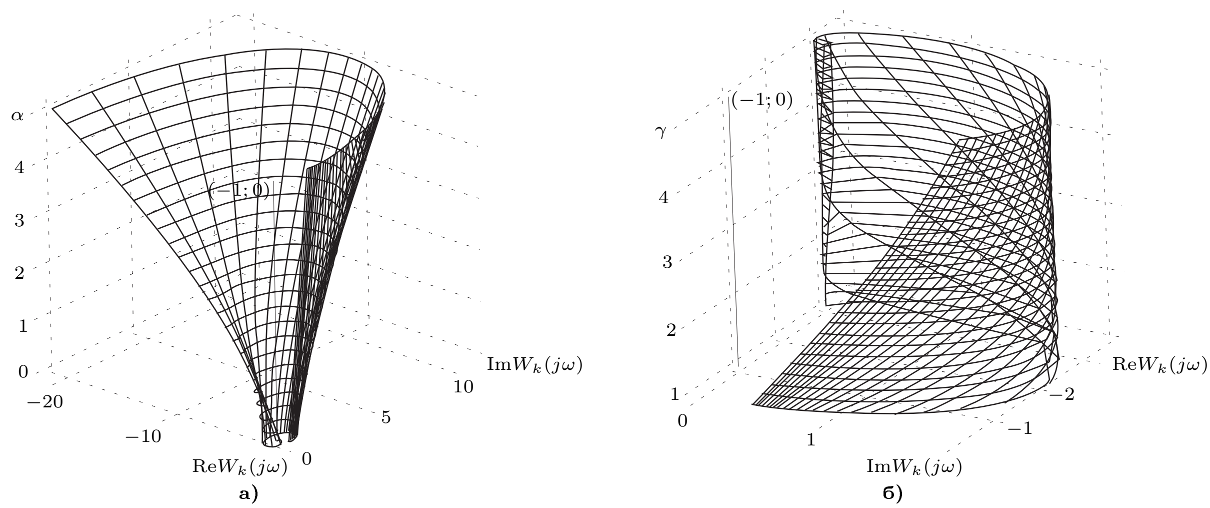


Рис. 6.30. Вид преобразования Фурье производных ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 1$, $\alpha \in [0;5]$; б) $\gamma \in [1;5]$, $\alpha = 1$

[6.86] $W_k^{[1]\left\{\frac{\partial P_k^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = j\omega\sum_{s=0}^{k}\binom{k}{s}\times$

$$\times\binom{k+s-1/2}{s-1/2}(-1)^s\frac{1}{(4s+1)\gamma/2+j\omega} - (-1)^k.$$

[6.87] $W_k^{[2]\left\{\frac{\partial P_k^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$=\begin{cases}\dfrac{j\omega}{\gamma/2+j\omega}-1, & \text{если } k=0;\\ \dfrac{j\omega}{(4k+1)\gamma/2+j\omega}\times\\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(4s+1)\gamma/2-j\omega}{(4s+1)\gamma/2+j\omega}-(-1)^k, & \text{если } k>0.\end{cases}$$

[6.88] $W_k^{[3]\left\{\frac{\partial P_k^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$=\begin{cases} j\sin\varphi_0\exp\bigl(-j\varphi_0\bigr)-1, & \text{если } k=0;\\ j\sin\varphi_k\exp\Biggl(-j\biggl(\varphi_k+\\ +2\displaystyle\sum_{s=0}^{k-1}\varphi_s\biggr)\Biggr)-(-1)^k, & \text{если } k>0,\end{cases}$$

$$\varphi_k = \arctan\frac{2\omega}{(4k+1)\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$$W_0^{\left\{\frac{\partial P_0^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = -\frac{\gamma/2}{\gamma/2+j\omega};$$

$$W_1^{\left\{\frac{\partial P_1^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma/2-j\omega)}{(\gamma/2+j\omega)(5\gamma/2+j\omega)}+1;$$

$$W_2^{\left\{\frac{\partial P_2^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma/2-j\omega)(5\gamma/2-j\omega)}{(\gamma/2+j\omega)(5\gamma/2+j\omega)(9\gamma/2+j\omega)}-$$

$$-\,1;$$

$$W_3^{\left\{\frac{\partial P_3^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{(13\gamma/2+j\omega)}\frac{(\gamma/2-j\omega)}{(\gamma/2+j\omega)}\times$$

$$\times\frac{(5\gamma/2-j\omega)(9\gamma/2-j\omega)}{(5\gamma/2+j\omega)(9\gamma/2+j\omega)}+1;$$

$$W_4^{\left\{\frac{\partial P_4^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{(17\gamma/2+j\omega)}\frac{(\gamma/2-j\omega)}{(\gamma/2+j\omega)}\times$$

$$\times\frac{(5\gamma/2-j\omega)(9\gamma/2-j\omega)}{(5\gamma/2+j\omega)(9\gamma/2+j\omega)}\frac{(13\gamma/2-j\omega)}{(13\gamma/2+j\omega)}-1;$$

$$W_5^{\left\{\frac{\partial P_5^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{(21\gamma/2+j\omega)}\frac{(\gamma/2-j\omega)}{(\gamma/2+j\omega)}\times$$

$$\times\frac{(5\gamma/2-j\omega)(9\gamma/2-j\omega)}{(5\gamma/2+j\omega)(9\gamma/2+j\omega)}\frac{(13\gamma/2-j\omega)(17\gamma/2-j\omega)}{(13\gamma/2+j\omega)(17\gamma/2+j\omega)}+1.$$

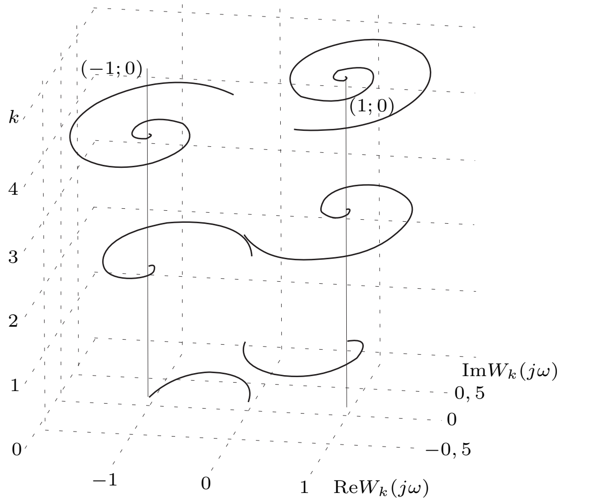


Рис. 6.31. Вид преобразования Фурье производных ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = -1/2$, $\beta = 0$

[6.89] $W_k^{[1]\left\{\frac{\partial Leg_k(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$= j\omega\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{(2s+1)\gamma+j\omega}-(-1)^k.$$

[6.90] $W_k^{[2]\left\{\frac{\partial Leg_k(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$=\begin{cases}\dfrac{j\omega}{\gamma+j\omega}-1, & \text{если } k=0;\\ \dfrac{j\omega}{(2k+1)\gamma+j\omega}\times\\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+1)\gamma-j\omega}{(2s+1)\gamma+j\omega}-(-1)^k, & \text{если } k>0.\end{cases}$$

[6.91] $W_k^{[3]\left\{\frac{\partial Leg_k(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$=\begin{cases} j\sin\varphi_0\exp\bigl(-j\varphi_0\bigr)-1, & \text{если } k=0;\\ j\sin\varphi_k\exp\Biggl(-j\biggl(\varphi_k+\\ +2\displaystyle\sum_{s=0}^{k-1}\varphi_s\biggr)\Biggr)-(-1)^k, & \text{если } k>0,\end{cases}$$

$$\varphi_k = \arctan\frac{\omega}{(2k+1)\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$$W_0^{\left\{\frac{\partial Leg_0(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = -\frac{\gamma}{\gamma+j\omega};$$

$$W_1^{\left\{\frac{\partial Leg_1(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)}+1;$$

$$W_2^{\left\{\frac{\partial Leg_2(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma-j\omega)(3\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)(5\gamma+j\omega)}-1;$$

$$W_3^{\left\{\frac{\partial Leg_3(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega(\gamma-j\omega)(3\gamma-j\omega)(5\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)(5\gamma+j\omega)(7\gamma+j\omega)}+1;$$

$$W_4^{\left\{\frac{\partial Leg_4(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(9\gamma+j\omega)}\frac{(\gamma-j\omega)(3\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)} \times$$
$$\times \frac{(5\gamma-j\omega)(7\gamma-j\omega)}{(5\gamma+j\omega)(7\gamma+j\omega)} - 1;$$
$$W_5^{\left\{\frac{\partial Leg_5(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(11\gamma+j\omega)}\frac{(\gamma-j\omega)(3\gamma-j\omega)}{(\gamma+j\omega)(3\gamma+j\omega)} \times$$
$$\times \frac{(5\gamma-j\omega)(7\gamma-j\omega)(9\gamma-j\omega)}{(5\gamma+j\omega)(7\gamma+j\omega)(9\gamma+j\omega)} + 1.$$

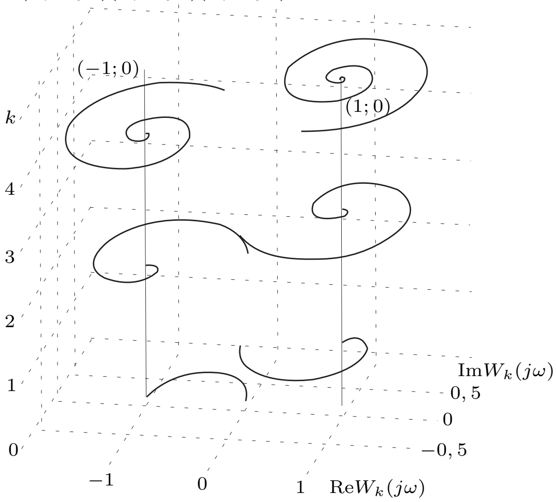


Рис. 6.32. Вид преобразования Фурье производных ортогональных функций Лежандра 0-5 порядков; $\gamma = 0,25$, $c = 2$

[6.92] $$W_k^{[1]\left\{\frac{\partial P_k^{(1/2,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = j\omega \sum_{s=0}^{k} \binom{k}{s} \times$$
$$\times \binom{k+s+1/2}{s+1/2}(-1)^s \frac{1}{(4s+3)\gamma/2+j\omega} - (-1)^k.$$

[6.93] $$W_k^{[2]\left\{\frac{\partial P_k^{(1/2,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) =$$
$$= \begin{cases} \dfrac{j\omega}{3\gamma/2+j\omega} - 1, & \text{если } k = 0; \\ \dfrac{j\omega}{(4k+3)\gamma/2+j\omega} \times \\ \times \displaystyle\prod_{s=0}^{k-1} \frac{(4s+3)\gamma/2-j\omega}{(4s+3)\gamma/2+j\omega} - (-1)^k, & \text{если } k > 0. \end{cases}$$

[6.94] $$W_k^{[3]\left\{\frac{\partial P_k^{(1/2,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) =$$
$$= \begin{cases} j\sin\varphi_0 \exp\bigl(-j\varphi_0\bigr) - 1, & \text{если } k = 0; \\ j\sin\varphi_k \exp\Biggl(-j\biggl(\varphi_k + \\ +2\displaystyle\sum_{s=0}^{k-1}\varphi_s\biggr)\Biggr) - (-1)^k, & \text{если } k > 0, \end{cases}$$
$$\varphi_k = \arctan\frac{2\omega}{(4k+3)\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$$W_0^{\left\{\frac{\partial P_0^{(1/2,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = -\frac{3\gamma/2}{3\gamma/2+j\omega};$$
$$W_1^{\left\{\frac{\partial P_1^{(1/2,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega(3\gamma/2-j\omega)}{(3\gamma/2+j\omega)(7\gamma/2+j\omega)} + 1;$$
$$W_2^{\left\{\frac{\partial P_2^{(1/2,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega(3\gamma/2-j\omega)(7\gamma/2-j\omega)}{(3\gamma/2+j\omega)(7\gamma/2+j\omega)} \times$$
$$\times \frac{1}{(11\gamma/2+j\omega)} - 1;$$
$$W_3^{\left\{\frac{\partial P_3^{(1/2,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(15\gamma/2+j\omega)}\frac{(3\gamma/2-j\omega)}{(3\gamma/2+j\omega)} \times$$
$$\times \frac{(7\gamma/2-j\omega)(11\gamma/2-j\omega)}{(7\gamma/2+j\omega)(11\gamma/2+j\omega)} + 1;$$
$$W_4^{\left\{\frac{\partial P_4^{(1/2,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(19\gamma/2+j\omega)}\frac{(3\gamma/2-j\omega)}{(3\gamma/2+j\omega)} \times$$
$$\times \frac{(7\gamma/2-j\omega)(11\gamma/2-j\omega)}{(7\gamma/2+j\omega)(11\gamma/2+j\omega)}\frac{(15\gamma/2-j\omega)}{(15\gamma/2+j\omega)} - 1;$$
$$W_5^{\left\{\frac{\partial P_5^{(1/2,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \frac{j\omega}{(23\gamma/2+j\omega)}\frac{(3\gamma/2-j\omega)}{(3\gamma/2+j\omega)} \times$$
$$\times \frac{(7\gamma/2-j\omega)(11\gamma/2-j\omega)}{(7\gamma/2+j\omega)(11\gamma/2+j\omega)}\frac{(15\gamma/2-j\omega)(19\gamma/2-j\omega)}{(15\gamma/2+j\omega)(19\gamma/2+j\omega)} + 1.$$

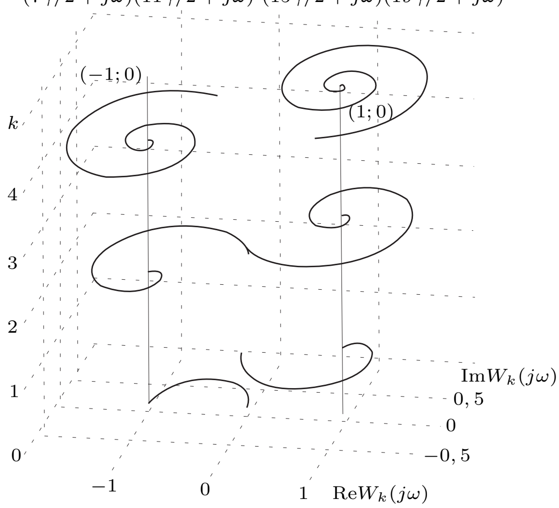


Рис. 6.33. Вид преобразования Фурье производных ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[6.95] $$W_k^{[1]\left\{\frac{\partial P_k^{(1,0)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) =$$
$$= j\omega \sum_{s=0}^{k} \binom{k}{s}\binom{k+s+1}{s+1}(-1)^s \frac{1}{(s+1)\gamma+j\omega} - (-1)^k.$$

[6.96] $W_k^{[2]\left\{\frac{\partial P_k^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$= \begin{cases} \dfrac{j\omega}{\gamma + j\omega} - 1, & \text{если } k = 0; \\ \dfrac{j\omega}{(k+1)\gamma + j\omega}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\dfrac{(s+1)\gamma - j\omega}{(s+1)\gamma + j\omega} - (-1)^k, & \text{если } k > 0. \end{cases}$$

[6.97] $W_k^{[3]\left\{\frac{\partial P_k^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$= \begin{cases} j\sin\varphi_0\exp\big(-j\varphi_0\big) - 1, & \text{если } k = 0; \\ j\sin\varphi_k\exp\Bigg(-j\Bigg(\varphi_k + \\ +2\displaystyle\sum_{s=0}^{k-1}\varphi_s\Bigg)\Bigg) - (-1)^k, & \text{если } k > 0, \end{cases}$$

$$\varphi_k = \arctan\frac{\omega}{(k+1)\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$W_0^{\left\{\frac{\partial P_0^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = -\dfrac{\gamma}{\gamma + j\omega};$

$W_1^{\left\{\frac{\partial P_1^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \dfrac{j\omega(\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)} + 1;$

$W_2^{\left\{\frac{\partial P_2^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \dfrac{j\omega(\gamma - j\omega)(2\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)(3\gamma + j\omega)} - 1;$

$W_3^{\left\{\frac{\partial P_3^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \dfrac{j\omega(\gamma - j\omega)(2\gamma - j\omega)(3\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)(3\gamma + j\omega)(4\gamma + j\omega)} +$
$+ 1;$

$W_4^{\left\{\frac{\partial P_4^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \dfrac{j\omega}{(5\gamma + j\omega)}\,\dfrac{(\gamma - j\omega)(2\gamma - j\omega)(3\gamma - j\omega)}{(\gamma + j\omega)(2\gamma + j\omega)(3\gamma + j\omega)}\times$
$\times\,\dfrac{(4\gamma - j\omega)}{(4\gamma + j\omega)} - 1;$

$W_5^{\left\{\frac{\partial P_5^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \dfrac{j\omega}{(6\gamma + j\omega)}\,\dfrac{(\gamma - j\omega)}{(\gamma + j\omega)}\,\dfrac{(2\gamma - j\omega)(3\gamma - j\omega)}{(2\gamma + j\omega)(3\gamma + j\omega)}\times$
$\times\,\dfrac{(4\gamma - j\omega)(5\gamma - j\omega)}{(4\gamma + j\omega)(5\gamma + j\omega)} + 1.$

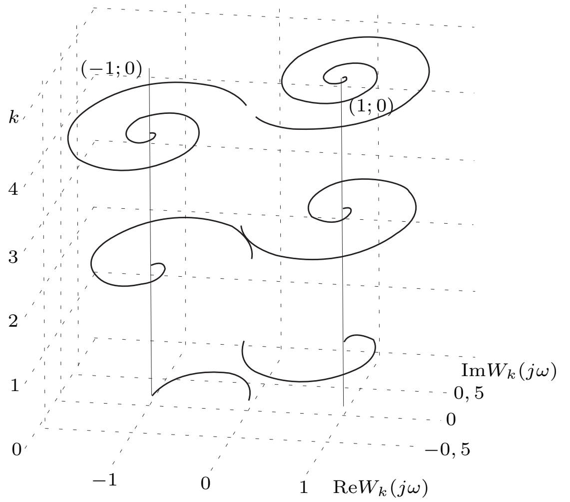


Рис. 6.34. Вид преобразования Фурье производных ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 1$, $\alpha = 1$, $\beta = 0$

[6.98] $W_k^{[1]\left\{\frac{\partial P_k^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$= j\omega\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s\frac{1}{(2s+3)\gamma + j\omega} - (-1)^k.$$

[6.99] $W_k^{[2]\left\{\frac{\partial P_k^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$= \begin{cases} \dfrac{j\omega}{3\gamma + j\omega} - 1, & \text{если } k = 0; \\ \dfrac{j\omega}{(2k+3)\gamma + j\omega}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\dfrac{(2s+3)\gamma - j\omega}{(2s+3)\gamma + j\omega} - (-1)^k, & \text{если } k > 0. \end{cases}$$

[6.100] $W_k^{[3]\left\{\frac{\partial P_k^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) =$

$$= \begin{cases} j\sin\varphi_0\exp\big(-j\varphi_0\big) - 1, & \text{если } k = 0; \\ j\sin\varphi_k\times \\ \exp\Bigg(-j\Bigg(\varphi_k + \\ +2\displaystyle\sum_{s=0}^{k-1}\varphi_s\Bigg)\Bigg) - (-1)^k, & \text{если } k > 0, \end{cases}$$

$$\varphi_k = \arctan\frac{\omega}{(2k+3)\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$W_0^{\left\{\frac{\partial P_0^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = -\dfrac{3\gamma}{3\gamma + j\omega};$

$W_1^{\left\{\frac{\partial P_1^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \dfrac{j\omega(3\gamma - j\omega)}{(3\gamma + j\omega)(5\gamma + j\omega)} + 1;$

$$W_2^{\left\{\frac{\partial P_2^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega(3\gamma-j\omega)(5\gamma-j\omega)}{(3\gamma+j\omega)(5\gamma+j\omega)(7\gamma+j\omega)}-1;$$

$$W_3^{\left\{\frac{\partial P_3^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega}{(9\gamma+j\omega)}\,\frac{(3\gamma-j\omega)(5\gamma-j\omega)}{(3\gamma+j\omega)(5\gamma+j\omega)}\,\frac{(7\gamma-j\omega)}{(7\gamma+j\omega)}+$$

$$+1;$$

$$W_4^{\left\{\frac{\partial P_4^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega}{(11\gamma+j\omega)}\,\frac{(3\gamma-j\omega)(5\gamma-j\omega)}{(3\gamma+j\omega)(5\gamma+j\omega)}\times$$

$$\times\frac{(7\gamma-j\omega)(9\gamma-j\omega)}{(7\gamma+j\omega)(9\gamma+j\omega)}-1;$$

$$W_5^{\left\{\frac{\partial P_5^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega}{(13\gamma+j\omega)}\,\frac{(3\gamma-j\omega)(5\gamma-j\omega)}{(3\gamma+j\omega)(5\gamma+j\omega)}\times$$

$$\times\frac{(7\gamma-j\omega)(9\gamma-j\omega)(11\gamma-j\omega)}{(7\gamma+j\omega)(9\gamma+j\omega)(11\gamma+j\omega)}+1.$$

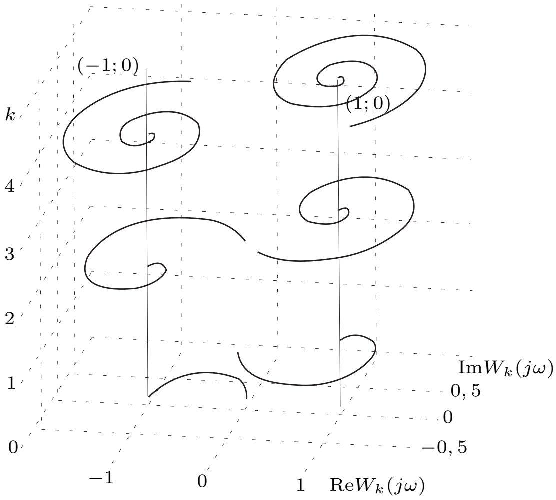


Рис. 6.35. Вид преобразования Фурье производных ортогональных функций Якоби 0-5 порядков; $\gamma=0,25$, $c=2$, $\alpha=2$, $\beta=0$

[6.101]
$$W_k^{[1]\left\{\frac{\partial P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=j\omega\sum_{s=0}^{k}\binom{k}{s}\times$$
$$\times\binom{k+s+\alpha}{s+\alpha}(-1)^s\frac{1}{(2s+\alpha+1)c\gamma/2+j\omega}-(-1)^k.$$

[6.102]
$$W_k^{[2]\left\{\frac{\partial P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=$$
$$=\begin{cases}\dfrac{j\omega}{(\alpha+1)c\gamma/2+j\omega}-1, & \text{если } k=0;\\ \dfrac{j\omega}{(2k+\alpha+1)c\gamma/2+j\omega}\times & \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+\alpha+1)c\gamma/2-j\omega}{(2s+\alpha+1)c\gamma/2+j\omega}-(-1)^k, & \text{если } k>0.\end{cases}$$

[6.103]
$$W_k^{[3]\left\{\frac{\partial P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=$$
$$=\begin{cases}j\sin\varphi_0\exp\bigl(-j\varphi_0\bigr)-1, & \text{если } k=0;\\ j\sin\varphi_k\times & \\ \times\exp\Biggl(-j\Biggl(\varphi_k+ & \\ +2\displaystyle\sum_{s=0}^{k-1}\varphi_s\Biggr)\Biggr)-(-1)^k, & \text{если } k>0,\end{cases}$$
$$\varphi_k=\arctan\frac{2\omega}{(2k+\alpha+1)c\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$$W_0^{\left\{\frac{\partial P_0^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=-\frac{(\alpha+1)c\gamma/2}{(\alpha+1)c\gamma/2+j\omega};$$

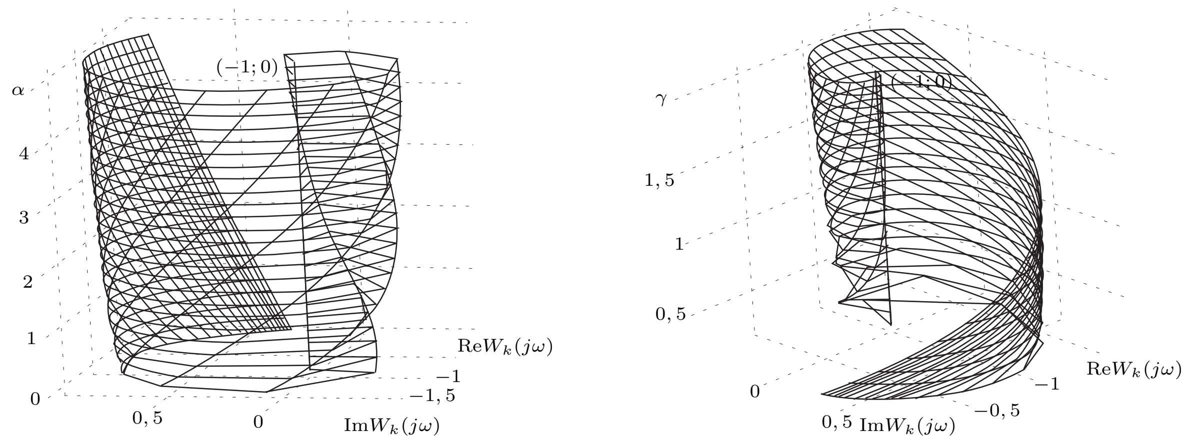
$$W_1^{\left\{\frac{\partial P_1^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega\bigl((\alpha+1)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2+j\omega\bigr)}\times$$
$$\times\frac{1}{\bigl((\alpha+3)c\gamma/2+j\omega\bigr)}+1;$$

$$W_2^{\left\{\frac{\partial P_2^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega}{\bigl((\alpha+5)c\gamma/2+j\omega\bigr)}\times$$
$$\times\frac{\bigl((\alpha+1)c\gamma/2-j\omega\bigr)\bigl((\alpha+3)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2+j\omega\bigr)\bigl((\alpha+3)c\gamma/2+j\omega\bigr)}-1;$$

$$W_3^{\left\{\frac{\partial P_3^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega}{\bigl((\alpha+7)c\gamma/2+j\omega\bigr)}\times$$
$$\times\frac{\bigl((\alpha+1)c\gamma/2-j\omega\bigr)\bigl((\alpha+3)c\gamma/2-j\omega\bigr)\bigl((\alpha+5)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2+j\omega\bigr)\bigl((\alpha+3)c\gamma/2+j\omega\bigr)\bigl((\alpha+5)c\gamma/2+j\omega\bigr)}+1;$$

$$W_4^{\left\{\frac{\partial P_4^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega}{\bigl((\alpha+9)c\gamma/2+j\omega\bigr)}\times$$
$$\times\frac{\bigl((\alpha+1)c\gamma/2-j\omega\bigr)\bigl((\alpha+3)c\gamma/2-j\omega\bigr)\bigl((\alpha+5)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2+j\omega\bigr)\bigl((\alpha+3)c\gamma/2+j\omega\bigr)\bigl((\alpha+5)c\gamma/2+j\omega\bigr)}\times$$
$$\times\frac{\bigl((\alpha+7)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+7)c\gamma/2+j\omega\bigr)}-1;$$

$$W_5^{\left\{\frac{\partial P_5^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)=\frac{j\omega}{\bigl((\alpha+11)c\gamma/2+j\omega\bigr)}\times$$
$$\times\frac{\bigl((\alpha+1)c\gamma/2-j\omega\bigr)\bigl((\alpha+3)c\gamma/2-j\omega\bigr)\bigl((\alpha+5)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+1)c\gamma/2+j\omega\bigr)\bigl((\alpha+3)c\gamma/2+j\omega\bigr)\bigl((\alpha+5)c\gamma/2+j\omega\bigr)}\times$$
$$\times\frac{\bigl((\alpha+7)c\gamma/2-j\omega\bigr)\bigl((\alpha+9)c\gamma/2-j\omega\bigr)}{\bigl((\alpha+7)c\gamma/2+j\omega\bigr)\bigl((\alpha+9)c\gamma/2+j\omega\bigr)}+1.$$



а) б)

Рис. 6.36. Вид преобразования Фурье производных ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,25$, $c = 2$, $\alpha \in [0; 5]$, $\beta = 0$; б) $\gamma \in [0,25; 2]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[6.104] $$W_k^{[1]\left\{\frac{\partial P_k^{(0,1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = j\omega\sum_{s=0}^{k}\binom{k}{s}\times$$
$$\times\binom{k+s+1}{s}(-1)^s\frac{1}{(2s+1)\gamma+j\omega} - (-1)^k(k+1).$$

[6.105] $$W_k^{[2]\left\{\frac{\partial P_k^{(0,1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}\times$$
$$\times(-1)^s j\sin\varphi_s\exp\bigl(-j\varphi_s\bigr) - (-1)^k(k+1),$$
$$\varphi_k = \arctan\frac{\omega}{(2k+1)\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$$W_0^{\left\{\frac{\partial P_0^{(0,1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = -\frac{\gamma}{\gamma+j\omega};$$

$$W_1^{\left\{\frac{\partial P_1^{(0,1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{\gamma+j\omega} - \frac{3j\omega}{3\gamma+j\omega} + 2;$$

$$W_2^{\left\{\frac{\partial P_2^{(0,1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{\gamma+j\omega} - \frac{8j\omega}{3\gamma+j\omega} + \frac{10j\omega}{5\gamma+j\omega} - 3;$$

$$W_3^{\left\{\frac{\partial P_3^{(0,1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{\gamma+j\omega} - \frac{15j\omega}{3\gamma+j\omega} + \frac{45j\omega}{5\gamma+j\omega} - \frac{35j\omega}{7\gamma+j\omega} +$$
$$+ 4;$$

$$W_4^{\left\{\frac{\partial P_4^{(0,1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{\gamma+j\omega} - \frac{24j\omega}{3\gamma+j\omega} + \frac{126j\omega}{5\gamma+j\omega} -$$
$$- \frac{224j\omega}{7\gamma+j\omega} + \frac{126j\omega}{9\gamma+j\omega} - 5;$$

$$W_5^{\left\{\frac{\partial P_5^{(0,1)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \frac{j\omega}{\gamma+j\omega} - \frac{35j\omega}{3\gamma+j\omega} + \frac{280j\omega}{5\gamma+j\omega} -$$
$$- \frac{840j\omega}{7\gamma+j\omega} + \frac{1050j\omega}{9\gamma+j\omega} - \frac{462j\omega}{11\gamma+j\omega} + 6.$$

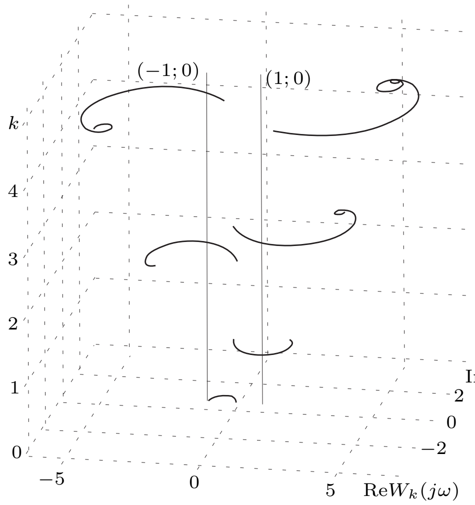


Рис. 6.37. Вид преобразования Фурье производных ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 1$

[6.106] $$W_k^{[1]\left\{\frac{\partial P_k^{(0,2)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = j\omega\sum_{s=0}^{k}\binom{k}{s}\times$$
$$\times\binom{k+s+2}{s}(-1)^s\frac{1}{(2s+1)\gamma+j\omega} -$$
$$-(-1)^k\frac{(k+1)(k+2)}{2}.$$

[6.107] $$W_k^{[2]\left\{\frac{\partial P_k^{(0,2)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega) = \sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}\times$$
$$\times(-1)^s j\sin\varphi_s\exp\bigl(-j\varphi_s\bigr) - (-1)^k\frac{(k+1)(k+2)}{2},$$
$$\varphi_k = \arctan\frac{\omega}{(2k+1)\gamma}.$$

Частные случаи для преобразования Фурье производных функций 0-5 порядков:

$W_0^{\{P_0^{(0,2)}(\tau,\gamma)\}}(j\omega) = -\dfrac{\gamma}{\gamma + j\omega};$

$W_1^{\{P_1^{(0,2)}(\tau,\gamma)\}}(j\omega) = \dfrac{j\omega}{\gamma + j\omega} - \dfrac{4j\omega}{3\gamma + j\omega} + 3;$

$W_2^{\{P_2^{(0,2)}(\tau,\gamma)\}}(j\omega) = \dfrac{j\omega}{\gamma + j\omega} - \dfrac{10j\omega}{3\gamma + j\omega} + \dfrac{15j\omega}{5\gamma + j\omega} - 6;$

$W_3^{\{P_3^{(0,2)}(\tau,\gamma)\}}(j\omega) = \dfrac{j\omega}{\gamma + j\omega} - \dfrac{18j\omega}{3\gamma + j\omega} + \dfrac{63j\omega}{5\gamma + j\omega} - \dfrac{56j\omega}{7\gamma + j\omega} +$
$+ 10;$

$W_4^{\{P_4^{(0,2)}(\tau,\gamma)\}}(j\omega) = \dfrac{j\omega}{\gamma + j\omega} - \dfrac{28j\omega}{3\gamma + j\omega} + \dfrac{168j\omega}{5\gamma + j\omega} -$
$- \dfrac{336j\omega}{7\gamma + j\omega} + \dfrac{210j\omega}{9\gamma + j\omega} - 15;$

$W_5^{\{P_5^{(0,2)}(\tau,\gamma)\}}(j\omega) = \dfrac{j\omega}{\gamma + j\omega} - \dfrac{40j\omega}{3\gamma + j\omega} + \dfrac{360j\omega}{5\gamma + j\omega} -$
$- \dfrac{1200j\omega}{7\gamma + j\omega} + \dfrac{1650j\omega}{9\gamma + j\omega} - \dfrac{792j\omega}{11\gamma + j\omega} + 21.$

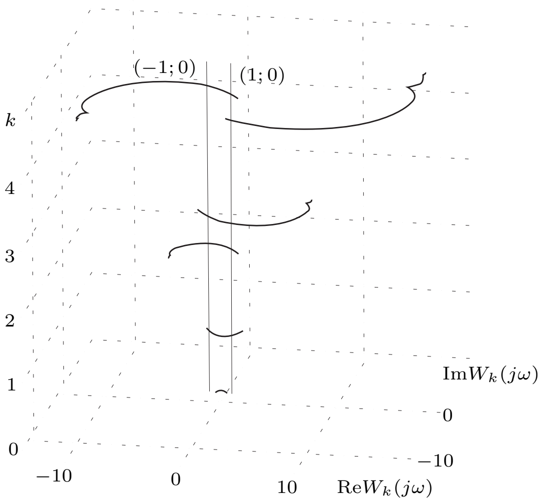


Рис. 6.38. Вид преобразования Фурье производных ортогональных функций Якоби 0-5 порядков; $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta = 2$

[6.108] $$W_k^{[1]\left\{\frac{\partial P_k^{(0,\beta)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = j\omega \sum_{s=0}^{k} \binom{k}{s} \times$$
$$\times \binom{k+s+\beta}{s} (-1)^s \frac{1}{(2s+1)c\gamma/2 + j\omega} - (-1)^k \binom{k+\beta}{k}.$$

[6.109] $$W_k^{[2]\left\{\frac{\partial P_k^{(0,\beta)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \sum_{s=0}^{k} \binom{k}{s} \binom{k+s+\beta}{s} \times$$
$$\times (-1)^s j \sin\varphi_s \exp\bigl(-j\varphi_s\bigr) - (-1)^k \binom{k+\beta}{k},$$
$$\varphi_k = \arctan \frac{2\omega}{(2k+1)c\gamma}.$$

Частные случаи для преобразования Фурье функций 0-5 порядков:

$W_0^{\left\{\frac{\partial P_0^{(0,\beta)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = -\dfrac{c\gamma/2}{c\gamma/2 + j\omega};$

$W_1^{\left\{\frac{\partial P_1^{(0,\beta)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \dfrac{j\omega}{c\gamma/2 + j\omega} - \dfrac{j\omega(\beta+2)}{3c\gamma/2 + j\omega} + \beta + 1;$

$W_2^{\left\{\frac{\partial P_2^{(0,\beta)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \dfrac{j\omega}{c\gamma/2 + j\omega} - \dfrac{2j\omega(\beta+3)}{3c\gamma/2 + j\omega} +$
$+ \dfrac{j\omega(\beta+3)(\beta+4)/2}{5c\gamma/2 + j\omega} - (\beta+1)(\beta+2)/2;$

$W_3^{\left\{\frac{\partial P_3^{(0,\beta)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \dfrac{j\omega}{c\gamma/2 + j\omega} - \dfrac{3j\omega(\beta+4)}{3c\gamma/2 + j\omega} +$
$+ \dfrac{3j\omega(\beta+4)(\beta+5)/2}{5c\gamma/2 + j\omega} - \dfrac{j\omega(\beta+4)(\beta+5)(\beta+6)/6}{7c\gamma/2 + j\omega} + (\beta+1) \times$
$\times (\beta+2)(\beta+3)/6;$

$W_4^{\left\{\frac{\partial P_4^{(0,\beta)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \dfrac{j\omega}{c\gamma/2 + j\omega} - \dfrac{4j\omega(\beta+5)}{3c\gamma/2 + j\omega} +$
$+ \dfrac{3j\omega(\beta+5)(\beta+6)}{5c\gamma/2 + j\omega} - \dfrac{2j\omega(\beta+5)(\beta+6)(\beta+7)/3}{7c\gamma/2 + j\omega} +$
$+ \dfrac{j\omega(\beta+8)!}{24(\beta+4)!(9c\gamma/2 + j\omega)} - (\beta+1)(\beta+2)(\beta+3)(\beta+4)/24;$

$W_5^{\left\{\frac{\partial P_5^{(0,\beta)}(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) = \dfrac{j\omega}{c\gamma/2 + j\omega} - \dfrac{5j\omega(\beta+6)}{3c\gamma/2 + j\omega} +$
$+ \dfrac{5j\omega(\beta+6)(\beta+7)}{5c\gamma/2 + j\omega} - \dfrac{5j\omega(\beta+6)(\beta+7)(\beta+8)/3}{7c\gamma/2 + j\omega} +$
$+ \dfrac{5j\omega(\beta+9)!}{24(\beta+5)!(9c\gamma/2 + j\omega)} - \dfrac{j\omega(\beta+10)!}{120(\beta+5)!(11c\gamma/2 + j\omega)} + (\beta+1) \times$
$\times (\beta+2)(\beta+3)(\beta+4)(\beta+5)/120.$

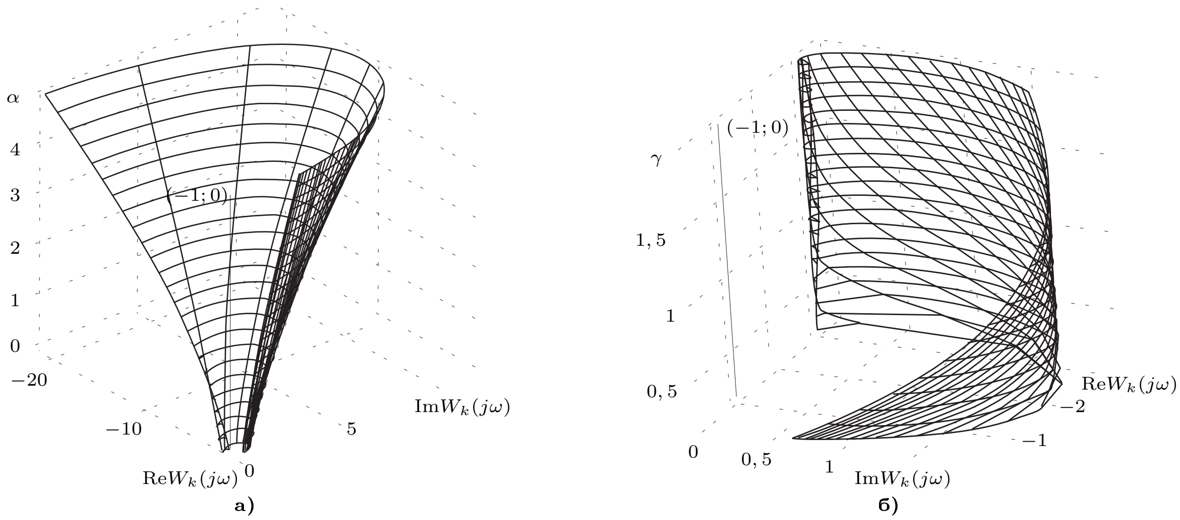


Рис. 6.39. Вид преобразования Фурье ортогональных функций Якоби 2-ого порядка: а) $\gamma = 0,25$, $c = 2$, $\alpha = 0$, $\beta \in [0;5]$; б) $\gamma \in [0,25;2]$, $c = 2$, $\alpha = 0$, $\beta = 1$

6.4 Производные преобразований Фурье ортогональных функций

[6.110]
$$\frac{\partial W_k^{[1]\{L_k(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(j\omega+\gamma/2)^2}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s (s+1).$$

[6.111]
$$\frac{\partial W_k^{[2]\{L_k(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\big(j\omega-\gamma(2k+1)/2\big)}{(j\omega+\gamma/2)^3}\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^{k-1}.$$

[6.112]
$$\frac{\partial W_k^{[3]\{L_k(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{4j}{\gamma^2}(-1)^k(\cos\varphi)^2\times \exp(-j(2k+1)\varphi)\big((2k+1)\cos\varphi - j\sin\varphi\big),$$
$$\varphi = \arctan\frac{2\omega}{\gamma}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{L_0(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\omega + j\gamma/2}{(j\omega-\gamma/2)(j\omega+\gamma/2)^2};$$

$$\frac{\partial W_1^{\{L_1(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\omega + 3j\gamma/2}{(j\omega+\gamma/2)^3};$$

$$\frac{\partial W_2^{\{L_2(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{(\omega + 5j\gamma/2)(j\omega-\gamma/2)}{(j\omega+\gamma/2)^4};$$

$$\frac{\partial W_3^{\{L_3(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{(\omega + 7j\gamma/2)(j\omega-\gamma/2)^2}{(j\omega+\gamma/2)^5};$$

$$\frac{\partial W_4^{\{L_4(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{(\omega + 9j\gamma/2)(j\omega-\gamma/2)^3}{(j\omega+\gamma/2)^6};$$

$$\frac{\partial W_5^{\{L_5(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{(\omega + 11j\gamma/2)(j\omega-\gamma/2)^4}{(j\omega+\gamma/2)^7}.$$

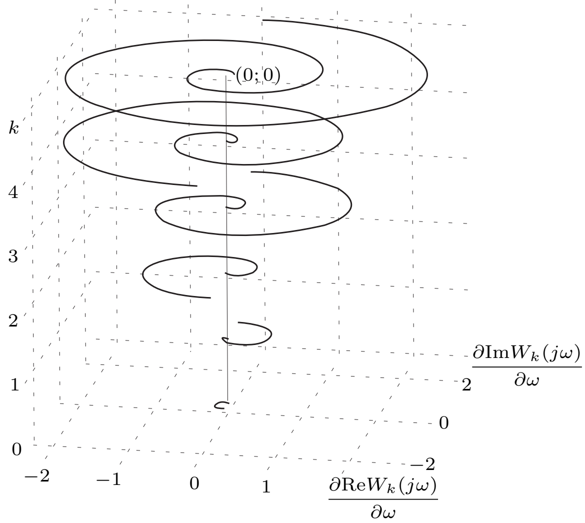


Рис. 6.40. Вид производных преобразования Фурье ортогональных функций Лагерра 0-5 порядков; $\gamma = 4$

[6.113]
$$\frac{\partial W_k^{[1]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{1}{(j\omega+\gamma/2)^2}\sum_{s=0}^{k}\binom{k+1}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

[6.114]
$$\frac{\partial W_k^{[2]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}= -\frac{j(k+1)\big(j\omega-\gamma/2\big)}{(j\omega+\gamma/2)^3}\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^{k-1}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{L_0^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{\big(j\omega+\gamma/2\big)^2};$$

$$\frac{\partial W_1^{\{L_1^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{2j\big(j\omega-\gamma/2\big)}{\big(j\omega+\gamma/2\big)^3};$$

$$\frac{\partial W_2^{\{L_2^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{3j\big(j\omega-\gamma/2\big)^2}{\big(j\omega+\gamma/2\big)^4};$$

$$\frac{\partial W_3^{\{L_3^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{4j\big(j\omega-\gamma/2\big)^3}{\big(j\omega+\gamma/2\big)^5};$$

$$\frac{\partial W_4^{\{L_4^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{5j\big(j\omega-\gamma/2\big)^4}{\big(j\omega+\gamma/2\big)^6};$$

$$\frac{\partial W_5^{\{L_5^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{6j\big(j\omega-\gamma/2\big)^5}{\big(j\omega+\gamma/2\big)^7}.$$

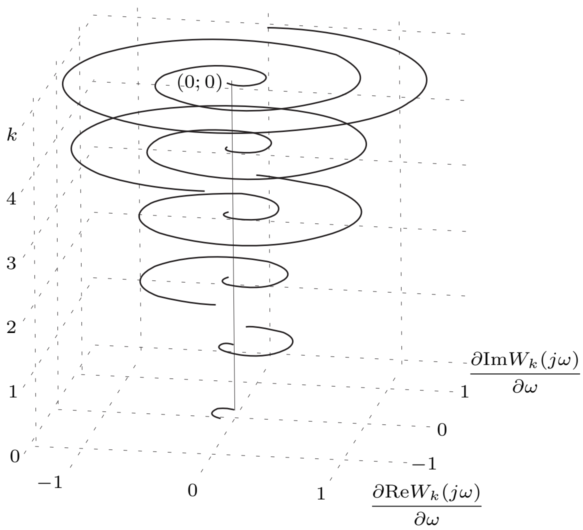


Рис. 6.41. Вид производных преобразования Фурье ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma=4,\ \alpha=1$

[6.115]
$$\frac{\partial W_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}= \frac{1}{(j\omega+\gamma/2)^2}\sum_{s=0}^{k}\binom{k+2}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{L_0^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{\big(j\omega+\gamma/2\big)^2};$$

$$\frac{\partial W_1^{\{L_1^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j\big(3j\omega-\gamma/2\big)}{\big(j\omega+\gamma/2\big)^3};$$

$$\frac{\partial W_2^{\{L_2^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{2j\big(j\omega\gamma-\gamma^2/4+3\omega^2\big)}{\big(j\omega+\gamma/2\big)^4};$$

$$\frac{\partial W_3^{\{L_3^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=\frac{2j}{\big(j\omega+\gamma/2\big)^5}\big(\gamma^3/8-10j\gamma^2\omega/8-20\gamma\omega^2/8+ + 5j\omega^3\big);$$

$$\frac{\partial W_4^{\{L_4^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{\big(j\omega+\gamma/2\big)^6}\big(\gamma^4/16-6j\gamma^3\omega/4-20\gamma^2\times \times\omega^2+10j\gamma\omega^3+240\omega^4\big);$$

$$\frac{\partial W_5^{\{L_5^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=\frac{j}{\big(j\omega+\gamma/2\big)^7}\big(\gamma^5/32-21j\gamma^4\omega/4-21\gamma^3\times \times\omega^2/4+35j\gamma^2\omega^3/2+35\gamma\omega^4/2-672j\omega^5\big).$$

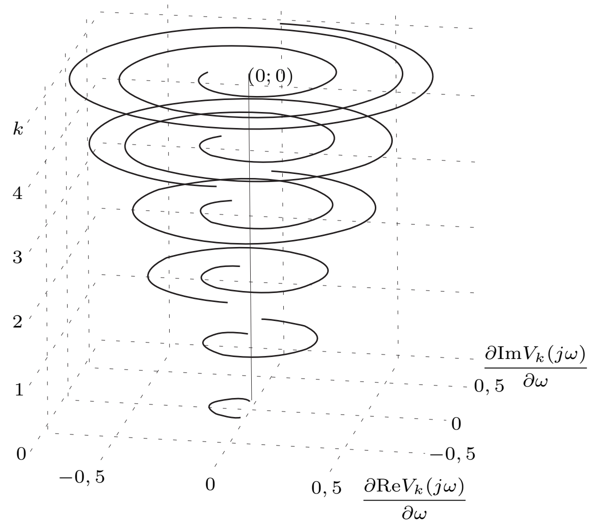


Рис. 6.42. Вид производных преобразования Фурье ортогональных функций Сонина-Лагерра 0-5 порядков; $\gamma=4,\ \alpha=2$

[6.116]
$$\frac{\partial W_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}= \frac{1}{(j\omega+\gamma/2)^2}\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{L_0^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{(j\omega+\gamma/2)^2};$$

$$\frac{\partial W_1^{\{L_1^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j(\alpha+1)}{(j\omega+\gamma/2)^2}+\frac{2j\gamma}{(j\omega+\gamma/2)^3};$$

$$\frac{\partial W_2^{\{L_2^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j(\alpha+1)(\alpha+2)}{2(j\omega+\gamma/2)^2}+\frac{2j\gamma(\alpha+2)}{(j\omega+\gamma/2)^3}- -\frac{3j\gamma^2}{(j\omega+\gamma/2)^4};$$

$$\frac{\partial W_3^{\{L_3^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j(\alpha+3)!}{6\alpha!(j\omega+\gamma/2)^2} + \frac{j\gamma(\alpha+2)(\alpha+3)}{(j\omega+\gamma/2)^3} - \frac{3j\gamma^2(\alpha+3)}{(j\omega+\gamma/2)^4} + \frac{4j\gamma^3}{(j\omega+\gamma/2)^5};$$

$$\frac{\partial W_4^{\{L_4^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j(\alpha+4)!}{24\alpha!(j\omega+\gamma/2)^2} + \frac{j\gamma(\alpha+4)!}{3(\alpha+1)!}\times \frac{1}{(j\omega+\gamma/2)^3} - \frac{3j\gamma^2(\alpha+3)(\alpha+4)}{2(j\omega+\gamma/2)^4} + \frac{4j\gamma^3(\alpha+4)}{(j\omega+\gamma/2)^5} - \frac{5j\gamma^4}{(j\omega+\gamma/2)^6};$$

$$\frac{\partial W_5^{\{L_5^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j(\alpha+5)!}{120\alpha!(j\omega+\gamma/2)^2} + \frac{j\gamma(\alpha+5)!}{12(\alpha+1)!}\times \frac{1}{(j\omega+\gamma/2)^3} - \frac{j\gamma^2(\alpha+5)!}{2(\alpha+2)!(j\omega+\gamma/2)^4} + \frac{2j\gamma^3(\alpha+4)(\alpha+5)}{(j\omega+\gamma/2)^5} - \frac{5j\gamma^4(\alpha+5)}{(j\omega+\gamma/2)^6} + \frac{6j\gamma^5}{(j\omega+\gamma/2)^7}.$$

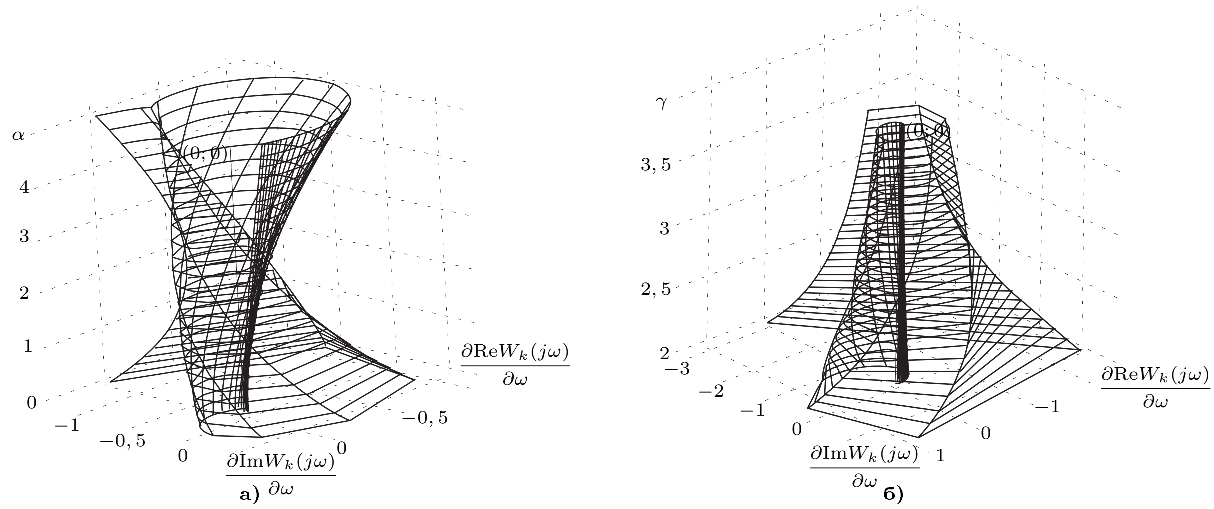


Рис. 6.43. Вид производных преобразования Фурье ортогональных функций Сонина-Лагерра 2-ого порядка: а) $\gamma = 4$, $\alpha \in [0;5]$; б) $\gamma \in [2;4]$, $\alpha = 1$

[6.117]
$$\frac{\partial W_k^{[1]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -j\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\frac{1}{\big((4s+1)\gamma/2+j\omega\big)^2}.$$

[6.118]
$$\frac{\partial W_k^{[2]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{j}{(\gamma/2+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{j}{(4k+1)\gamma/2+j\omega}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(4s+1)\gamma/2-j\omega}{(4s+1)\gamma/2+j\omega}\times \\ \times\Bigg(\dfrac{1}{(4k+1)\gamma/2+j\omega}+ \\ +\gamma\displaystyle\sum_{s=0}^{k-1}\frac{4s+1}{\big((4s+1)\gamma/2\big)^2+\omega^2}\Bigg), & \text{если } k>0.\end{cases}$$

[6.119]
$$\frac{\partial W_k^{[3]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{4j(\cos\varphi_0)^2}{\gamma^2}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -\dfrac{2j\cos\varphi_k}{(4k+1)\gamma}\times \\ \times\exp\Bigg(-j\Bigg(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\Bigg)\Bigg)\times \\ \times\Bigg(\dfrac{2\cos\varphi_k\exp\big(-j\varphi_k\big)}{(4k+1)\gamma}+ \\ +\dfrac{4}{\gamma}\displaystyle\sum_{s=0}^{k-1}\frac{(cos\varphi_k)^2}{4s+1}\Bigg), & \text{если } k>0,\end{cases}$$

$$\varphi_k = \arctan\frac{2\omega}{(4k+1)\gamma}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{P_0^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma/2+j\omega)^2};$$

$$\frac{\partial W_1^{\{P_1^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{2(\gamma/2+j\omega)^2} + \frac{3j}{2(5\gamma/2+j\omega)^2};$$

$$\frac{\partial W_2^{\{P_2^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{3j}{8(\gamma/2+j\omega)^2} + \frac{15j}{4(5\gamma/2+j\omega)^2} -$$

$$-\frac{35j}{8(9\gamma/2+j\omega)^2};$$

$$\frac{\partial W_3^{\{P_3^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{5j}{16(\gamma/2+j\omega)^2}+\frac{105j}{16(5\gamma/2+j\omega)^2}-$$
$$-\frac{315j}{16(9\gamma/2+j\omega)^2}+\frac{231j}{16(13\gamma/2+j\omega)^2};$$

$$\frac{\partial W_4^{\{P_4^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{35j}{128(\gamma/2+j\omega)^2}+\frac{315j}{32}\times$$
$$\times\frac{1}{(5\gamma/2+j\omega)^2}-\frac{3465j}{64(9\gamma/2+j\omega)^2}+\frac{3003j}{32(13\gamma/2+j\omega)^2}-$$
$$-\frac{6435j}{128(17\gamma/2+j\omega)^2};$$

$$\frac{\partial W_5^{\{P_5^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{63j}{256(\gamma/2+j\omega)^2}+\frac{3465j}{256}\times$$
$$\times\frac{1}{(5\gamma/2+j\omega)^2}-\frac{15015j}{128(9\gamma/2+j\omega)^2}+\frac{45045j}{128(13\gamma/2+j\omega)^2}-$$
$$-\frac{109395j}{256(17\gamma/2+j\omega)^2}+\frac{46189j}{256(21\gamma/2+j\omega)^2}.$$

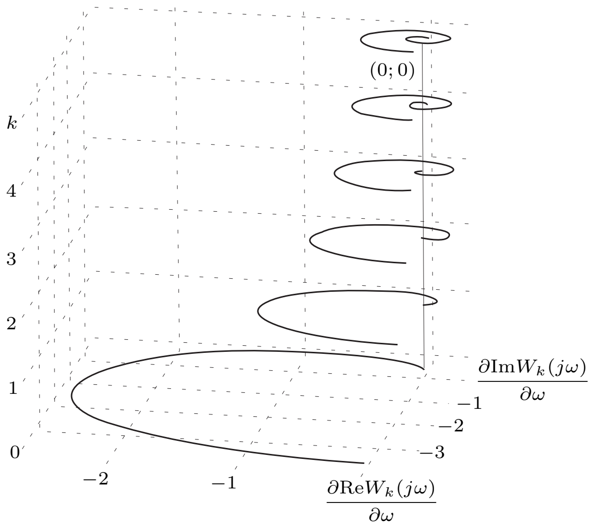


Рис. 6.44. Вид производных преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma=1$, $c=2$, $\alpha=-1/2$, $\beta=0$

[6.120]
$$\frac{\partial W_k^{[1]\{Leg_k(\tau,\gamma)\}}(j\omega)}{\partial\omega}=$$
$$=-j\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{\big((2s+1)\gamma+j\omega\big)^2}.$$

[6.121]
$$\frac{\partial W_k^{[2]\{Leg_k(\tau,\gamma)\}}(j\omega)}{\partial\omega}=$$
$$=\begin{cases}-\dfrac{j}{(\gamma+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{j}{(2k+1)\gamma+j\omega}\times\\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+1)\gamma-j\omega}{(2s+1)\gamma+j\omega}\times\\ \times\Bigg(\dfrac{1}{(2k+1)\gamma+j\omega}+\\ +2\gamma\displaystyle\sum_{s=0}^{k-1}\frac{2s+1}{\big((2s+1)\gamma\big)^2+\omega^2}\Bigg), & \text{если } k>0.\end{cases}$$

[6.122]
$$\frac{\partial W_k^{[3]\{Leg_k(\tau,\gamma)\}}(j\omega)}{\partial\omega}=$$
$$=\begin{cases}-\dfrac{j(\cos\varphi_0)^2}{\gamma^2}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -\dfrac{j\cos\varphi_k}{(2k+1)\gamma}\times\\ \times\exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right)\times\\ \times\left(\dfrac{\cos\varphi_k\exp\big(-j\varphi_k\big)}{(2k+1)\gamma}+\right.\\ \left.+\dfrac{2}{\gamma}\displaystyle\sum_{s=0}^{k-1}\frac{(cos\varphi_k)^2}{2s+1}\right), & \text{если } k>0,\end{cases}$$
$$\varphi_k=\arctan\frac{\omega}{(2k+1)\gamma}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{Leg_0(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{(\gamma+j\omega)^2};$$

$$\frac{\partial W_1^{\{Leg_1(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{(\gamma+j\omega)^2}+\frac{2j}{(3\gamma+j\omega)^2};$$

$$\frac{\partial W_2^{\{Leg_2(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{(\gamma+j\omega)^2}+\frac{6j}{(3\gamma+j\omega)^2}-\frac{6j}{(5\gamma+j\omega)^2};$$

$$\frac{\partial W_3^{\{Leg_3(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{(\gamma+j\omega)^2}+\frac{12j}{(3\gamma+j\omega)^2}-$$
$$-\frac{30j}{(5\gamma+j\omega)^2}+\frac{20j}{(7\gamma+j\omega)^2};$$

$$\frac{\partial W_4^{\{Leg_4(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{(\gamma+j\omega)^2}+\frac{20j}{(3\gamma+j\omega)^2}-$$
$$-\frac{90j}{(5\gamma+j\omega)^2}+\frac{140j}{(7\gamma+j\omega)^2}-\frac{70j}{(9\gamma+j\omega)^2};$$

$$\frac{\partial W_5^{\{Leg_5(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j}{(\gamma+j\omega)^2}+\frac{30j}{(3\gamma+j\omega)^2}-$$
$$-\frac{210j}{(5\gamma+j\omega)^2}+\frac{560j}{(7\gamma+j\omega)^2}-\frac{630j}{(9\gamma+j\omega)^2}+\frac{252j}{(11\gamma+j\omega)^2}.$$

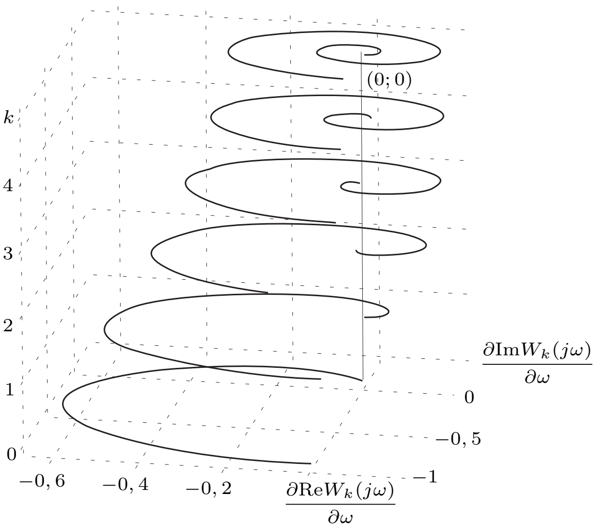


Рис. 6.45. Вид производных преобразования Фурье ортогональных функций Лежандра 0-5 порядков; $\gamma = 1$, $c = 2$

[6.123]
$$\frac{\partial W_k^{[1]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial \omega} = -j\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s\frac{1}{\big((4s+3)\gamma/2+j\omega\big)^2}.$$

[6.124]
$$\frac{\partial W_k^{[2]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial \omega} = \begin{cases} -\dfrac{j}{(3\gamma/2+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{j}{(4k+3)\gamma/2+j\omega}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(4s+3)\gamma/2-j\omega}{(4s+3)\gamma/2+j\omega}\times \\ \times\Bigg(\dfrac{1}{(4k+3)\gamma/2+j\omega}+ \\ +\gamma\displaystyle\sum_{s=0}^{k-1}\frac{4s+3}{\big((4s+3)\gamma/2\big)^2+\omega^2}\Bigg), & \text{если } k>0. \end{cases}$$

[6.125]
$$\frac{\partial W_k^{[3]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial \omega} = \begin{cases} -\dfrac{4j(\cos\varphi_0)^2}{\gamma^2}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -\dfrac{2j\cos\varphi_k}{(4k+3)\gamma}\times \\ \times\exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right)\times \\ \times\Bigg(\dfrac{2\cos\varphi_k\exp\big(-j\varphi_k\big)}{(4k+3)\gamma}+ \\ +\dfrac{4}{\gamma}\displaystyle\sum_{s=0}^{k-1}\frac{(cos\varphi_k)^2}{4s+3}\Bigg), & \text{если } k>0, \end{cases}$$
$$\varphi_k = \arctan\frac{2\omega}{(4k+3)\gamma}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{P_0^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(3\gamma/2+j\omega)^2};$$

$$\frac{\partial W_1^{\{P_1^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{3j}{2(3\gamma/2+j\omega)^2}+\frac{5j}{2(7\gamma/2+j\omega)^2};$$

$$\frac{\partial W_2^{\{P_2^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{15j}{8(3\gamma/2+j\omega)^2}+\frac{35j}{4(7\gamma/2+j\omega)^2}- -\frac{63j}{8(11\gamma/2+j\omega)^2};$$

$$\frac{\partial W_3^{\{P_3^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{35j}{16(3\gamma/2+j\omega)^2}+\frac{315j}{16(7\gamma/2+j\omega)^2}- -\frac{693j}{16(11\gamma/2+j\omega)^2}+\frac{429j}{16(15\gamma/2+j\omega)^2};$$

$$\frac{\partial W_4^{\{P_4^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{315j}{128(3\gamma/2+j\omega)^2}+\frac{1155j}{32}\times \times\frac{1}{(7\gamma/2+j\omega)^2}-\frac{9009j}{64(11\gamma/2+j\omega)^2}+\frac{6435j}{32(15\gamma/2+j\omega)^2}- -\frac{12155j}{128(19\gamma/2+j\omega)^2};$$

$$\frac{\partial W_5^{\{P_5^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{693j}{256(3\gamma/2+j\omega)^2}+\frac{15015j}{256}\times \times\frac{1}{(7\gamma/2+j\omega)^2}-\frac{45045j}{128(11\gamma/2+j\omega)^2}+\frac{109395j}{128(15\gamma/2+j\omega)^2}- -\frac{230945j}{256(19\gamma/2+j\omega)^2}+\frac{88179j}{256(23\gamma/2+j\omega)^2}.$$

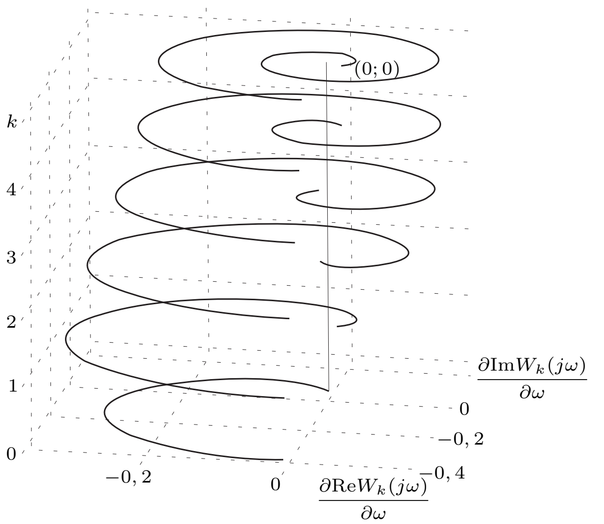


Рис. 6.46. Вид производных преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[6.126]
$$\frac{\partial W_k^{[1]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -j\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s\frac{1}{\big((s+1)\gamma+j\omega\big)^2}.$$

[6.127]
$$\frac{\partial W_k^{[2]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{j}{(\gamma+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{j}{(k+1)\gamma+j\omega}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(s+1)\gamma-j\omega}{(s+1)\gamma+j\omega}\times \\ \times\Big(\dfrac{1}{(k+1)\gamma+j\omega}+ \\ +2\gamma\displaystyle\sum_{s=0}^{k-1}\frac{s+1}{\big((s+1)\gamma\big)^2+\omega^2}\Big), & \text{если } k>0. \end{cases}$$

[6.128]
$$\frac{\partial W_k^{[3]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{j(\cos\varphi_0)^2}{\gamma^2}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -\dfrac{j\cos\varphi_k}{(k+1)\gamma}\times \\ \times\exp\Bigg(-j\bigg(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\bigg)\Bigg)\times \\ \times\bigg(\dfrac{\cos\varphi_k\exp\big(-j\varphi_k\big)}{(k+1)\gamma}+ \\ +\dfrac{2}{\gamma}\displaystyle\sum_{s=0}^{k-1}\frac{(cos\varphi_k)^2}{s+1}\bigg), & \text{если } k>0, \end{cases}$$
$$\varphi_k = \arctan\frac{\omega}{(k+1)\gamma}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{P_0^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma+j\omega)^2};$$

$$\frac{\partial W_1^{\{P_1^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{2j}{(\gamma+j\omega)^2}+\frac{3j}{(2\gamma+j\omega)^2};$$

$$\frac{\partial W_2^{\{P_2^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{3j}{(\gamma+j\omega)^2}+\frac{12j}{(2\gamma+j\omega)^2}-\frac{10j}{(3\gamma+j\omega)^2};$$

$$\frac{\partial W_3^{\{P_3^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{4j}{(\gamma+j\omega)^2}+\frac{30j}{(2\gamma+j\omega)^2}- -\frac{60j}{(3\gamma+j\omega)^2}+\frac{35j}{(4\gamma+j\omega)^2};$$

$$\frac{\partial W_4^{\{P_4^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{5j}{(\gamma+j\omega)^2}+\frac{60j}{(2\gamma+j\omega)^2}- -\frac{210j}{(3\gamma+j\omega)^2}+\frac{280j}{(4\gamma+j\omega)^2}-\frac{126j}{(5\gamma+j\omega)^2};$$

$$\frac{\partial W_5^{\{P_5^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{6j}{(\gamma+j\omega)^2}+\frac{105j}{(2\gamma+j\omega)^2}- -\frac{560j}{(3\gamma+j\omega)^2}+\frac{1260j}{(4\gamma+j\omega)^2}-\frac{1260j}{(5\gamma+j\omega)^2}+\frac{462j}{(6\gamma+j\omega)^2}.$$

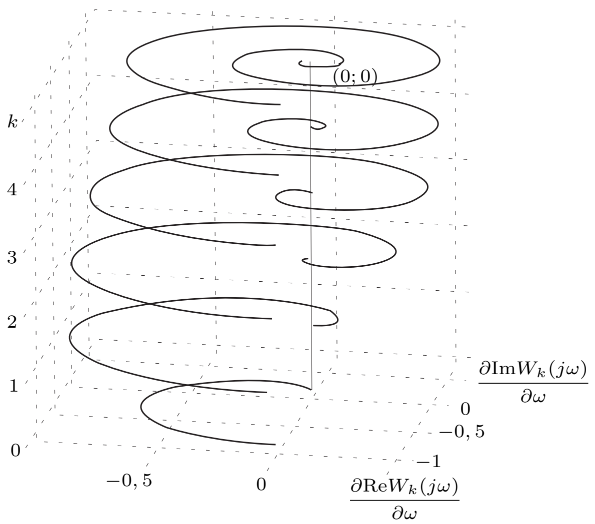


Рис. 6.47. Вид производных преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 1$, $\alpha = 1$, $\beta = 0$

[6.129]

$$\frac{\partial W_k^{[1]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -j\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s\frac{1}{\big((2s+3)\gamma+j\omega\big)^2}.$$

[6.130]

$$\frac{\partial W_k^{[2]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{j}{(\gamma+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{j}{(2k+3)\gamma+j\omega}\times \prod_{s=0}^{k-1}\dfrac{(2s+3)\gamma-j\omega}{(2s+3)\gamma+j\omega}\times \left(\dfrac{1}{(2k+3)\gamma+j\omega}+ 2\gamma\sum_{s=0}^{k-1}\dfrac{2s+3}{\big((2s+3)\gamma\big)^2+\omega^2}\right), & \text{если } k>0. \end{cases}$$

[6.131]

$$\frac{\partial W_k^{[3]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{j(\cos\varphi_0)^2}{\gamma^2}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -\dfrac{j\cos\varphi_k}{(2k+3)\gamma}\times \exp\left(-j\left(\varphi_k+2\sum_{s=0}^{k-1}\varphi_s\right)\right)\times \left(\dfrac{\cos\varphi_k\exp\big(-j\varphi_k\big)}{(2k+3)\gamma}+ \dfrac{2}{\gamma}\sum_{s=0}^{k-1}\dfrac{(cos\varphi_k)^2}{2s+3}\right), & \text{если } k>0, \end{cases}$$

$$\varphi_k = \arctan\frac{\omega}{(2k+3)\gamma}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{P_0^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(3\gamma+j\omega)^2};$$

$$\frac{\partial W_1^{\{P_1^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{3j}{(3\gamma+j\omega)^2}+\frac{4j}{(5\gamma+j\omega)^2};$$

$$\frac{\partial W_2^{\{P_2^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{6j}{(3\gamma+j\omega)^2}+\frac{20j}{(5\gamma+j\omega)^2}-\frac{15j}{(7\gamma+j\omega)^2};$$

$$\frac{\partial W_3^{\{P_3^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{10j}{(3\gamma+j\omega)^2}+\frac{60j}{(5\gamma+j\omega)^2}-\frac{105j}{(7\gamma+j\omega)^2}+\frac{56j}{(9\gamma+j\omega)^2};$$

$$\frac{\partial W_4^{\{P_4^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{15j}{(3\gamma+j\omega)^2}+\frac{140j}{(5\gamma+j\omega)^2}-\frac{420j}{(7\gamma+j\omega)^2}+\frac{504j}{(9\gamma+j\omega)^2}-\frac{210j}{(11\gamma+j\omega)^2};$$

$$\frac{\partial W_5^{\{P_5^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{21j}{(3\gamma+j\omega)^2}+\frac{280j}{(5\gamma+j\omega)^2}-\frac{1260j}{(7\gamma+j\omega)^2}+\frac{2520j}{(9\gamma+j\omega)^2}-\frac{2310j}{(11\gamma+j\omega)^2}+\frac{792j}{(13\gamma+j\omega)^2}.$$

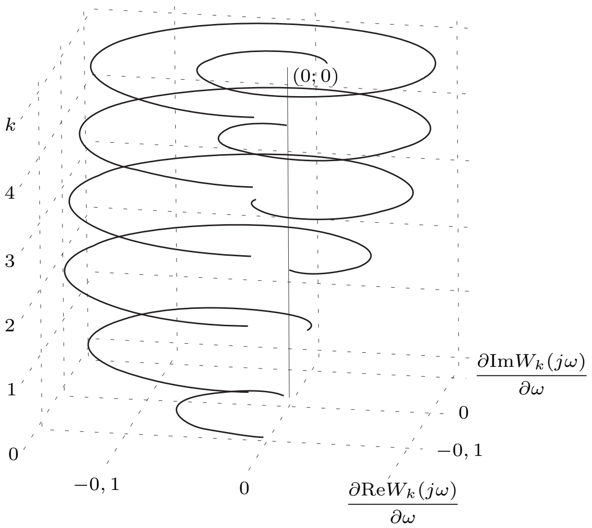


Рис. 6.48. Вид производных преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 2$, $\beta = 0$

[6.132]
$$\frac{\partial W_k^{[1]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -j\sum_{s=0}^{k}\binom{k}{s}\times \binom{k+s+\alpha}{s+\alpha}(-1)^s\frac{1}{\big((2s+\alpha+1)c\gamma/2+j\omega\big)^2}.$$

[6.133]
$$\frac{\partial W_k^{[2]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{j}{\big((\alpha+1)c\gamma/2+j\omega\big)^2}, & \text{если } k=0;\\ -\dfrac{j}{(2k+\alpha+1)c\gamma/2+j\omega}\times\\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+\alpha+1)c\gamma/2-j\omega}{(2s+\alpha+1)c\gamma/2+j\omega}\times\\ \times\Bigg(\dfrac{1}{(2k+\alpha+1)c\gamma/2+j\omega}+2c\gamma/2\times\\ \times\displaystyle\sum_{s=0}^{k-1}\frac{2s+\alpha+1}{\big((2s+\alpha+1)c\gamma/2\big)^2+\omega^2}\Bigg), & \text{если } k>0. \end{cases}$$

[6.134]
$$\frac{\partial W_k^{[3]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{4j(\cos\varphi_0)^2}{(\alpha+1)^2c^2\gamma^2}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -\dfrac{2j\cos\varphi_k}{(2k+\alpha+1)c\gamma}\times\\ \times\exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right)\times\\ \times\left(\dfrac{\cos\varphi_k\exp\big(-j\varphi_k\big)}{(2k+\alpha+1)c\gamma/2}+\right.\\ \left.+\dfrac{4}{c\gamma}\displaystyle\sum_{s=0}^{k-1}\frac{(cos\varphi_k)^2}{2s+\alpha+1}\right), & \text{если } k>0, \end{cases}$$
$$\varphi_k = \arctan\frac{2\omega}{(2k+\alpha+1)c\gamma}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{P_0^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{\big(c\gamma(\alpha+1)/2+j\omega\big)^2};$$

$$\frac{\partial W_1^{\{P_1^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j(\alpha+1)}{\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+j(\alpha+2)\times \times\frac{1}{\big(c\gamma(\alpha+3)/2+j\omega\big)^2};$$

$$\frac{\partial W_2^{\{P_2^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j(\alpha+1)(\alpha+2)}{2\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+j(\alpha+2)\times \times\frac{(\alpha+3)}{2\big(c\gamma(\alpha+3)/2+j\omega\big)^2}-\frac{j(\alpha+3)(\alpha+4)}{2\big(c\gamma(\alpha+5)/2+j\omega\big)^2};$$

$$\frac{\partial W_3^{\{P_3^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j(\alpha+3)!}{6\alpha!\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+\frac{j(\alpha+4)!}{2(\alpha+1)!}\times \times\frac{1}{\big(c\gamma(\alpha+3)/2+j\omega\big)^2}-\frac{j(\alpha+5)!}{2(\alpha+2)!\big(c\gamma(\alpha+5)/2+j\omega\big)^2}+ +\frac{j(\alpha+6)!}{6(\alpha+3)!\big(c\gamma(\alpha+7)/2+j\omega\big)^2};$$

$$\frac{\partial W_4^{\{P_4^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j(\alpha+4)!}{24\alpha!\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+\frac{j}{6}\times \times\frac{(\alpha+5)!}{(\alpha+1)!\big(c\gamma(\alpha+3)/2+j\omega\big)^2}-\frac{j(\alpha+6)!}{4(\alpha+2)!\big(c\gamma(\alpha+5)/2+j\omega\big)^2}+ +\frac{j(\alpha+7)!}{6(\alpha+3)!\big(c\gamma(\alpha+7)/2+j\omega\big)^2}-\frac{j(\alpha+8)!}{24(\alpha+4)!\big(c\gamma(\alpha+9)/2+j\omega\big)^2};$$

$$\frac{\partial W_5^{\{P_5^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j(\alpha+5)!}{120\alpha!\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+\frac{j}{24}\times \times\frac{(\alpha+6)!}{(\alpha+1)!\big(c\gamma(\alpha+3)/2+j\omega\big)^2}-\frac{j(\alpha+7)!}{12(\alpha+2)!\big(c\gamma(\alpha+5)/2+j\omega\big)^2}+ +\frac{j(\alpha+8)!}{12(\alpha+3)!\big(c\gamma(\alpha+7)/2+j\omega\big)^2}-\frac{j(\alpha+9)!}{24(\alpha+4)!\big(c\gamma(\alpha+9)/2+j\omega\big)^2}+ +\frac{j(\alpha+10)!}{120(\alpha+5)!\big(c\gamma(\alpha+11)/2+j\omega\big)^2}.$$

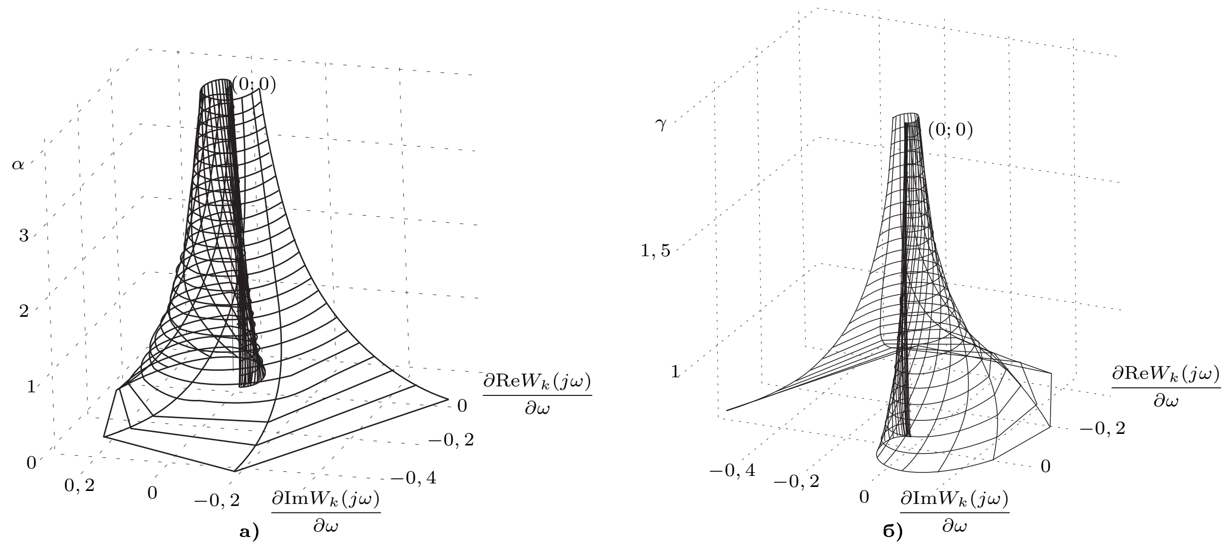


Рис. 6.49. Вид производных преобразования Фурье ортогональных функций Якоби 2-ого порядка: а) $\gamma = 1$, $c = 2$, $\alpha \in [0; 4]$, $\beta = 0$; б) $\gamma \in [0, 75; 2]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[6.135]
$$\frac{\partial W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -j\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s\frac{1}{\big((2s+1)\gamma + j\omega\big)^2}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{P_0^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma + j\omega)^2};$$

$$\frac{\partial W_1^{\{P_1^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma + j\omega)^2} + \frac{3j}{(3\gamma + j\omega)^2};$$

$$\frac{\partial W_2^{\{P_2^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma + j\omega)^2} + \frac{8j}{(3\gamma + j\omega)^2} - \frac{10j}{(5\gamma + j\omega)^2};$$

$$\frac{\partial W_3^{\{P_3^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma + j\omega)^2} + \frac{15j}{(3\gamma + j\omega)^2} - \frac{45j}{(5\gamma + j\omega)^2} + \frac{35j}{(7\gamma + j\omega)^2};$$

$$\frac{\partial W_4^{\{P_4^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma + j\omega)^2} + \frac{24j}{(3\gamma + j\omega)^2} - \frac{126j}{(5\gamma + j\omega)^2} + \frac{224j}{(7\gamma + j\omega)^2} - \frac{126j}{(9\gamma + j\omega)^2};$$

$$\frac{\partial W_5^{\{P_5^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma + j\omega)^2} + \frac{35j}{(3\gamma + j\omega)^2} - \frac{280j}{(5\gamma + j\omega)^2} + \frac{840j}{(7\gamma + j\omega)^2} - \frac{1050j}{(9\gamma + j\omega)^2} + \frac{462j}{(11\gamma + j\omega)^2}.$$

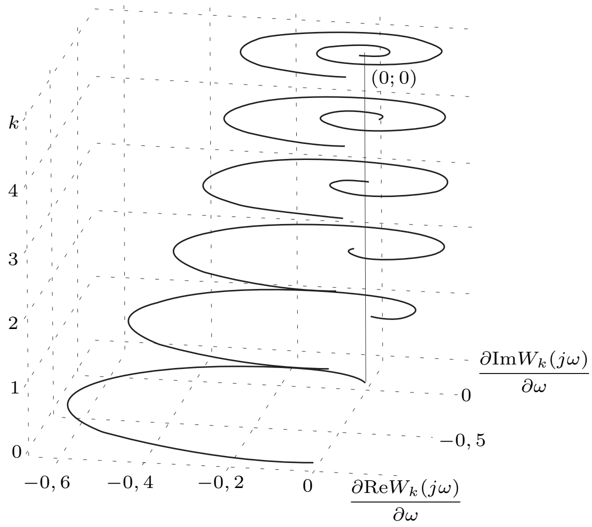


Рис. 6.50. Вид производных преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta = 1$

[6.136]
$$\frac{\partial W_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -j\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s\frac{1}{\big((2s+1)\gamma + j\omega\big)^2}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{P_0^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma + j\omega)^2};$$

$$\frac{\partial W_1^{\{P_1^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma+j\omega)^2} + \frac{4j}{(3\gamma+j\omega)^2};$$

$$\frac{\partial W_2^{\{P_2^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma+j\omega)^2} + \frac{10j}{(3\gamma+j\omega)^2} - \frac{15j}{(5\gamma+j\omega)^2};$$

$$\frac{\partial W_3^{\{P_3^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma+j\omega)^2} + \frac{18j}{(3\gamma+j\omega)^2} -$$
$$-\frac{63j}{(5\gamma+j\omega)^2} + \frac{56j}{(7\gamma+j\omega)^2};$$

$$\frac{\partial W_4^{\{P_4^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma+j\omega)^2} + \frac{28j}{(3\gamma+j\omega)^2} -$$
$$-\frac{168j}{(5\gamma+j\omega)^2} + \frac{336j}{(7\gamma+j\omega)^2} - \frac{210j}{(9\gamma+j\omega)^2};$$

$$\frac{\partial W_5^{\{P_5^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(\gamma+j\omega)^2} + \frac{40j}{(3\gamma+j\omega)^2} -$$
$$-\frac{360j}{(5\gamma+j\omega)^2} + \frac{1200j}{(7\gamma+j\omega)^2} - \frac{1650j}{(9\gamma+j\omega)^2} + \frac{792j}{(11\gamma+j\omega)^2}.$$

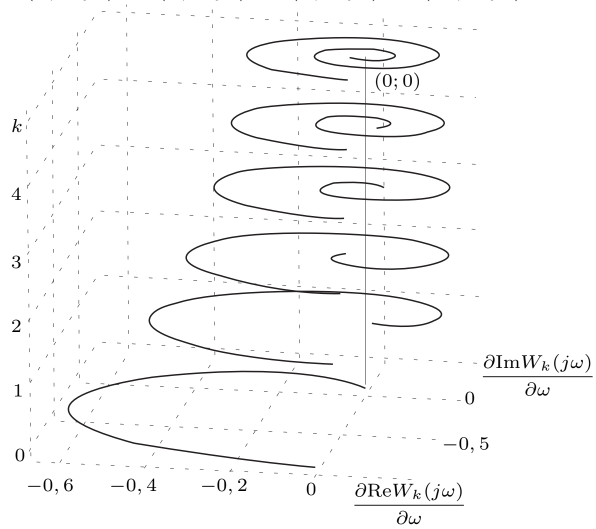


Рис. 6.51. Вид производных преобразования Фурье ортогональных функций Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta = 2$

[6.137]
$$\frac{\partial W_k^{\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -j\sum_{s=0}^{k}\binom{k}{s}\times$$
$$\times\binom{k+s+\beta}{s}(-1)^s\frac{1}{\big((2s+1)c\gamma/2+j\omega\big)^2}.$$

Частные случаи для производных преобразования Фурье функций 0-5 порядков:

$$\frac{\partial W_0^{\{P_0^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(c\gamma/2+j\omega)^2};$$

$$\frac{\partial W_1^{\{P_1^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(c\gamma/2+j\omega)^2} + \frac{j(\beta+2)}{(3c\gamma/2+j\omega)^2};$$

$$\frac{\partial W_2^{\{P_2^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{2(c\gamma/2+j\omega)^2} + \frac{2j(\beta+3)}{(3c\gamma/2+j\omega)^2} -$$
$$-\frac{j(\beta+3)(\beta+4)}{2(5c\gamma/2+j\omega)^2};$$

$$\frac{\partial W_3^{\{P_3^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(c\gamma/2+j\omega)^2} + \frac{3j(\beta+4)}{(3c\gamma/2+j\omega)^2} -$$
$$-\frac{3j(\beta+3)(\beta+4)}{2(5c\gamma/2+j\omega)^2} + \frac{j(\beta+6)!}{6(\beta+3)!(7c\gamma/2+j\omega)^2};$$

$$\frac{\partial W_4^{\{P_4^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(c\gamma/2+j\omega)^2} + \frac{4j(\beta+5)}{(3c\gamma/2+j\omega)^2} -$$
$$-\frac{3j(\beta+5)(\beta+6)}{(5c\gamma/2+j\omega)^2} + \frac{2j(\beta+7)!}{3(\beta+4)!(7c\gamma/2+j\omega)^2} -$$
$$-\frac{j(\beta+8)!}{24(\beta+4)!(9c\gamma/2+j\omega)^2};$$

$$\frac{\partial W_5^{\{P_5^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j}{(c\gamma/2+j\omega)^2} + \frac{5j(\beta+6)}{(3c\gamma/2+j\omega)^2} -$$
$$-\frac{5j(\beta+6)(\beta+7)}{(5c\gamma/2+j\omega)^2} + \frac{5j(\alpha+8)!}{3(\alpha+5)!(7c\gamma/2+j\omega)^2} -$$
$$-\frac{5j(\beta+9)!}{24(\beta+5)!(9c\gamma/2+j\omega)^2} + \frac{j(\beta+10)!}{120(\beta+5)!(11c\gamma/2+j\omega)^2}.$$

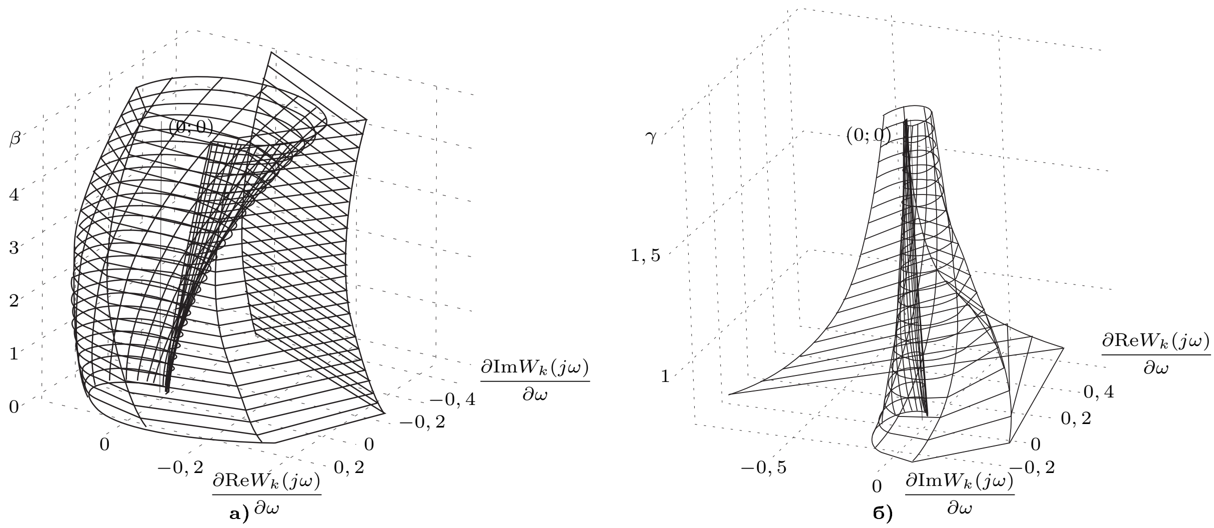


Рис. 6.52. Вид производных преобразования Фурье ортогональных функций Якоби 2-ого порядка: а) $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta \in [0; 5]$; б) $\gamma \in [0,75; 2]$, $c = 2$, $\alpha = 0$, $\beta = 1$

6.5 Производные преобразований Фурье ортогональных фильтров

[6.138]
$$\frac{\partial V_k^{[1]\{L_k(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma}{(j\omega+\gamma/2)^2}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s(s+1).$$

[6.139]
$$\frac{\partial V_k^{[2]\{L_k(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma\big(j\omega-\gamma(2k+1)/2\big)}{(j\omega+\gamma/2)^3}\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^{k-1}.$$

[6.140]
$$\frac{\partial V_k^{[3]\{L_k(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{4j}{\gamma}(-1)^k(\cos\varphi)^2\times \exp(-j(2k+1)\varphi)\big((2k+1)\cos\varphi - j\sin\varphi\big),$$
$$\varphi = \arctan\frac{2\omega}{\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{L_0(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma(\omega+j\gamma/2)}{(j\omega-\gamma/2)\big(j\omega+\gamma/2\big)^2};$$
$$\frac{\partial V_1^{\{L_1(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma(\omega+3j\gamma/2)}{\big(j\omega+\gamma/2\big)^3};$$
$$\frac{\partial V_2^{\{L_2(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma(\omega+5j\gamma/2)\big(j\omega-\gamma/2\big)}{\big(j\omega+\gamma/2\big)^4};$$
$$\frac{\partial V_3^{\{L_3(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma(\omega+7j\gamma/2)\big(j\omega-\gamma/2\big)^2}{\big(j\omega+\gamma/2\big)^5};$$
$$\frac{\partial V_4^{\{L_4(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma(\omega+9j\gamma/2)\big(j\omega-\gamma/2\big)^3}{\big(j\omega+\gamma/2\big)^6};$$
$$\frac{\partial V_5^{\{L_5(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma(\omega+11j\gamma/2)\big(j\omega-\gamma/2\big)^4}{\big(j\omega+\gamma/2\big)^7}.$$

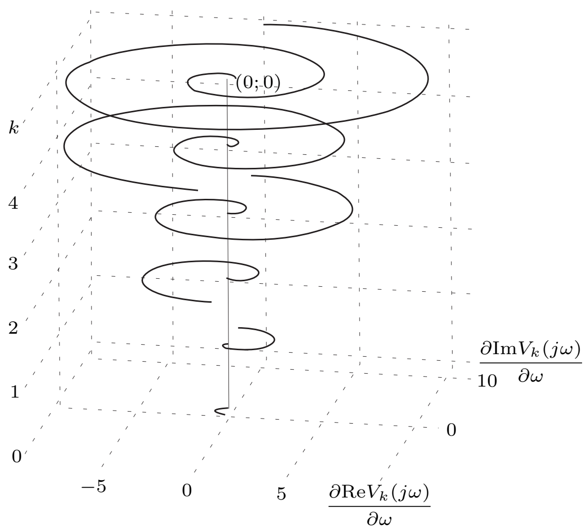


Рис. 6.53. Вид производных преобразования Фурье ортогональных фильтров Лагерра 0-5 порядков; $\gamma = 4$

[6.141]
$$\frac{\partial V_k^{[1]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^2}{(j\omega+\gamma/2)^3}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s(s+2).$$

[6.142] $$\frac{\partial V_k^{[2]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^2\big(2j\omega-\gamma(2k+2)/2\big)}{(j\omega+\gamma/2)^4}\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^{k-1}.$$

[6.143] $$\frac{\partial V_k^{[3]\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{8j}{\gamma}(-1)^k(\cos\varphi)^3\times$$
$$\times\exp(-j(2k+2)\varphi)\big((2k+2)\cos\varphi-2j\sin\varphi\big),$$
$$\varphi=\arctan\frac{2\omega}{\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{L_0^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{2\gamma^2(\omega+j\gamma/2)}{(j\omega-\gamma/2)\big(j\omega+\gamma/2\big)^3};$$

$$\frac{\partial V_1^{\{L_1^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{2\gamma^2(\omega+j\gamma)}{\big(j\omega+\gamma/2\big)^4};$$

$$\frac{\partial V_2^{\{L_2^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{2\gamma^2\big(\omega+3j\gamma/2\big)\big(j\omega-\gamma/2\big)}{\big(j\omega+\gamma/2\big)^5};$$

$$\frac{\partial V_3^{\{L_3^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{2\gamma^2\big(\omega+2j\gamma\big)\big(j\omega-\gamma/2\big)^2}{\big(j\omega+\gamma/2\big)^6};$$

$$\frac{\partial V_4^{\{L_4^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{2\gamma^2\big(\omega+5j\gamma/2\big)\big(j\omega-\gamma/2\big)^3}{\big(j\omega+\gamma/2\big)^7};$$

$$\frac{\partial V_5^{\{L_5^{(1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{2\gamma^2\big(\omega+3j\gamma\big)\big(j\omega-\gamma/2\big)^4}{\big(j\omega+\gamma/2\big)^8}.$$

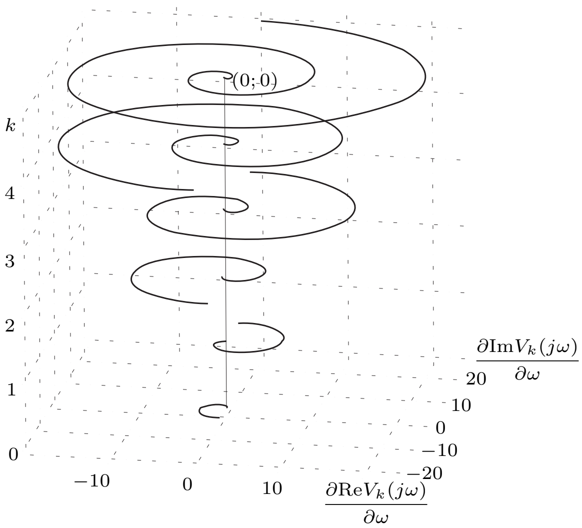


Рис. 6.54. Вид производных преобразования Фурье ортогональных фильтров Сонина-Лагерра 0-5 порядков; $\gamma=4$, $\alpha=1$

[6.144] $$\frac{\partial V_k^{[1]\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^3}{(j\omega+\gamma/2)^4}\sum_{s=0}^{k}\binom{k}{k-s}\left(\frac{-\gamma}{j\omega+\gamma/2}\right)^s(s+3).$$

[6.145] $$\frac{\partial V_k^{[2]\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^3\big(3j\omega-\gamma(2k+3)/2\big)}{(j\omega+\gamma/2)^5}\left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^{k-1}.$$

[6.146] $$\frac{\partial V_k^{[3]\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{16j}{\gamma}(-1)^k(\cos\varphi)^4\times$$
$$\times\exp(-j(2k+3)\varphi)\big((2k+3)\cos\varphi-3j\sin\varphi\big),$$
$$\varphi=\arctan\frac{2\omega}{\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{L_0^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{3\gamma^3(\omega+j\gamma/2)}{(j\omega-\gamma/2)\big(j\omega+\gamma/2\big)^4};$$

$$\frac{\partial V_1^{\{L_1^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^3(3\omega+5j\gamma/2)}{\big(j\omega+\gamma/2\big)^5};$$

$$\frac{\partial V_2^{\{L_2^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^3\big(3\omega+7j\gamma/2\big)\big(j\omega-\gamma/2\big)}{\big(j\omega+\gamma/2\big)^6};$$

$$\frac{\partial V_3^{\{L_3^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^3\big(3\omega+9j\gamma/2\big)\big(j\omega-\gamma/2\big)^2}{\big(j\omega+\gamma/2\big)^7};$$

$$\frac{\partial V_4^{\{L_4^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^3\big(3\omega+11j\gamma/2\big)\big(j\omega-\gamma/2\big)^3}{\big(j\omega+\gamma/2\big)^8};$$

$$\frac{\partial V_5^{\{L_5^{(2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^3\big(3\omega+13j\gamma/2\big)\big(j\omega-\gamma/2\big)^4}{\big(j\omega+\gamma/2\big)^9}.$$

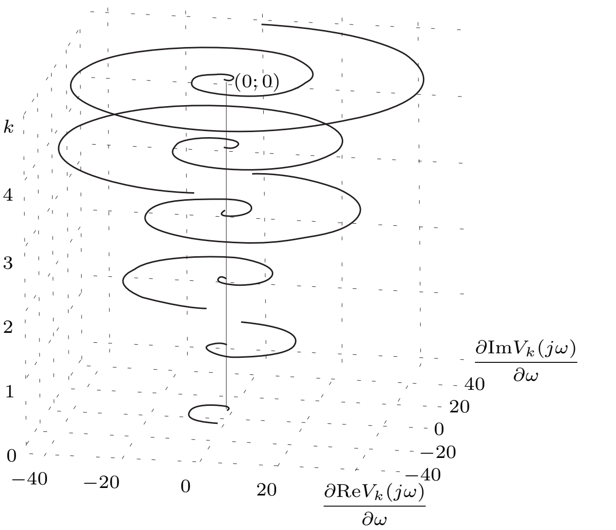


Рис. 6.55. Вид производных преобразования Фурье ортогональных фильтров Сонина-Лагерра 0-5 порядков; $\gamma = 4,\ \alpha = 2$

[6.147]
$$\frac{\partial V_k^{[1]\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^{\alpha+1}}{(j\omega+\gamma/2)^{\alpha+2}} \sum_{s=0}^{k} \binom{k}{k-s} \left(\frac{-\gamma}{j\omega+\gamma/2}\right)^{s} (s+\alpha+1).$$

[6.148]
$$\frac{\partial V_k^{[2]\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -j\gamma^{\alpha+1} \times \frac{\big((\alpha+1)j\omega - \gamma(2k+\alpha+1)/2\big)}{(j\omega+\gamma/2)^{\alpha+3}} \left(\frac{j\omega-\gamma/2}{j\omega+\gamma/2}\right)^{k-1}.$$

[6.149]
$$\frac{\partial V_k^{[3]\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{2^{\alpha+2}j}{\gamma}(-1)^k \times (\cos\varphi)^{\alpha+2}\exp(-j(2k+\alpha+1)\varphi) \times \big((2k+\alpha+1)\cos\varphi - j(\alpha+1)\sin\varphi\big), \quad \varphi = \arctan\frac{2\omega}{\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{L_0^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^{\alpha+1}\big((\alpha+1)\omega + j(\alpha+1)\gamma/2\big)}{(j\omega-\gamma/2)\big(j\omega+\gamma/2\big)^{\alpha+2}};$$

$$\frac{\partial V_1^{\{L_1^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^{\alpha+1}\big((\alpha+1)\omega + j(\alpha+3)\gamma/2\big)}{\big(j\omega+\gamma/2\big)^{\alpha+3}};$$

$$\frac{\partial V_2^{\{L_2^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^{\alpha+1}\big((\alpha+1)\omega + j(\alpha+5)\gamma/2\big)}{\big(j\omega+\gamma/2\big)^{\alpha+4}} \times (j\omega-\gamma/2);$$

$$\frac{\partial V_3^{\{L_3^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^{\alpha+1}\big((\alpha+1)\omega + j(\alpha+7)\gamma/2\big)}{\big(j\omega+\gamma/2\big)^{\alpha+5}} \times (j\omega-\gamma/2)^2;$$

$$\frac{\partial V_4^{\{L_4^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^{\alpha+1}\big((\alpha+1)\omega + j(\alpha+9)\gamma/2\big)}{\big(j\omega+\gamma/2\big)^{\alpha+6}} \times (j\omega-\gamma/2)^3;$$

$$\frac{\partial V_5^{\{L_5^{(\alpha)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \frac{\gamma^{\alpha+1}\big((\alpha+1)\omega + j(\alpha+11)\gamma/2\big)}{\big(j\omega+\gamma/2\big)^{\alpha+7}} \times (j\omega-\gamma/2)^4.$$

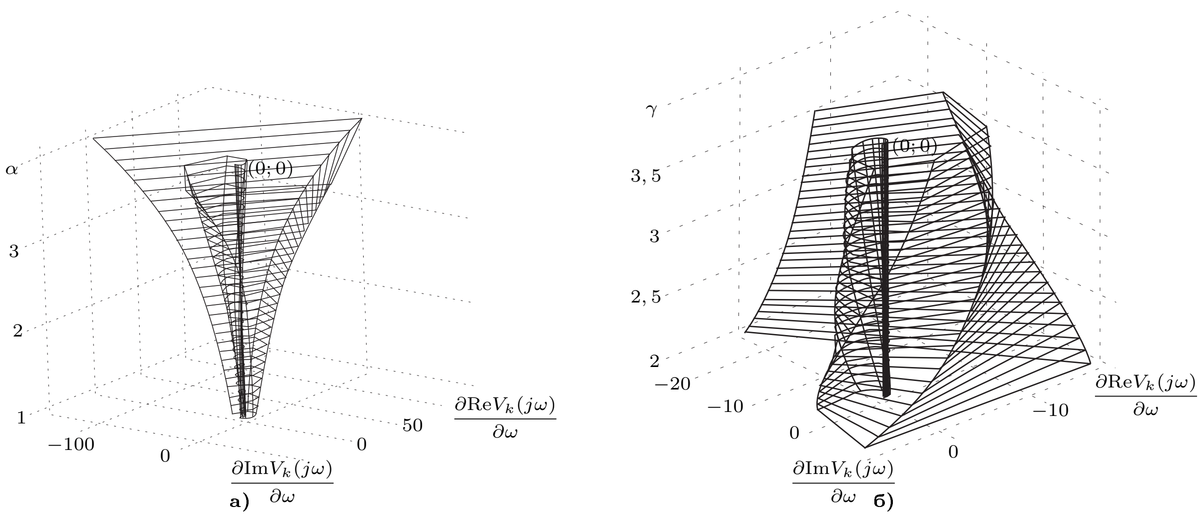


Рис. 6.56. Вид производных преобразования Фурье ортогональных фильтров Сонина-Лагерра 2-ого порядка: а) $\gamma = 4,\ \alpha \in [1; 4]$; б) $\gamma \in [2; 4],\ \alpha = 1$

[6.150]
$$\frac{\partial V_k^{[1]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-j(4k+1)\gamma\times$$
$$\times\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\frac{1}{\big((4s+1)\gamma/2+j\omega\big)^2}.$$

[6.151]
$$\frac{\partial V_k^{[2]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=$$
$$=\begin{cases}-\dfrac{j\gamma}{(\gamma/2+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{j(4k+1)\gamma}{(4k+1)\gamma/2+j\omega}\times\\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(4s+1)\gamma/2-j\omega}{(4s+1)\gamma/2+j\omega}\times\\ \times\Big(\dfrac{1}{(4k+1)\gamma/2+j\omega}+\\ +\gamma\displaystyle\sum_{s=0}^{k-1}\frac{4s+1}{\big((4s+1)\gamma/2\big)^2+\omega^2}\Big), & \text{если } k>0.\end{cases}$$

[6.152]
$$\frac{\partial V_k^{[3]\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=$$
$$=\begin{cases}-\dfrac{4j(\cos\varphi_0)^2}{\gamma}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -2j\cos\varphi_k\times\\ \times\exp\Big(-j\Big(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\Big)\Big)\times\\ \times\Big(\dfrac{2\cos\varphi_k\exp\big(-j\varphi_k\big)}{(4k+1)\gamma}+\\ +\dfrac{4}{\gamma}\displaystyle\sum_{s=0}^{k-1}\frac{(cos\varphi_k)^2}{4s+1}\Big), & \text{если } k>0,\end{cases}$$
$$\varphi_k=\arctan\frac{2\omega}{(4k+1)\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{P_0^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{j\gamma}{(\gamma/2+j\omega)^2};$$
$$\frac{\partial V_1^{\{P_1^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{5j\gamma}{2(\gamma/2+j\omega)^2}+\frac{15j\gamma}{2(5\gamma/2+j\omega)^2};$$
$$\frac{\partial V_2^{\{P_2^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{27j\gamma}{8(\gamma/2+j\omega)^2}+\frac{135j\gamma}{4(5\gamma/2+j\omega)^2}-$$
$$-\frac{315j\gamma}{8(9\gamma/2+j\omega)^2};$$
$$\frac{\partial V_3^{\{P_3^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{65j\gamma}{16(\gamma/2+j\omega)^2}+\frac{1365j\gamma}{16(5\gamma/2+j\omega)^2}-$$
$$-\frac{4095j\gamma}{16(9\gamma/2+j\omega)^2}+\frac{3003j\gamma}{16(13\gamma/2+j\omega)^2};$$
$$\frac{\partial V_4^{\{P_4^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{595j\gamma}{128(\gamma/2+j\omega)^2}+\frac{5355j\gamma}{32}\times$$
$$\times\frac{1}{(5\gamma/2+j\omega)^2}-\frac{58905j\gamma}{64(9\gamma/2+j\omega)^2}+\frac{51051j\gamma}{32(13\gamma/2+j\omega)^2}-$$
$$-\frac{109395j\gamma}{128(17\gamma/2+j\omega)^2};$$
$$\frac{\partial V_5^{\{P_5^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{1323j\gamma}{256(\gamma/2+j\omega)^2}+\frac{72765j\gamma}{256}\times$$
$$\times\frac{1}{(5\gamma/2+j\omega)^2}-\frac{315315j\gamma}{128(9\gamma/2+j\omega)^2}+\frac{945945j\gamma}{128(13\gamma/2+j\omega)^2}-$$
$$-\frac{2297295j\gamma}{256(17\gamma/2+j\omega)^2}+\frac{969969j\gamma}{256(21\gamma/2+j\omega)^2}.$$

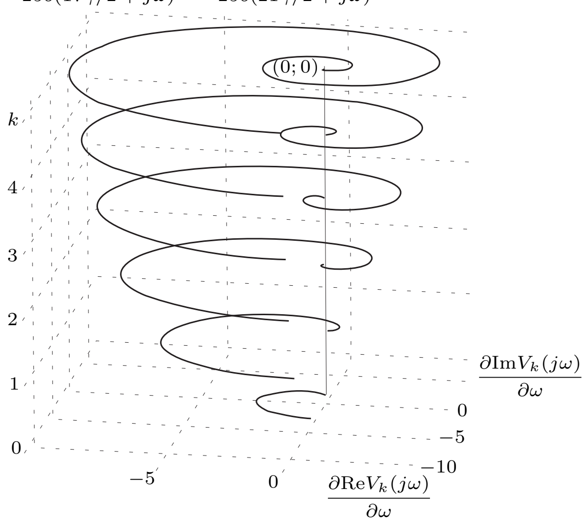


Рис. 6.57. Вид производных преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma=1$, $c=2$, $\alpha=-1/2$, $\beta=0$

[6.153]
$$\frac{\partial V_k^{[1]\{Leg_k(\tau,\gamma)\}}(j\omega)}{\partial\omega}=$$
$$=-2j(2k+1)\gamma\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{\big((2s+1)\gamma+j\omega\big)^2}.$$

[6.154]
$$\frac{\partial V_k^{[2]\{Leg_k(\tau,\gamma)\}}(j\omega)}{\partial\omega}=$$
$$=\begin{cases}-\dfrac{2j\gamma}{(\gamma+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{2j(2k+1)\gamma}{(2k+1)\gamma+j\omega}\times\\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+1)\gamma-j\omega}{(2s+1)\gamma+j\omega}\times\\ \times\Big(\dfrac{1}{(2k+1)\gamma+j\omega}+\\ +2\gamma\displaystyle\sum_{s=0}^{k-1}\frac{2s+1}{\big((2s+1)\gamma\big)^2+\omega^2}\Big), & \text{если } k>0.\end{cases}$$

[6.155]
$$\frac{\partial V_k^{[3]\{Leg_k(\tau,\gamma)\}}(j\omega)}{\partial \omega}=$$
$$=\begin{cases}-\dfrac{2j(\cos\varphi_0)^2}{\gamma}\exp\bigl(-2j\varphi_0\bigr), & \text{если } k=0;\\ -2j\cos\varphi_k\times\\ \times\exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right)\times\\ \times\left(\dfrac{\cos\varphi_k\exp\bigl(-j\varphi_k\bigr)}{(2k+1)\gamma}+\right.\\ \left.+\dfrac{2}{\gamma}\displaystyle\sum_{s=0}^{k-1}\dfrac{(cos\varphi_k)^2}{2s+1}\right), & \text{если } k>0,\end{cases}$$
$$\varphi_k=\arctan\frac{\omega}{(2k+1)\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{Leg_0(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{2j\gamma}{(\gamma+j\omega)^2};$$
$$\frac{\partial V_1^{\{Leg_1(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{6j\gamma}{(\gamma+j\omega)^2}+\frac{12j\gamma}{(3\gamma+j\omega)^2};$$
$$\frac{\partial V_2^{\{Leg_2(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{10j\gamma}{(\gamma+j\omega)^2}+\frac{60j\gamma}{(3\gamma+j\omega)^2}-\frac{6j\gamma}{(5\gamma+j\omega)^2};$$
$$\frac{\partial V_3^{\{Leg_3(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{14j\gamma}{(\gamma+j\omega)^2}+\frac{168j\gamma}{(3\gamma+j\omega)^2}-$$
$$-\frac{420j\gamma}{(5\gamma+j\omega)^2}+\frac{280j\gamma}{(7\gamma+j\omega)^2};$$
$$\frac{\partial V_4^{\{Leg_4(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{18j\gamma}{(\gamma+j\omega)^2}+\frac{360j\gamma}{(3\gamma+j\omega)^2}-$$
$$-\frac{1620j\gamma}{(5\gamma+j\omega)^2}+\frac{2520j\gamma}{(7\gamma+j\omega)^2}-\frac{1260j\gamma}{(9\gamma+j\omega)^2};$$
$$\frac{\partial V_5^{\{Leg_5(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{22j\gamma}{(\gamma+j\omega)^2}+\frac{660j\gamma}{(3\gamma+j\omega)^2}-$$
$$-\frac{4620j\gamma}{(5\gamma+j\omega)^2}+\frac{12320j\gamma}{(7\gamma+j\omega)^2}-\frac{13860j\gamma}{(9\gamma+j\omega)^2}+\frac{5544j\gamma}{(11\gamma+j\omega)^2}.$$

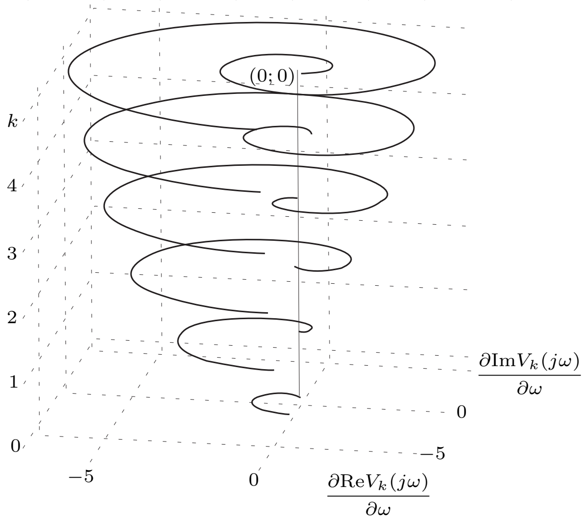


Рис. 6.58. Вид производных преобразования Фурье ортогональных фильтров Лежандра 0-5 порядков; $\gamma=1$, $c=2$

[6.156]
$$\frac{\partial V_k^{[1]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-j(4k+3)\gamma\times$$
$$\times\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s\frac{1}{\bigl((4s+3)\gamma/2+j\omega\bigr)^2}.$$

[6.157]
$$\frac{\partial V_k^{[2]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=$$
$$=\begin{cases}-\dfrac{3j\gamma}{(3\gamma/2+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{j(4k+3)\gamma}{(4k+3)\gamma/2+j\omega}\times\\ \times\displaystyle\prod_{s=0}^{k-1}\dfrac{(4s+3)\gamma/2-j\omega}{(4s+3)\gamma/2+j\omega}\times\\ \times\left(\dfrac{1}{(4k+3)\gamma/2+j\omega}+\right.\\ \left.+\gamma\displaystyle\sum_{s=0}^{k-1}\dfrac{4s+3}{\bigl((4s+3)\gamma/2\bigr)^2+\omega^2}\right), & \text{если } k>0.\end{cases}$$

[6.158]
$$\frac{\partial V_k^{[3]\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=$$
$$=\begin{cases}-\dfrac{4j(\cos\varphi_0)^2}{3\gamma}\exp\bigl(-2j\varphi_0\bigr), & \text{если } k=0;\\ -2j\cos\varphi_k\times\\ \times\exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right)\times\\ \times\left(\dfrac{2\cos\varphi_k\exp\bigl(-j\varphi_k\bigr)}{(4k+3)\gamma}+\right.\\ \left.+\dfrac{4}{\gamma}\displaystyle\sum_{s=0}^{k-1}\dfrac{(cos\varphi_k)^2}{4s+3}\right), & \text{если } k>0,\end{cases}$$
$$\varphi_k=\arctan\frac{2\omega}{(4k+3)\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{P_0^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{3j\gamma}{(3\gamma/2+j\omega)^2};$$
$$\frac{\partial V_1^{\{P_1^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{35j\gamma}{2(3\gamma/2+j\omega)^2}+\frac{105j\gamma}{2(7\gamma/2+j\omega)^2};$$
$$\frac{\partial V_2^{\{P_2^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{297j\gamma}{8(3\gamma/2+j\omega)^2}+\frac{1485j\gamma}{4(7\gamma/2+j\omega)^2}-$$
$$-\frac{3465j\gamma}{8(11\gamma/2+j\omega)^2};$$
$$\frac{\partial V_3^{\{P_3^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{975j\gamma}{16(3\gamma/2+j\omega)^2}+\frac{20475j\gamma}{16(7\gamma/2+j\omega)^2}-$$
$$-\frac{61425j\gamma}{16(11\gamma/2+j\omega)^2}+\frac{45045j\gamma}{16(15\gamma/2+j\omega)^2};$$
$$\frac{\partial V_4^{\{P_4^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega}=-\frac{11305j\gamma}{128(3\gamma/2+j\omega)^2}+\frac{101745j\gamma}{32}\times$$
$$\times\frac{1}{(7\gamma/2+j\omega)^2}-\frac{1119195j\gamma}{64(11\gamma/2+j\omega)^2}+\frac{969969j\gamma}{32(15\gamma/2+j\omega)^2}-$$
$$-\frac{2078505j\gamma}{128(19\gamma/2+j\omega)^2};$$

$$\frac{\partial V_5^{\{P_5^{(1/2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{30429j\gamma}{256(3\gamma/2+j\omega)^2} + \frac{1673595j\gamma}{256}\times$$
$$\times\frac{1}{(7\gamma/2+j\omega)^2} - \frac{7252245j\gamma}{128(11\gamma/2+j\omega)^2} + \frac{21756735j\gamma}{128(15\gamma/2+j\omega)^2} -$$
$$-\frac{52837785j\gamma}{256(19\gamma/2+j\omega)^2} + \frac{22309287j\gamma}{256(23\gamma/2+j\omega)^2}.$$

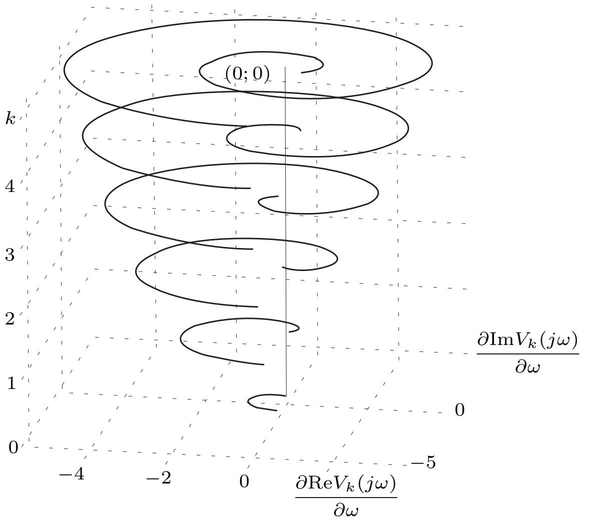


Рис. 6.59. Вид производных преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 1/2$, $\beta = 0$

[6.159]
$$\frac{\partial V_k^{[1]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -2j(k+1)\gamma\times$$
$$\times\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s\frac{1}{\big((s+1)\gamma+j\omega\big)^2}.$$

[6.160]
$$\frac{\partial V_k^{[2]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} =$$
$$= \begin{cases} -\dfrac{2j\gamma}{(\gamma+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{2j(k+1)\gamma}{(k+1)\gamma+j\omega}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(s+1)\gamma-j\omega}{(s+1)\gamma+j\omega}\times \\ \times\Big(\dfrac{1}{(k+1)\gamma+j\omega}+ \\ +2\gamma\displaystyle\sum_{s=0}^{k-1}\frac{s+1}{\big((s+1)\gamma\big)^2+\omega^2}\Big), & \text{если } k>0. \end{cases}$$

[6.161]
$$\frac{\partial V_k^{[3]\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} =$$
$$= \begin{cases} -\dfrac{2j(\cos\varphi_0)^2}{\gamma}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -2j\cos\varphi_k\times \\ \times\exp\left(-j\left(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\right)\right)\times \\ \times\Big(\dfrac{\cos\varphi_k\exp\big(-j\varphi_k\big)}{(k+1)\gamma}+ \\ +\dfrac{2}{\gamma}\displaystyle\sum_{s=0}^{k-1}\frac{(cos\varphi_k)^2}{s+1}\Big), & \text{если } k>0, \end{cases}$$
$$\varphi_k = \arctan\frac{\omega}{(k+1)\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{P_0^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{2j\gamma}{(\gamma+j\omega)^2};$$

$$\frac{\partial V_1^{\{P_1^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{8j\gamma}{(\gamma+j\omega)^2} + \frac{12j\gamma}{(2\gamma+j\omega)^2};$$

$$\frac{\partial V_2^{\{P_2^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{18j\gamma}{(\gamma+j\omega)^2} + \frac{72j\gamma}{(2\gamma+j\omega)^2} - \frac{60j\gamma}{(3\gamma+j\omega)^2};$$

$$\frac{\partial V_3^{\{P_3^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{32j\gamma}{(\gamma+j\omega)^2} + \frac{240j\gamma}{(2\gamma+j\omega)^2} -$$
$$-\frac{480j\gamma}{(3\gamma+j\omega)^2} + \frac{280j\gamma}{(4\gamma+j\omega)^2};$$

$$\frac{\partial V_4^{\{P_4^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{50j\gamma}{(\gamma+j\omega)^2} + \frac{600j\gamma}{(2\gamma+j\omega)^2} -$$
$$-\frac{2100j\gamma}{(3\gamma+j\omega)^2} + \frac{2800j\gamma}{(4\gamma+j\omega)^2} - \frac{1260j\gamma}{(5\gamma+j\omega)^2};$$

$$\frac{\partial V_5^{\{P_5^{(1,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{72j\gamma}{(\gamma+j\omega)^2} + \frac{1260j\gamma}{(2\gamma+j\omega)^2} -$$
$$-\frac{6720j\gamma}{(3\gamma+j\omega)^2} + \frac{15120j\gamma}{(4\gamma+j\omega)^2} - \frac{15120j\gamma}{(5\gamma+j\omega)^2} + \frac{5544j\gamma}{(6\gamma+j\omega)^2}.$$

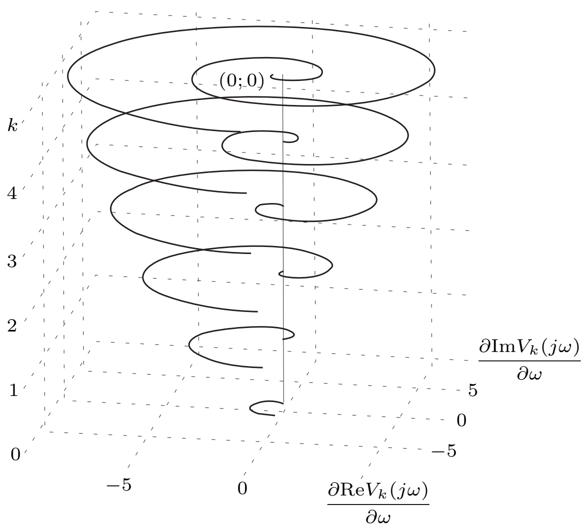


Рис. 6.60. Вид производных преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 1$, $\alpha = 1$, $\beta = 0$

[6.162]
$$\frac{\partial V_k^{[1]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -2j(2k+3)\gamma\times$$
$$\times\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s\frac{1}{\big((2s+3)\gamma+j\omega\big)^2}.$$

[6.163]
$$\frac{\partial V_k^{[2]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} =$$
$$=\begin{cases} -\dfrac{2j\gamma}{(\gamma+j\omega)^2}, & \text{если } k=0;\\ -\dfrac{2j(2k+3)\gamma}{(2k+3)\gamma+j\omega}\times \\ \times\displaystyle\prod_{s=0}^{k-1}\frac{(2s+3)\gamma-j\omega}{(2s+3)\gamma+j\omega}\times \\ \times\Bigg(\dfrac{1}{(2k+3)\gamma+j\omega}+ \\ +2\gamma\displaystyle\sum_{s=0}^{k-1}\frac{2s+3}{\big((2s+3)\gamma\big)^2+\omega^2}\Bigg), & \text{если } k>0. \end{cases}$$

[6.164]
$$\frac{\partial V_k^{[3]\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} =$$
$$=\begin{cases} -\dfrac{2j(\cos\varphi_0)^2}{\gamma}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -2j\cos\varphi_k\times \\ \times\exp\Bigg(-j\Bigg(\varphi_k+2\displaystyle\sum_{s=0}^{k-1}\varphi_s\Bigg)\Bigg)\times \\ \times\Bigg(\dfrac{\cos\varphi_k\exp\big(-j\varphi_k\big)}{(2k+3)\gamma}+ \\ +\dfrac{2}{\gamma}\displaystyle\sum_{s=0}^{k-1}\frac{(cos\varphi_k)^2}{2s+3}\Bigg), & \text{если } k>0, \end{cases}$$
$$\varphi_k = \arctan\frac{\omega}{(2k+3)\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{P_0^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{6j\gamma}{(3\gamma+j\omega)^2};$$
$$\frac{\partial V_1^{\{P_1^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{30j\gamma}{(3\gamma+j\omega)^2}+\frac{40j\gamma}{(5\gamma+j\omega)^2};$$
$$\frac{\partial V_2^{\{P_2^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{84j\gamma}{(3\gamma+j\omega)^2}+\frac{280j\gamma}{(5\gamma+j\omega)^2}-\frac{210j\gamma}{(7\gamma+j\omega)^2};$$
$$\frac{\partial V_3^{\{P_3^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{180j\gamma}{(3\gamma+j\omega)^2}+\frac{1080j\gamma}{(5\gamma+j\omega)^2}-$$
$$-\frac{1890j\gamma}{(7\gamma+j\omega)^2}+\frac{1008j\gamma}{(9\gamma+j\omega)^2};$$
$$\frac{\partial V_4^{\{P_4^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{330j}{(3\gamma+j\omega)^2}+\frac{3080j}{(5\gamma+j\omega)^2}-$$
$$-\frac{9240j\gamma}{(7\gamma+j\omega)^2}+\frac{11880j\gamma}{(9\gamma+j\omega)^2}-\frac{4620j\gamma}{(11\gamma+j\omega)^2};$$
$$\frac{\partial V_5^{\{P_5^{(2,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{546j\gamma}{(3\gamma+j\omega)^2}+\frac{7280j\gamma}{(5\gamma+j\omega)^2}-$$
$$-\frac{32760j\gamma}{(7\gamma+j\omega)^2}+\frac{65520j\gamma}{(9\gamma+j\omega)^2}-\frac{60060j\gamma}{(11\gamma+j\omega)^2}+\frac{20592j\gamma}{(13\gamma+j\omega)^2}.$$

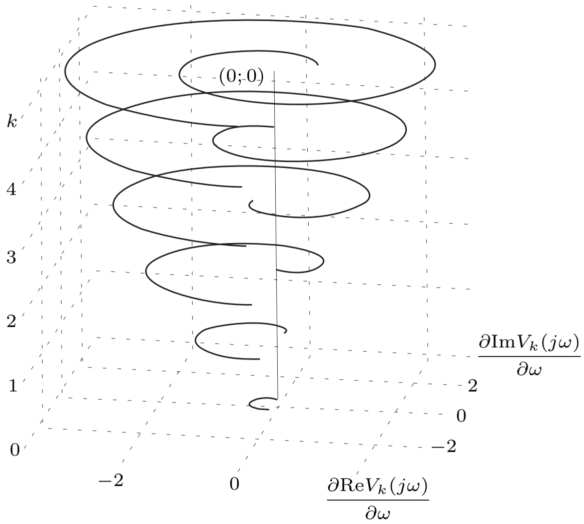


Рис. 6.61. Вид производных преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 2$, $\beta = 0$

[6.165]

$$\frac{\partial V_k^{[1]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -j(2k+\alpha+1)c\gamma\sum_{s=0}^{k}\binom{k}{s}\times \binom{k+s+\alpha}{s+\alpha}(-1)^s\frac{1}{\big((2s+\alpha+1)c\gamma/2+j\omega\big)^2}.$$

[6.166]

$$\frac{\partial V_k^{[2]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{j(\alpha+1)c\gamma}{\big((\alpha+1)c\gamma/2+j\omega\big)^2}, & \text{если } k=0;\\ -\dfrac{j(2k+\alpha+1)c\gamma}{(2k+\alpha+1)c\gamma/2+j\omega}\times \prod_{s=0}^{k-1}\dfrac{(2s+\alpha+1)c\gamma/2-j\omega}{(2s+\alpha+1)c\gamma/2+j\omega}\times \left(\dfrac{1}{(2k+\alpha+1)c\gamma/2+j\omega}+2c\gamma/2\times \sum_{s=0}^{k-1}\dfrac{2s+\alpha+1}{\big((2s+\alpha+1)c\gamma/2\big)^2+\omega^2}\right), & \text{если } k>0. \end{cases}$$

[6.167]

$$\frac{\partial V_k^{[3]\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = \begin{cases} -\dfrac{4j(\cos\varphi_0)^2}{(\alpha+1)c\gamma}\exp\big(-2j\varphi_0\big), & \text{если } k=0;\\ -2j\cos\varphi_k\times \exp\left(-j\left(\varphi_k+2\sum_{s=0}^{k-1}\varphi_s\right)\right)\times \left(\dfrac{\cos\varphi_k\exp\big(-j\varphi_k\big)}{(2k+\alpha+1)c\gamma/2}+ +\dfrac{4}{c\gamma}\sum_{s=0}^{k-1}\dfrac{(cos\varphi_k)^2}{2s+\alpha+1}\right), & \text{если } k>0, \end{cases}$$

$$\varphi_k = \arctan\frac{2\omega}{(2k+\alpha+1)c\gamma}.$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{P_0^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{jc\gamma(\alpha+1)}{\big(c\gamma(\alpha+1)/2+j\omega\big)^2};$$

$$\frac{\partial V_1^{\{P_1^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{jc\gamma(\alpha+1)(\alpha+3)}{\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+jc\gamma(\alpha+2)\times \times\frac{(\alpha+3)}{\big(c\gamma(\alpha+3)/2+j\omega\big)^2};$$

$$\frac{\partial V_2^{\{P_2^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{jc\gamma(\alpha+1)(\alpha+2)(\alpha+5)}{2\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+jc\gamma\times \times\frac{(\alpha+2)(\alpha+3)(\alpha+5)}{2\big(c\gamma(\alpha+3)/2+j\omega\big)^2}-\frac{jc\gamma(\alpha+3)(\alpha+4)(\alpha+5)}{2\big(c\gamma(\alpha+5)/2+j\omega\big)^2};$$

$$\frac{\partial V_3^{\{P_3^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{jc\gamma(\alpha+3)!(\alpha+7)}{6\alpha!\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+jc\gamma(\alpha+7)\times \times\frac{(\alpha+4)!}{2(\alpha+1)!\big(c\gamma(\alpha+3)/2+j\omega\big)^2}-\frac{jc\gamma(\alpha+5)!(\alpha+7)}{2(\alpha+2)!\big(c\gamma(\alpha+5)/2+j\omega\big)^2}+ +\frac{jc\gamma(\alpha+6)!(\alpha+7)}{6(\alpha+3)!\big(c\gamma(\alpha+7)/2+j\omega\big)^2};$$

$$\frac{\partial V_4^{\{P_4^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{jc\gamma(\alpha+4)!(\alpha+9)}{24\alpha!\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+\frac{j}{6}\times \times\frac{c\gamma(\alpha+5)!(\alpha+9)}{(\alpha+1)!\big(c\gamma(\alpha+3)/2+j\omega\big)^2}-\frac{jc\gamma(\alpha+6)!(\alpha+9)}{4(\alpha+2)!\big(c\gamma(\alpha+5)/2+j\omega\big)^2}+ +\frac{jc\gamma(\alpha+7)!(\alpha+9)}{6(\alpha+3)!\big(c\gamma(\alpha+7)/2+j\omega\big)^2}-\frac{jc\gamma(\alpha+8)!(\alpha+9)}{24(\alpha+4)!\big(c\gamma(\alpha+9)/2+j\omega\big)^2};$$

$$\frac{\partial V_5^{\{P_5^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{jc\gamma(\alpha+5)!(\alpha+11)}{120\alpha!\big(c\gamma(\alpha+1)/2+j\omega\big)^2}+\frac{j}{24}\times \times\frac{c\gamma(\alpha+6)!(\alpha+11)}{(\alpha+1)!\big(c\gamma(\alpha+3)/2+j\omega\big)^2}-\frac{jc\gamma(\alpha+7)!(\alpha+11)}{12(\alpha+2)!\big(c\gamma(\alpha+5)/2+j\omega\big)^2}+ +\frac{jc\gamma(\alpha+8)!(\alpha+11)}{12(\alpha+3)!\big(c\gamma(\alpha+7)/2+j\omega\big)^2}-\frac{jc\gamma(\alpha+9)!(\alpha+11)}{24(\alpha+4)!\big(c\gamma(\alpha+9)/2+j\omega\big)^2}+ +\frac{jc\gamma(\alpha+10)!(\alpha+11)}{120(\alpha+5)!\big(c\gamma(\alpha+11)/2+j\omega\big)^2}.$$

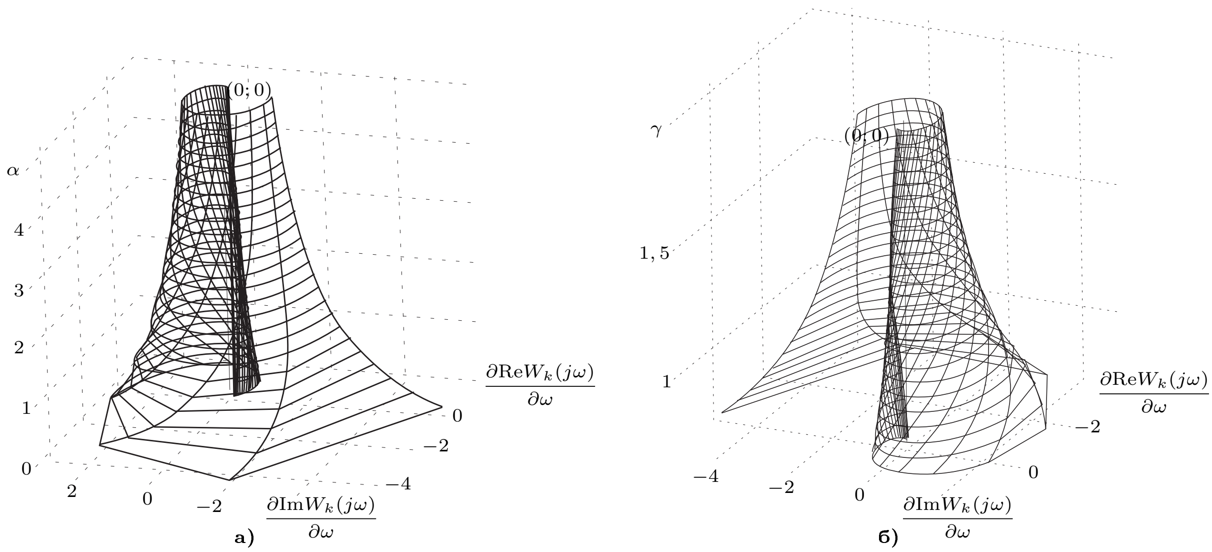


Рис. 6.62. Вид производных преобразования Фурье ортогональных фильтров Якоби 2-ого порядка: а) $\gamma = 1$, $c = 2$, $\alpha \in [0; 5]$, $\beta = 0$; б) $\gamma \in [0,75; 2]$, $c = 2$, $\alpha = 1$, $\beta = 0$

[6.168]
$$\frac{\partial V_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{8j\gamma^2(k+1)^2}{(2k+3)\gamma+j\omega}\times$$
$$\times\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{(2s+1)\gamma+j\omega}\times$$
$$\times\left(\frac{1}{(2s+1)\gamma+j\omega}+\frac{1}{(2k+3)\gamma+j\omega}\right).$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{P_0^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{16j\gamma^2(2\gamma+j\omega)}{(\gamma+j\omega)^2(3\gamma+j\omega)^2};$$

$$\frac{\partial V_1^{\{P_1^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{32j\gamma^2}{(5\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{5\gamma+j\omega}\right)\times\right.$$
$$\left.\times\ \frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{5\gamma+j\omega}\right)\frac{2}{3\gamma+j\omega}\right);$$

$$\frac{\partial V_2^{\{P_2^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{72j\gamma^2}{(7\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{7\gamma+j\omega}\right)\times\right.$$
$$\times\ \frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{7\gamma+j\omega}\right)\frac{6}{3\gamma+j\omega}+\left(\frac{1}{5\gamma+j\omega}\ +\right.$$
$$\left.\left.+\ \frac{1}{7\gamma+j\omega}\right)\frac{6}{5\gamma+j\omega}\right);$$

$$\frac{\partial V_3^{\{P_3^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{128j\gamma^2}{(9\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{9\gamma+j\omega}\right)\times\right.$$
$$\times\ \frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{9\gamma+j\omega}\right)\frac{12}{3\gamma+j\omega}+\left(\frac{1}{5\gamma+j\omega}\ +\right.$$
$$\left.\left.+\ \frac{1}{9\gamma+j\omega}\right)\frac{30}{5\gamma+j\omega}-\left(\frac{1}{7\gamma+j\omega}+\frac{1}{9\gamma+j\omega}\right)\frac{20}{7\gamma+j\omega}\right);$$

$$\frac{\partial V_4^{\{P_4^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{200j\gamma^2}{(11\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{11\gamma+j\omega}\right)\times\right.$$
$$\times\ \frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{11\gamma+j\omega}\right)\frac{20}{3\gamma+j\omega}+\left(\frac{1}{5\gamma+j\omega}\ +\right.$$
$$+\ \frac{1}{11\gamma+j\omega}\right)\frac{90}{5\gamma+j\omega}-\left(\frac{1}{7\gamma+j\omega}+\frac{1}{11\gamma+j\omega}\right)\frac{140}{7\gamma+j\omega}\ +$$
$$\left.+\left(\frac{1}{9\gamma+j\omega}+\frac{1}{11\gamma+j\omega}\right)\frac{70}{11\gamma+j\omega}\right);$$

$$\frac{\partial V_5^{\{P_5^{(0,1)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{288j\gamma^2}{(13\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\times\right.$$
$$\times\ \frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\frac{30}{3\gamma+j\omega}+\left(\frac{1}{5\gamma+j\omega}\ +\right.$$
$$+\ \frac{1}{13\gamma+j\omega}\right)\frac{210}{5\gamma+j\omega}-\left(\frac{1}{7\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\frac{560}{7\gamma+j\omega}\ +$$
$$+\left(\frac{1}{9\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\frac{630}{9\gamma+j\omega}-\left(\frac{1}{11\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\times$$
$$\left.\times\ \frac{252}{11\gamma+j\omega}\right).$$

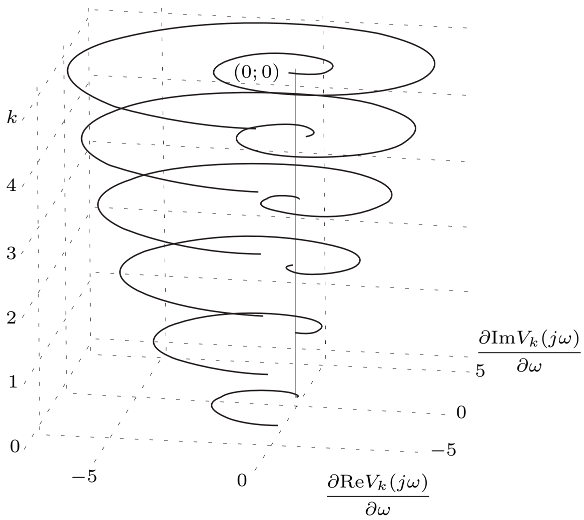


Рис. 6.63. Вид производных преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta = 1$

[6.169]
$$\frac{\partial V_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{8j\gamma^3(2k+3)}{((2k+3)\gamma+j\omega)}\times \frac{(k+1)(k+2)}{((2k+5)\gamma+j\omega)}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\times \frac{1}{(2s+1)\gamma+j\omega}\left(\frac{1}{(2s+1)\gamma+j\omega}+\frac{1}{(2k+3)\gamma+j\omega}+\frac{1}{(2k+5)\gamma+j\omega}\right).$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{P_0^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{144j\gamma^3(3\gamma+j\omega)}{(\gamma+j\omega)^2(3\gamma+j\omega)^2(5\gamma+j\omega)^2};$$

$$\frac{\partial V_1^{\{P_1^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{240j\gamma^3}{(5\gamma+j\omega)(7\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{5\gamma+j\omega}+\frac{1}{7\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{5\gamma+j\omega}+\frac{1}{7\gamma+j\omega}\right)\frac{2}{3\gamma+j\omega}\right);$$

$$\frac{\partial V_2^{\{P_2^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{672j\gamma^3}{(7\gamma+j\omega)(9\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{7\gamma+j\omega}+\frac{1}{9\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{7\gamma+j\omega}+\frac{1}{9\gamma+j\omega}\right)\frac{6}{3\gamma+j\omega}+\left(\frac{1}{5\gamma+j\omega}+\frac{1}{7\gamma+j\omega}+\frac{1}{9\gamma+j\omega}\right)\times\frac{6}{5\gamma+j\omega}\right);$$

$$\frac{\partial V_3^{\{P_3^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{1440j\gamma^3}{(9\gamma+j\omega)(11\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{9\gamma+j\omega}+\frac{1}{11\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{9\gamma+j\omega}+\frac{1}{11\gamma+j\omega}\right)\frac{12}{3\gamma+j\omega}+\left(\frac{1}{5\gamma+j\omega}+\frac{1}{9\gamma+j\omega}+\frac{1}{11\gamma+j\omega}\right)\times\frac{30}{5\gamma+j\omega}-\left(\frac{1}{7\gamma+j\omega}+\frac{1}{9\gamma+j\omega}+\frac{1}{11\gamma+j\omega}\right)\frac{20}{7\gamma+j\omega}\right);$$

$$\frac{\partial V_4^{\{P_4^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{2640j\gamma^3}{(11\gamma+j\omega)(13\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{11\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{11\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\frac{20}{3\gamma+j\omega}+\left(\frac{1}{5\gamma+j\omega}+\frac{1}{11\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\times\frac{90}{5\gamma+j\omega}-\left(\frac{1}{7\gamma+j\omega}+\frac{1}{11\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\frac{140}{7\gamma+j\omega}+\left(\frac{1}{9\gamma+j\omega}+\frac{1}{11\gamma+j\omega}+\frac{1}{13\gamma+j\omega}\right)\frac{70}{11\gamma+j\omega}\right);$$

$$\frac{\partial V_5^{\{P_5^{(0,2)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{4368j\gamma^3}{(13\gamma+j\omega)(15\gamma+j\omega)}\left(\left(\frac{1}{\gamma+j\omega}+\frac{1}{13\gamma+j\omega}+\frac{1}{15\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\left(\frac{1}{3\gamma+j\omega}+\frac{1}{13\gamma+j\omega}+\frac{1}{15\gamma+j\omega}\right)\frac{30}{3\gamma+j\omega}+\left(\frac{1}{5\gamma+j\omega}+\frac{1}{13\gamma+j\omega}+\frac{1}{15\gamma+j\omega}\right)\times\frac{210}{5\gamma+j\omega}-\left(\frac{1}{7\gamma+j\omega}+\frac{1}{13\gamma+j\omega}+\frac{1}{15\gamma+j\omega}\right)\frac{560}{7\gamma+j\omega}+\left(\frac{1}{9\gamma+j\omega}+\frac{1}{13\gamma+j\omega}+\frac{1}{15\gamma+j\omega}\right)\frac{630}{9\gamma+j\omega}-\left(\frac{1}{11\gamma+j\omega}+\frac{1}{13\gamma+j\omega}+\frac{1}{15\gamma+j\omega}\right)\frac{252}{11\gamma+j\omega}\right).$$

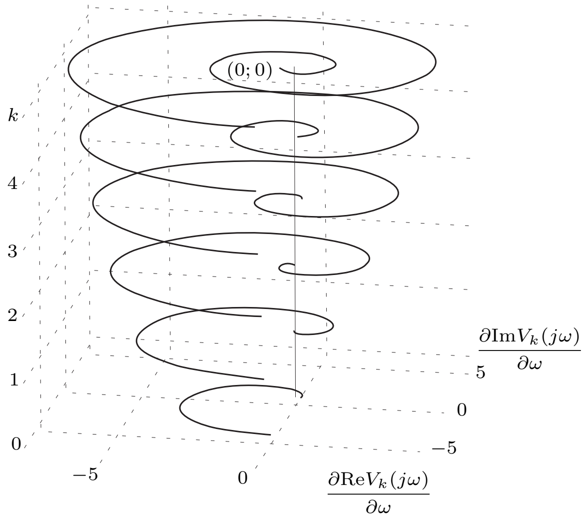


Рис. 6.64. Вид производных преобразования Фурье ортогональных фильтров Якоби 0-5 порядков; $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta = 2$

[6.170]
$$\frac{\partial V_k^{\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^{\beta+1}(k+\beta)!}{k!}\times$$
$$\times\frac{(2k+\beta+1)}{\prod\limits_{p=0}^{\beta}(2k+2p+1)\gamma+j\omega}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\times$$
$$\times\frac{1}{(2s+1)\gamma+j\omega}\left(\frac{1}{(2s+1)\gamma+j\omega}+\right.$$
$$\left.+\sum_{p=0}^{\beta}\frac{1}{(2k+2p+1)\gamma+j\omega}\right).$$

Частные случаи для производных преобразования Фурье фильтров 0-5 порядков:

$$\frac{\partial V_0^{\{P_0^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^{\beta+1}(\beta+1)!}{\prod\limits_{p=0}^{\beta}(2p+1)\gamma+j\omega}\left(\frac{1}{\gamma+j\omega}+\right.$$
$$\left.+\sum_{p=0}^{\beta}\frac{1}{(2p+1)\gamma+j\omega}\right)\frac{1}{\gamma+j\omega};$$

$$\frac{\partial V_1^{\{P_1^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^{\beta+1}(\beta+1)!(\beta+3)}{\prod\limits_{p=0}^{\beta}(2p+3)\gamma+j\omega}\times$$
$$\times\left(\left(\frac{1}{\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+3)\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\right.$$
$$\left.-\left(\frac{1}{3\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+3)\gamma+j\omega}\right)\frac{2}{3\gamma+j\omega}\right);$$

$$\frac{\partial V_2^{\{P_2^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^{\beta+1}(\beta+2)!(\beta+5)}{2\prod\limits_{p=0}^{\beta}(2p+5)\gamma+j\omega}\times$$
$$\times\left(\left(\frac{1}{\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+5)\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\right.$$
$$-\left(\frac{1}{3\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+5)\gamma+j\omega}\right)\frac{6}{3\gamma+j\omega}+$$
$$\left.+\left(\frac{1}{5\gamma+j\omega}\sum_{p=0}^{\beta}\frac{1}{(2p+5)\gamma+j\omega}\right)\frac{6}{5\gamma+j\omega}\right);$$

$$\frac{\partial V_3^{\{P_3^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^{\beta+1}(\beta+3)!(\beta+7)}{6\prod\limits_{p=0}^{\beta}(2p+7)\gamma+j\omega}\times$$
$$\times\left(\left(\frac{1}{\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+7)\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\right.$$
$$-\left(\frac{1}{3\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+7)\gamma+j\omega}\right)\frac{12}{3\gamma+j\omega}+$$
$$+\left(\frac{1}{5\gamma+j\omega}\sum_{p=0}^{\beta}\frac{1}{(2p+7)\gamma+j\omega}\right)\frac{30}{5\gamma+j\omega}-$$
$$\left.-\left(\frac{1}{7\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+7)\gamma+j\omega}\right)\frac{20}{7\gamma+j\omega}\right);$$

$$\frac{\partial V_4^{\{P_4^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^{\beta+1}(\beta+4)!(\beta+9)}{24\prod\limits_{p=0}^{\beta}(2p+9)\gamma+j\omega}\times$$
$$\times\left(\left(\frac{1}{\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+9)\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\right.$$
$$-\left(\frac{1}{3\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+9)\gamma+j\omega}\right)\frac{20}{3\gamma+j\omega}+$$
$$+\left(\frac{1}{5\gamma+j\omega}\sum_{p=0}^{\beta}\frac{1}{(2p+9)\gamma+j\omega}\right)\frac{90}{5\gamma+j\omega}-$$
$$-\left(\frac{1}{7\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+9)\gamma+j\omega}\right)\frac{140}{7\gamma+j\omega}+$$
$$\left.+\left(\frac{1}{9\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+9)\gamma+j\omega}\right)\frac{70}{11\gamma+j\omega}\right);$$

$$\frac{\partial V_5^{\{P_5^{(0,\beta)}(\tau,\gamma)\}}(j\omega)}{\partial\omega} = -\frac{j\gamma^{\beta+1}(\beta+5)!(\beta+11)}{120\prod\limits_{p=0}^{\beta}(2p+11)\gamma+j\omega}\times$$
$$\times\left(\left(\frac{1}{\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+11)\gamma+j\omega}\right)\frac{1}{\gamma+j\omega}-\right.$$
$$-\left(\frac{1}{3\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+11)\gamma+j\omega}\right)\frac{30}{3\gamma+j\omega}+$$
$$+\left(\frac{1}{5\gamma+j\omega}\sum_{p=0}^{\beta}\frac{1}{(2p+11)\gamma+j\omega}\right)\frac{210}{5\gamma+j\omega}-$$
$$-\left(\frac{1}{7\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+11)\gamma+j\omega}\right)\frac{560}{7\gamma+j\omega}+$$
$$+\left(\frac{1}{9\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+11)\gamma+j\omega}\right)\frac{630}{9\gamma+j\omega}-$$
$$\left.-\left(\frac{1}{11\gamma+j\omega}+\sum_{p=0}^{\beta}\frac{1}{(2p+11)\gamma+j\omega}\right)\frac{252}{11\gamma+j\omega}\right).$$

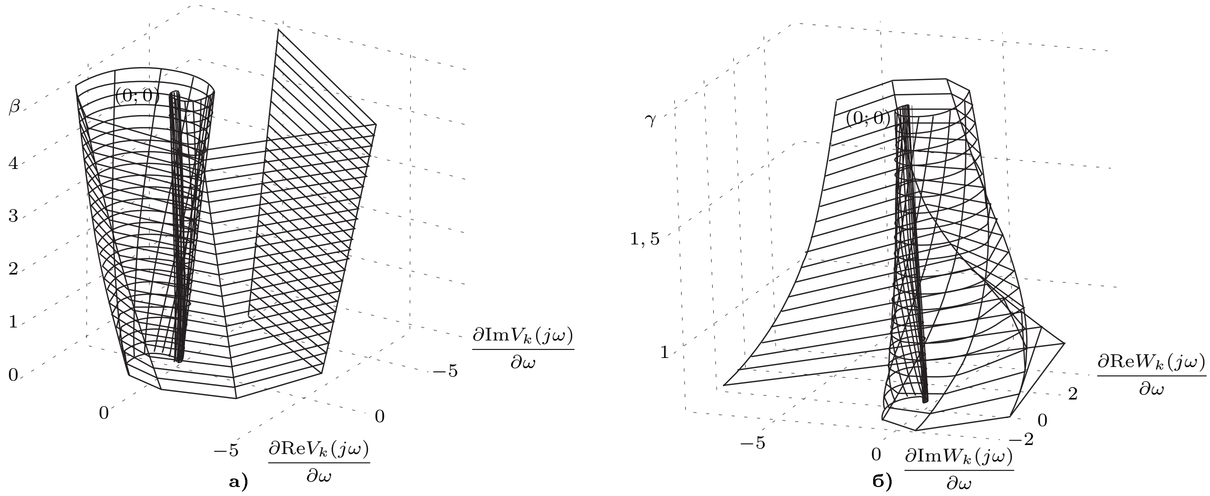


Рис. 6.65. Вид производных преобразования Фурье ортогональных фильтров Якоби 2-ого порядка: а) $\gamma = 1$, $c = 2$, $\alpha = 0$, $\beta \in [0; 5]$; б) $\gamma \in [0,75; 2]$, $c = 2$, $\alpha = 0$, $\beta = 1$

Глава 7

Основные и расширенные свойства в частотной области

Определение.

Ортогональные функции в частотной области, как и во временной, обладают рядом общих свойств [5]

$$\begin{cases} \int_0^\infty \mathrm{Re} W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)d\omega = \frac{\pi}{2}\psi_k(0,\gamma); \\ \int_0^\infty \mathrm{Im} W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)d\omega = 0. \end{cases}$$

Как и во временной, в частотной области были выделены дополнительные свойства [5]

$$\begin{cases} \mathrm{Re} W_k^{\left\{\frac{\partial \psi_k(\tau,\gamma)}{\partial \tau}\right\}}(0) = -\psi_k(0,\gamma); \\ \mathrm{Im} W_k^{\left\{\frac{\partial \psi_k(\tau,\gamma)}{\partial \tau}\right\}}(0) = 0. \end{cases}$$

Таблица 7.1. Основные и расширенные свойства в частотной области

$\psi_k(\tau,\gamma)$	$\int\limits_0^\infty \mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega$	$\int\limits_0^\infty \mathrm{Im}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega$	$\mathrm{Re}W_k^{\left\{\frac{\partial\psi_k(\tau,\gamma)}{\partial\tau}\right\}}(0)$	$\mathrm{Im}W_k^{\left\{\frac{\partial\psi_k(\tau,\gamma)}{\partial\tau}\right\}}(0)$
$L_k(\tau,\gamma)$	$\frac{\pi}{2}$	0	-1	0
$L_k^{(1)}(\tau,\gamma)$	$\frac{\pi(k+1)}{2}$	0	$-k-1$	0
$L_k^{(2)}(\tau,\gamma)$	$\frac{\pi(k+1)(k+2)}{4}$	0	$-\frac{(k+1)(k+2)}{2}$	0
$L_k^{(\alpha)}(\tau,\gamma)$	$\frac{\pi}{2}\binom{k+\alpha}{\alpha}$	0	$-\binom{k+\alpha}{\alpha}$	0
$P_k^{(-1/2,0)}(\tau,\gamma)$	$\frac{\pi}{2}(-1)^k$	0	$(-1)^{k+1}$	0
$Leg_k(\tau,\gamma)$	$\frac{\pi}{2}(-1)^k$	0	$(-1)^{k+1}$	0
$P_k^{(1/2,0)}(\tau,\gamma)$	$\frac{\pi}{2}(-1)^k$	0	$(-1)^{k+1}$	0
$P_k^{(1,0)}(\tau,\gamma)$	$\frac{\pi}{2}(-1)^k$	0	$(-1)^{k+1}$	0
$P_k^{(2,0)}(\tau,\gamma)$	$\frac{\pi}{2}(-1)^k$	0	$(-1)^{k+1}$	0
$P_k^{(\alpha,0)}(\tau,\gamma)$	$\frac{\pi}{2}(-1)^k$	0	$(-1)^{k+1}$	0
$P_k^{(0,1)}(\tau,\gamma)$	$\frac{\pi(k+1)}{2}(-1)^k$	0	$(-1)^{k+1}(k+1)$	0
$P_k^{(0,2)}(\tau,\gamma)$	$\frac{\pi(k+1)(k+2)}{4}(-1)^k$	0	$(-1)^{k+1}\frac{(k+1)(k+2)}{2}$	0
$P_k^{(0,\beta)}(\tau,\gamma)$	$(-1)^k\frac{\pi}{2}\binom{k+\beta}{\beta}$	0	$(-1)^{k+1}\binom{k+\beta}{\beta}$	0

Глава 8

Основные и расширенные соотношения ортогональности в частотной области

Определение.

Для ортогональных функций с единичной весовой функцией в частотной области также справедливы соотношения ортогональности [8, 10, 5]

$$\begin{cases} \int_0^\infty \mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\mathrm{Re}W_n^{\{\psi_n(\tau,\gamma)\}}(j\omega)d\omega = \frac{\pi}{2}\left\|\psi_k\right\|^2\delta_{k,n}; \\ \int_0^\infty \mathrm{Im}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\mathrm{Im}W_n^{\{\psi_n(\tau,\gamma)\}}(j\omega)d\omega = \frac{\pi}{2}\left\|\psi_k\right\|^2\delta_{k,n}. \end{cases}$$

Соотношение ортогональности дает возможность записать равенство, связывающее временную и частотную области (теорема Парсеваля) [5, 12]

$$\int_0^\infty \Big(\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\Big)^2 d\omega = \frac{\pi}{2}\int_0^\infty \Big(\psi_k(\tau,\gamma)\Big)^2 d\tau.$$

8.1 Основные соотношения ортогональности

[8.1] $$\int\limits_0^\infty \mathrm{Re}W_s^{\{L_s(\tau,\gamma)\}}(j\omega)\mathrm{Re}W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = $$

$$= \begin{cases} \frac{\pi}{2\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

[8.2] $$\int\limits_0^\infty \mathrm{Im}W_s^{\{L_s(\tau,\gamma)\}}(j\omega)\mathrm{Im}W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = $$

$$= \begin{cases} \frac{\pi}{2\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.1],[8.2]} = \frac{\pi}{2\gamma}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

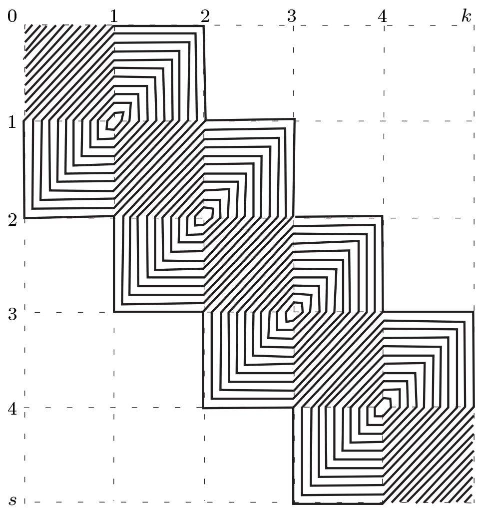


Рис. 8.1. Графическое представление соотношений [8.1], [8.2] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

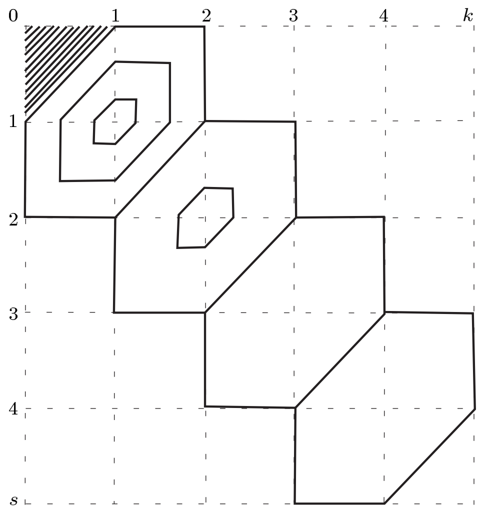


Рис. 8.2. Графическое представление соотношений [8.4], [??] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.3]
$$\int\limits_0^\infty \mathrm{Re}W_s^{\{P_s^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\times \times \mathrm{Re}W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{2(4k+1)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

[8.4]
$$\int\limits_0^\infty \mathrm{Im}W_s^{\{P_s^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\times \times \mathrm{Im}W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{2(4k+1)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[8.4],[??]} = \frac{\pi}{4\gamma}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/17 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/21 \end{pmatrix}.$$

[8.5]
$$\int\limits_0^\infty \mathrm{Re}W_s^{\{Leg_s(\tau,\gamma)\}}(j\omega)\mathrm{Re}W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{4(2k+1)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

[8.6]
$$\int\limits_0^\infty \mathrm{Im}W_s^{\{Leg_s(\tau,\gamma)\}}(j\omega)\mathrm{Im}W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{4(2k+1)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[8.5],[8.6]} = \frac{\pi}{4\gamma}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/11 \end{pmatrix}.$$

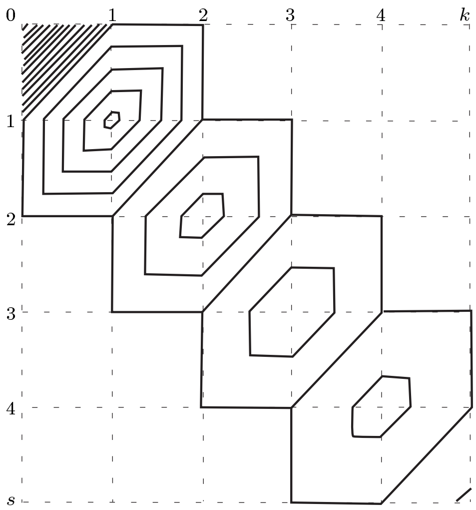


Рис. 8.3. Графическое представление соотношений [8.5], [8.6] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.7]
$$\int\limits_0^\infty \mathrm{Re}W_s^{\{P_s^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\times \mathrm{Re}W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{2(4k+3)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

[8.8]
$$\int\limits_0^\infty \mathrm{Im}W_s^{\{P_s^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\times \mathrm{Im}W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{2(4k+3)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[8.7],[8.8]} = \frac{\pi}{4\gamma}\begin{pmatrix} 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/15 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/19 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/23 \end{pmatrix}.$$

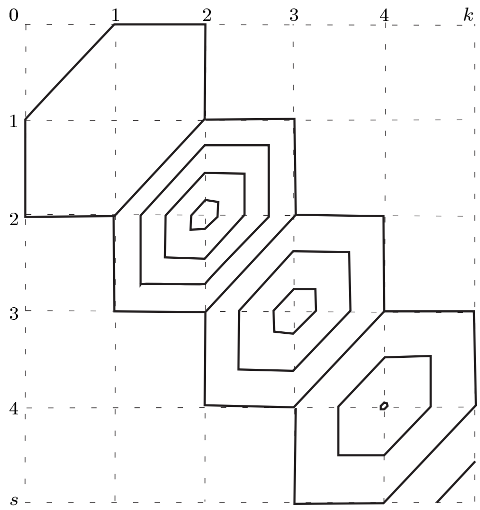


Рис. 8.4. Графическое представление соотношений [8.7], [8.8] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.9]
$$\int\limits_0^\infty \mathrm{Re}W_s^{\{P_s^{(1,0)}(\tau,\gamma)\}}(j\omega)\times \mathrm{Re}W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{4(k+1)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

[8.10]
$$\int\limits_0^\infty \mathrm{Im}W_s^{\{P_s^{(1,0)}(\tau,\gamma)\}}(j\omega)\times \mathrm{Im}W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{4(k+1)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[8.9],[8.10]} = \frac{\pi}{4\gamma}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/6 \end{pmatrix}.$$

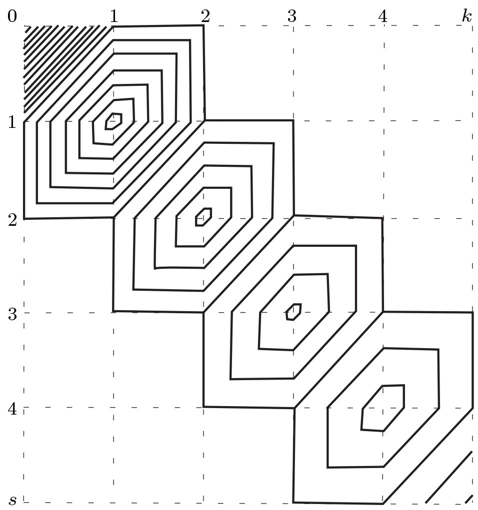


Рис. 8.5. Графическое представление соотношений [8.9], [8.10] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.11]
$$\int\limits_0^\infty \mathrm{Re}W_s^{\{P_s^{(2,0)}(\tau,\gamma)\}}(j\omega)(\tau,\gamma)\times \times\, \mathrm{Re}W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{4(2k+3)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

[8.12]
$$\int\limits_0^\infty \mathrm{Im}W_s^{\{P_s^{(2,0)}(\tau,\gamma)\}}(j\omega)(\tau,\gamma)\times \times\, \mathrm{Im}W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{4(2k+3)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[8.11],[8.12]} = \frac{\pi}{4\gamma}\begin{pmatrix} 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/13 \end{pmatrix}.$$

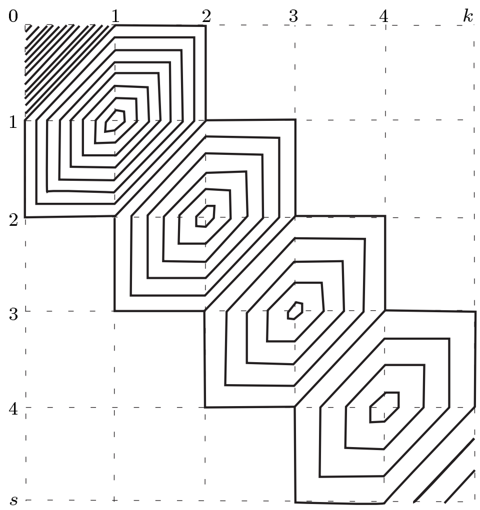


Рис. 8.6. Графическое представление соотношений [8.11], [8.12] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.13]
$$\int\limits_0^\infty \mathrm{Re}W_s^{\{P_s^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)(\tau,\gamma)\times \times\, \mathrm{Re}W_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{2c(2k+\alpha+1)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

[8.14]
$$\int\limits_0^\infty \mathrm{Im}W_s^{\{P_s^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)(\tau,\gamma)\times \times\, \mathrm{Im}W_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} \dfrac{\pi}{2c(2k+\alpha+1)\gamma}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[8.13],[8.14]} = \frac{\pi}{2c\gamma}\times$$
$$\times\begin{pmatrix} \frac{1}{(\alpha+1)} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(\alpha+3)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{(\alpha+5)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{(\alpha+7)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{(\alpha+9)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{(\alpha+11)} \end{pmatrix}.$$

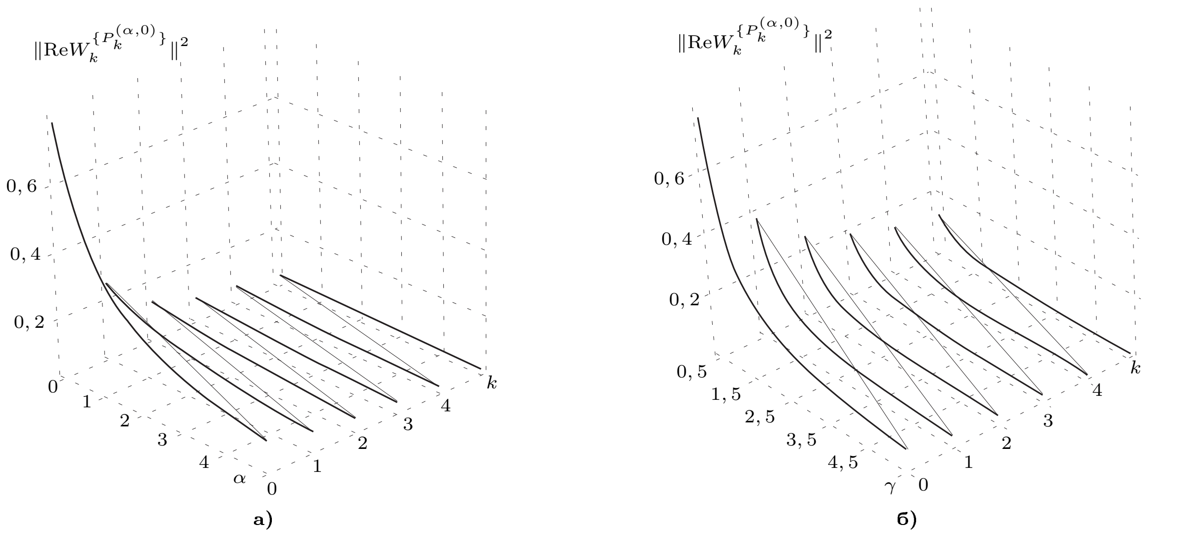


Рис. 8.7. Графическое представление соотношения [8.13], [8.14] при $k = 0..5$ и $k = s$: а) $\gamma = 1$, $c = 2$, $\alpha \in [0; 5]$; б) $\gamma \in [0,5; 5,5]$, $c = 2$, $\alpha = 1$

8.2 Расширенные соотношения ортогональности

$$[\mathbf{8.15}] \quad \int\limits_0^\infty \frac{\partial \mathrm{Im} W_s^{\{L_s(\tau,\gamma)\}}(j\omega)}{\partial \omega} \mathrm{Re} W_k^{\{L_k(\tau,\gamma)\}}(j\omega) \mathrm{d}\omega = \begin{cases} \dfrac{(k+1)\pi}{2\gamma^2}, & \text{если } k = s-1; \\ -\dfrac{(2k+1)\pi}{2\gamma^2}, & \text{если } k = s; \\ \dfrac{k\pi}{2\gamma^2}, & \text{если } k = s+1; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.15]} = \frac{\pi}{2\gamma^2} \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -3 & 2 & 0 & 0 & 0 \\ 0 & 2 & -5 & 3 & 0 & 0 \\ 0 & 0 & 3 & -7 & 4 & 0 \\ 0 & 0 & 0 & 4 & -9 & 5 \\ 0 & 0 & 0 & 0 & 5 & -11 \end{pmatrix}.$$

$$[\mathbf{8.16}] \quad \int\limits_0^\infty \frac{\partial \mathrm{Re} W_s^{\{L_s(\tau,\gamma)\}}(j\omega)}{\partial \omega} \mathrm{Im} W_k^{\{L_k(\tau,\gamma)\}}(j\omega) \mathrm{d}\omega = \begin{cases} -\dfrac{(k+1)\pi}{2\gamma^2}, & \text{если } k = s-1; \\ \dfrac{(2k+1)\pi}{2\gamma^2}, & \text{если } k = s; \\ -\dfrac{k\pi}{2\gamma^2}, & \text{если } k = s+1; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.16]} = \frac{\pi}{2\gamma^2} \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -2 & 0 & 0 & 0 \\ 0 & -2 & 5 & -3 & 0 & 0 \\ 0 & 0 & -3 & 7 & -4 & 0 \\ 0 & 0 & 0 & -4 & 9 & -5 \\ 0 & 0 & 0 & 0 & -5 & 11 \end{pmatrix}.$$

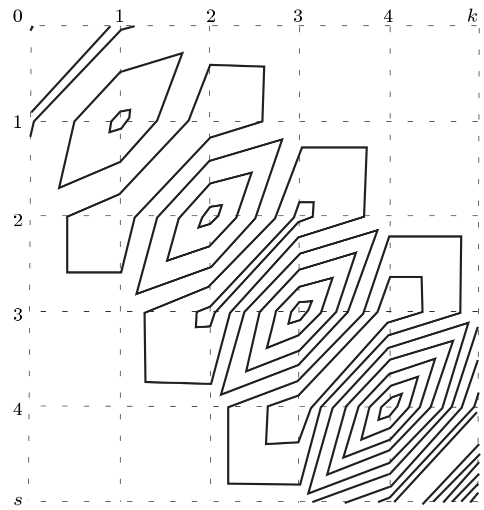


Рис. 8.8. Графическое представление соотношений [8.15], [8.16] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

$$[\mathbf{8.17}] \quad \int\limits_0^\infty \mathrm{Re} W_s^{\left\{\frac{\partial L_s(\tau,\gamma)}{\partial \tau}\right\}}(j\omega) \mathrm{Re} W_k^{\{L_k(\tau,\gamma)\}}(j\omega) \mathrm{d}\omega = \begin{cases} -\dfrac{\pi}{2}, & \text{если } k > s; \\ -\dfrac{\pi}{4}, & \text{если } k = s; \\ 0, & \text{иначе.} \end{cases}$$

[8.18]
$$\int_0^\infty \mathrm{Im}W_s^{\left\{\frac{\partial L_s(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\mathrm{Im}W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega =$$
$$= \begin{cases} -\dfrac{\pi}{2}, & \text{если } k > s;\\ -\dfrac{\pi}{4}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.17],[8.18]} = \frac{\pi}{2}\times$$
$$\times\begin{pmatrix} -1/2 & 0 & 0 & 0 & 0 & 0\\ -1 & -1/2 & 0 & 0 & 0 & 0\\ -1 & -1 & -1/2 & 0 & 0 & 0\\ -1 & -1 & -1 & -1/2 & 0 & 0\\ -1 & -1 & -1 & -1 & -1/2 & 0\\ -1 & -1 & -1 & -1 & -1 & -1/2 \end{pmatrix}.$$

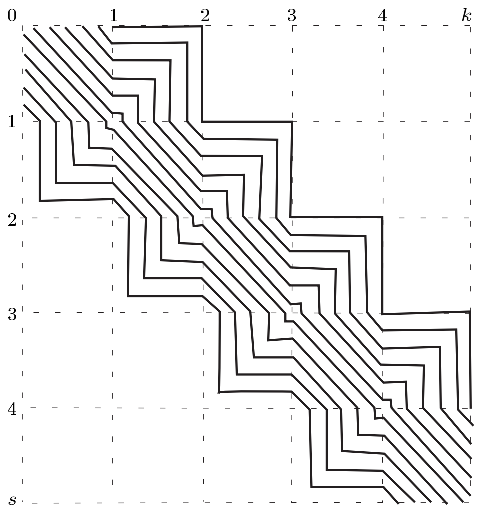


Рис. 8.9. Графическое представление соотношений [8.17], [8.18] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.19]
$$\int_0^\infty \mathrm{Im}W_s^{\left\{\frac{\partial L_s(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\frac{\partial \mathrm{Re}W_k^{\{L_k(\tau,\gamma)\}}(j\omega)}{\partial\omega}\mathrm{d}\omega =$$
$$= \begin{cases} -\dfrac{(k+1)\pi}{4\gamma}, & \text{если } k = s-1;\\ -\dfrac{\pi}{4\gamma}, & \text{если } k = s;\\ \dfrac{k\pi}{4\gamma}, & \text{если } k = s+1;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.19]} = \frac{\pi}{4\gamma}\begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0\\ -1 & -1 & 2 & 0 & 0 & 0\\ 0 & -2 & -1 & 3 & 0 & 0\\ 0 & 0 & -3 & -1 & 4 & 0\\ 0 & 0 & 0 & -4 & -1 & 5\\ 0 & 0 & 0 & 0 & -5 & -1 \end{pmatrix}.$$

[8.20]
$$\int_0^\infty \mathrm{Re}W_s^{\left\{\frac{\partial L_s(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\frac{\partial \mathrm{Im}W_k^{\{L_k(\tau,\gamma)\}}(j\omega)}{\partial\omega}\mathrm{d}\omega =$$
$$= \begin{cases} \dfrac{(k+1)\pi}{4\gamma}, & \text{если } k = s-1;\\ \dfrac{\pi}{4\gamma}, & \text{если } k = s;\\ -\dfrac{k\pi}{4\gamma}, & \text{если } k = s+1;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.20]} = \frac{\pi}{4\gamma}\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0\\ 1 & 1 & -2 & 0 & 0 & 0\\ 0 & 2 & 1 & -3 & 0 & 0\\ 0 & 0 & 3 & 1 & -4 & 0\\ 0 & 0 & 0 & 4 & 1 & -5\\ 0 & 0 & 0 & 0 & 5 & 1 \end{pmatrix}.$$

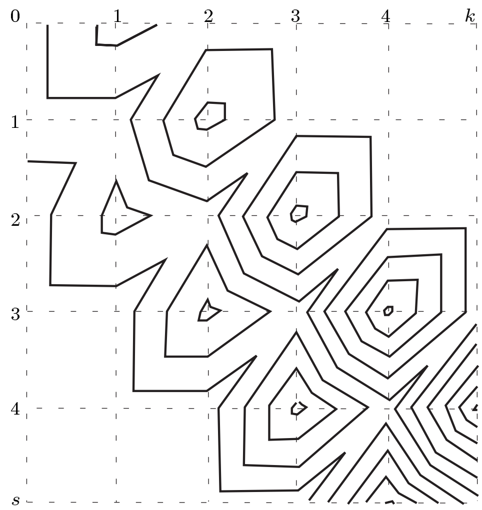


Рис. 8.10. Графическое представление соотношений [8.19], [8.20] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.21]
$$\int_0^\infty \mathrm{Re}W_s^{\left\{\frac{\partial L_s(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\,\mathrm{Re}\,W_k^{\left\{\frac{\partial L_k(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\mathrm{d}\omega -$$
$$-\frac{(2k+1)\gamma\pi}{4} = \begin{cases} -\dfrac{(k-s)\gamma\pi}{2}, & \text{если } k > s;\\ -\dfrac{\gamma\pi}{8}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

[8.22]
$$\int_0^\infty \mathrm{Im}W_s^{\left\{\frac{\partial L_s(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\,\mathrm{Im}\,W_k^{\left\{\frac{\partial L_k(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\mathrm{d}\omega -$$
$$-\frac{(2k+1)\gamma\pi}{4} = \begin{cases} -\dfrac{(k-s)\gamma\pi}{2}, & \text{если } k > s;\\ -\dfrac{\gamma\pi}{8}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.21],[8.22]} = \frac{\pi\gamma}{2}\times$$

$$\times\begin{pmatrix} -1/4 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1/4 & 0 & 0 & 0 & 0 \\ -2 & -1 & -1/4 & 0 & 0 & 0 \\ -3 & -2 & -1 & -1/4 & 0 & 0 \\ -4 & -3 & -2 & -1 & -1/4 & 0 \\ -5 & -4 & -3 & -2 & -1 & -1/4 \end{pmatrix}.$$

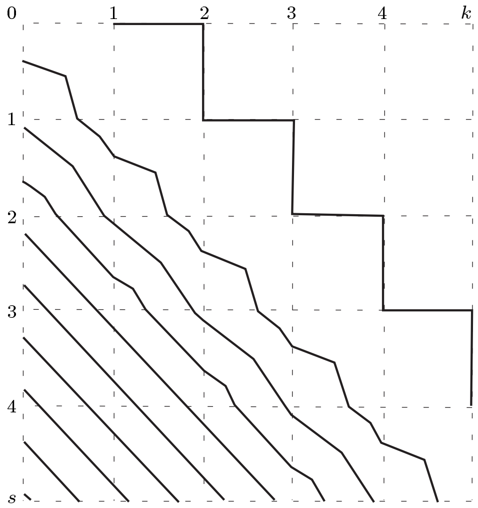


Рис. 8.11. Графическое представление соотношений [8.21], [8.22] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.23] $$\int\limits_0^\infty \mathrm{Im}W_s^{\{L_s(\tau,\gamma)\}}(j\omega)\mathrm{Re}W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega =$$

$$\begin{cases} \dfrac{1}{s-k}, & \text{если } (k+1) \mod 2 \neq 0; \\ -\dfrac{1}{2k+1}, & \text{если } k = s; \\ -\dfrac{1}{k+s+1}, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.23]} = \\ = \begin{pmatrix} -1 & -1 & -1/3 & -1/3 & -1/5 & -1/5 \\ 1 & -1/3 & -1 & -1/5 & -1/3 & -1/7 \\ -1/3 & 1 & -1/5 & -1 & -1/7 & -1/3 \\ 1/3 & -1/5 & 1 & -1/7 & -1 & -1/9 \\ -1/5 & 1/3 & -1/7 & 1 & -1/9 & -1 \\ 1/5 & -1/7 & 1/3 & -1/9 & 1 & -1/11 \end{pmatrix}.$$

[8.24] $$\int\limits_0^\infty \mathrm{Re}W_s^{\{L_s(\tau,\gamma)\}}(j\omega)\mathrm{Im}W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega =$$

$$\begin{cases} -\dfrac{1}{s-k}, & \text{если } (k+1) \mod 2 \neq 0; \\ \dfrac{1}{2k+1}, & \text{если } k = s; \\ \dfrac{1}{k+s+1}, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.24]} = \\ = \begin{pmatrix} -1 & 1 & -1/3 & 1/3 & -1/5 & 1/5 \\ -1 & -1/3 & 1 & -1/5 & 1/3 & -1/7 \\ -1/3 & -1 & -1/5 & 1 & -1/7 & 1/3 \\ -1/3 & -1/5 & -1 & -1/7 & 1 & -1/9 \\ -1/5 & -1/3 & -1/7 & -1 & -1/9 & 1 \\ -1/5 & -1/7 & -1/3 & -1/9 & -1 & -1/11 \end{pmatrix}.$$

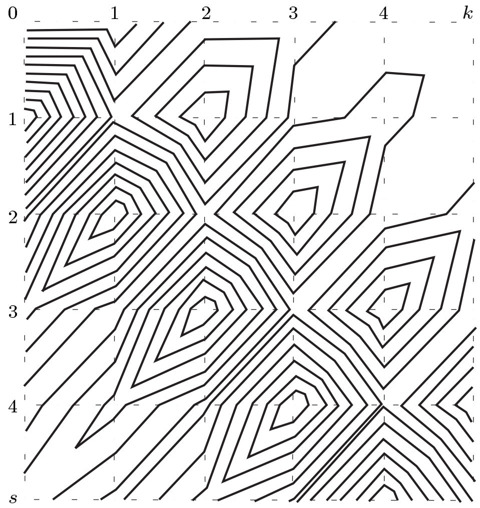


Рис. 8.12. Графическое представление соотношений [8.23], [8.24] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.25] $$\int\limits_0^\infty \mathrm{Re}W_s^{\{L_s^{(1)}(\tau,\gamma)\}}(j\omega)\mathrm{Re}W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega -$$

$$-\frac{\pi(k+1)}{2\gamma} = \begin{cases} -\dfrac{(k-s)\gamma\pi}{2}, & \text{если } k > s; \\ 0, & \text{иначе.} \end{cases}$$

[8.26] $$\int\limits_0^\infty \mathrm{Im}W_s^{\{L_s^{(1)}(\tau,\gamma)\}}(j\omega)\mathrm{Im}W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega -$$

$$-\frac{\pi(k+1)}{2\gamma} = \begin{cases} -\dfrac{(k-s)\gamma\pi}{2}, & \text{если } k > s; \\ 0, & \text{иначе.} \end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.25],[8.26]} = \frac{\pi}{2\gamma} \times \\ \times \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ -2 & -1 & 0 & 0 & 0 & 0 \\ -3 & -2 & -1 & 0 & 0 & 0 \\ -4 & -3 & -2 & -1 & 0 & 0 \\ -5 & -4 & -3 & -2 & -1 & 0 \end{pmatrix}.$$

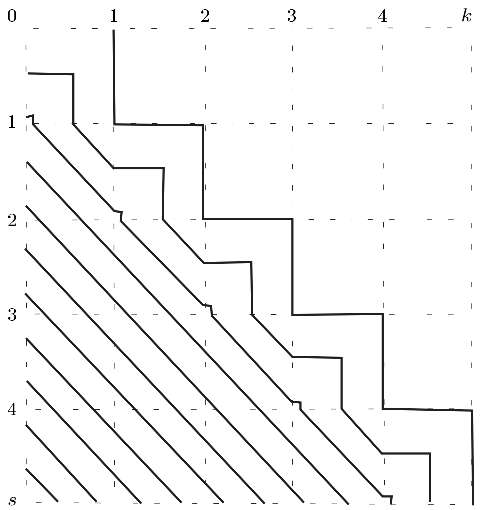


Рис. 8.13. Графическое представление соотношений [8.25], [8.26] при $k=0..5$, $s=0..5$; $\gamma=1$

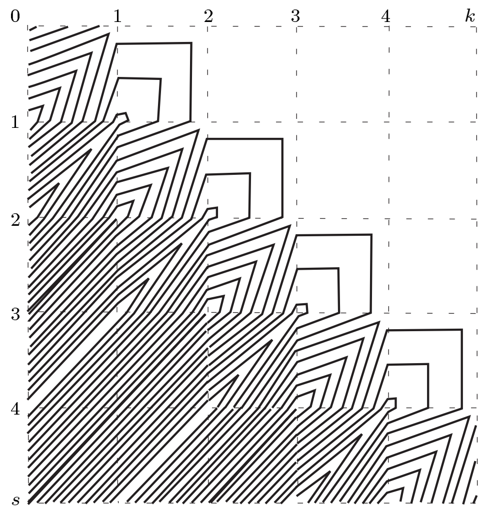


Рис. 8.14. Графическое представление соотношений [8.27], [8.28] при $k=0..5$, $s=0..5$; $\gamma=1$

[8.27]
$$\int\limits_0^\infty \mathrm{Re}W_s^{\left\{\frac{\partial P_s^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times \times\mathrm{Re}W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k>s;\\ -\dfrac{\pi}{4}, & \text{если } k=s;\\ 0, & \text{иначе.}\end{cases}$$

[8.28]
$$\int\limits_0^\infty \mathrm{Im}W_s^{\left\{\frac{\partial P_s^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times \times\mathrm{Im}W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k>s;\\ -\dfrac{\pi}{4}, & \text{если } k=s;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k=0..5$; $s=0..5$:
$$\mathcal{M}_{[8.27],[8.28]} = \frac{\pi}{2}\times \times\begin{pmatrix} -1/2 & 0 & 0 & 0 & 0 & 0\\ 1 & -1/2 & 0 & 0 & 0 & 0\\ -1 & 1 & -1/2 & 0 & 0 & 0\\ 1 & -1 & 1 & -1/2 & 0 & 0\\ -1 & 1 & -1 & 1 & -1/2 & 0\\ 1 & -1 & 1 & -1 & 1 & -1/2 \end{pmatrix}.$$

[8.29]
$$\int\limits_0^\infty \mathrm{Re}W_s^{\left\{\frac{\partial Leg_s(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times \times\mathrm{Re}W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k>s;\\ -\dfrac{\pi}{4}, & \text{если } k=s;\\ 0, & \text{иначе.}\end{cases}$$

[8.30]
$$\int\limits_0^\infty \mathrm{Im}W_s^{\left\{\frac{\partial Leg_s(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times \times\mathrm{Im}W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k>s;\\ -\dfrac{\pi}{4}, & \text{если } k=s;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k=0..5$; $s=0..5$:
$$\mathcal{M}_{[8.29],[8.30]} = \frac{\pi}{2}\times \times\begin{pmatrix} -1/2 & 0 & 0 & 0 & 0 & 0\\ 1 & -1/2 & 0 & 0 & 0 & 0\\ -1 & 1 & -1/2 & 0 & 0 & 0\\ 1 & -1 & 1 & -1/2 & 0 & 0\\ -1 & 1 & -1 & 1 & -1/2 & 0\\ 1 & -1 & 1 & -1 & 1 & -1/2 \end{pmatrix}.$$

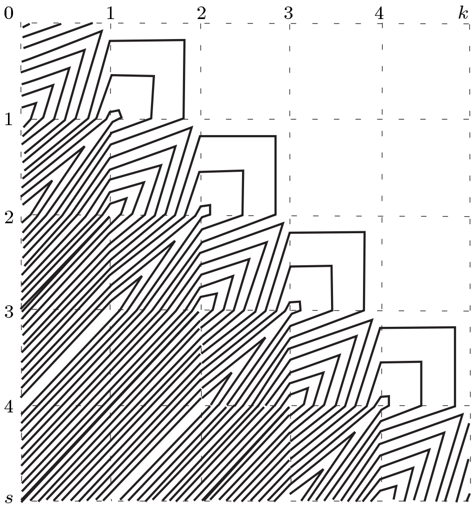


Рис. 8.15. Графическое представление соотношений [8.29], [8.30] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.31]
$$\int_0^\infty \mathrm{Re}W_s^{\left\{\frac{\partial P_s^{(1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times \times \mathrm{Re}W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k > s;\\ -\dfrac{\pi}{4}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

[8.32]
$$\int_0^\infty \mathrm{Im}W_s^{\left\{\frac{\partial P_s^{(1/2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times \times \mathrm{Im}W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k > s;\\ -\dfrac{\pi}{4}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[8.31],[8.32]} = \frac{\pi}{2}\times \times \begin{pmatrix} -1/2 & 0 & 0 & 0 & 0 & 0\\ 1 & -1/2 & 0 & 0 & 0 & 0\\ -1 & 1 & -1/2 & 0 & 0 & 0\\ 1 & -1 & 1 & -1/2 & 0 & 0\\ -1 & 1 & -1 & 1 & -1/2 & 0\\ 1 & -1 & 1 & -1 & 1 & -1/2 \end{pmatrix}.$$

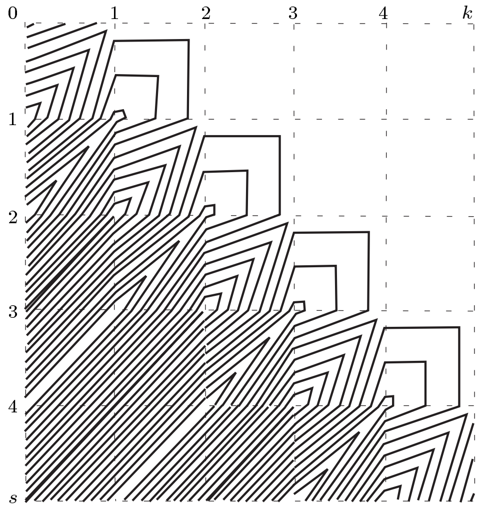


Рис. 8.16. Графическое представление соотношений [8.31], [8.32] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.33]
$$\int_0^\infty \mathrm{Re}W_s^{\left\{\frac{\partial P_s^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times \times \mathrm{Re}W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k > s;\\ -\dfrac{\pi}{4}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

[8.34]
$$\int_0^\infty \mathrm{Im}W_s^{\left\{\frac{\partial P_s^{(1,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times \times \mathrm{Im}W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k > s;\\ -\dfrac{\pi}{4}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:
$$\mathcal{M}_{[8.33],[8.34]} = \frac{\pi}{2}\times \times \begin{pmatrix} -1/2 & 0 & 0 & 0 & 0 & 0\\ 1 & -1/2 & 0 & 0 & 0 & 0\\ -1 & 1 & -1/2 & 0 & 0 & 0\\ 1 & -1 & 1 & -1/2 & 0 & 0\\ -1 & 1 & -1 & 1 & -1/2 & 0\\ 1 & -1 & 1 & -1 & 1 & -1/2 \end{pmatrix}.$$

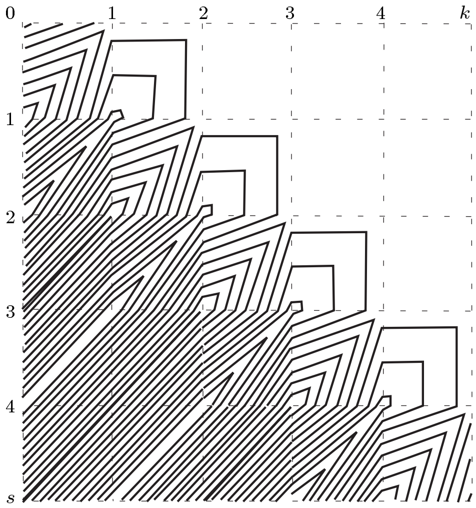


Рис. 8.17. Графическое представление соотношений [8.33], [8.34] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.35] $$\int\limits_0^\infty \mathrm{Re}W_s^{\left\{\frac{\partial P_s^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times$$

$$\times\,\mathrm{Re}W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k > s;\\ -\dfrac{\pi}{4}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

[8.36] $$\int\limits_0^\infty \mathrm{Im}W_s^{\left\{\frac{\partial P_s^{(2,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times$$

$$\times\,\mathrm{Im}W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k > s;\\ -\dfrac{\pi}{4}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

Матрица значений при $k = 0..5$; $s = 0..5$:

$$\mathcal{M}_{[8.35],[8.36]} = \frac{\pi}{2}\times$$

$$\times\begin{pmatrix} -1/2 & 0 & 0 & 0 & 0 & 0\\ 1 & -1/2 & 0 & 0 & 0 & 0\\ -1 & 1 & -1/2 & 0 & 0 & 0\\ 1 & -1 & 1 & -1/2 & 0 & 0\\ -1 & 1 & -1 & 1 & -1/2 & 0\\ 1 & -1 & 1 & -1 & 1 & -1/2 \end{pmatrix}.$$

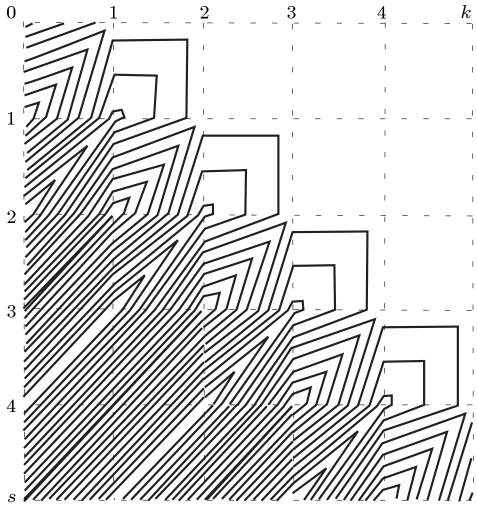


Рис. 8.18. Графическое представление соотношений [8.35], [8.36] при $k = 0..5$, $s = 0..5$; $\gamma = 1$

[8.37] $$\int\limits_0^\infty \mathrm{Re}W_s^{\left\{\frac{\partial P_s^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times$$

$$\times\,\mathrm{Re}W_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k > s;\\ -\dfrac{\pi}{4}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

[8.38] $$\int\limits_0^\infty \mathrm{Im}W_s^{\left\{\frac{\partial P_s^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}\right\}}(j\omega)\times$$

$$\times\,\mathrm{Im}W_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \begin{cases} -\dfrac{\pi(-1)^{k+s}}{2}, & \text{если } k > s;\\ -\dfrac{\pi}{4}, & \text{если } k = s;\\ 0, & \text{иначе.}\end{cases}$$

Глава 9

Рекуррентные соотношения

Определение.

Для ортогональных многочленов Якоби справедливо следующее рекуррентное соотношение [13, 15]:

$$2(k+1)(\alpha+\beta+k+1)(\alpha+\beta+2k)P_{k+1}^{(\alpha,\beta)}(x) =$$
$$= \Big((\alpha+\beta+2k)(\alpha+\beta+2k+2)x+\alpha^2-\beta^2\Big)(\alpha+\beta+2k+1)P_k^{(\alpha,\beta)}(x) -$$
$$-2(\alpha+k)(\beta+k)(\alpha+\beta+2k+2)P_{k-1}^{(\alpha,\beta)}(x).$$

Для обобщенных многочленов Лагерра справедливо следующее рекуррентное соотношение [13, 15]:

$$(k+1)L_{k+1}^{(\alpha)}(x) = (\alpha+2k+1-x)L_k^{(\alpha)}(x) - (\alpha+k)L_{k-1}^{(\alpha)}(x).$$

Аналогичные соотношения для ортогональных функций получены с учетом замен переменных, приведенных в Главе 1 [9].

9.1 Рекуррентные соотношения для ортогональных функций

[9.1] $$L_k(\tau,\gamma) = \frac{2k-1-\gamma\tau}{k}L_{k-1}(\tau,\gamma) - \frac{k-1}{k}L_{k-2}(\tau,\gamma).$$

[9.2] $$L_k^{(1)}(\tau,\gamma) = \frac{2k-\gamma\tau}{k}L_{k-1}^{(1)}(\tau,\gamma) - L_{k-2}^{(1)}(\tau,\gamma).$$

[9.3] $$L_k^{(2)}(\tau,\gamma) = \frac{2k+1-\gamma\tau}{k}L_{k-1}^{(2)}(\tau,\gamma) - \frac{k+1}{k}L_{k-2}^{(2)}(\tau,\gamma).$$

[9.4] $$L_k^{(\alpha)}(\tau,\gamma) = \frac{2k+\alpha-1-\gamma\tau}{k}L_{k-1}^{(\alpha)}(\tau,\gamma) - \frac{k+\alpha-1}{k}L_{k-2}^{(\alpha)}(\tau,\gamma).$$

[9.5] $$P_k^{(-1/2,0)}(\tau,\gamma) = \frac{\Big((4k-5)(4k-1)\big(1-2\exp(-2\gamma\tau)\big)+1\Big)}{4k(4k-5)(2k-1)}\times$$
$$\times (4k-3)P_{k-1}^{(-1/2,0)}(\tau,\gamma) - \frac{(k-1)(2k-3)(4k-1)}{k(2k-1)(4k-5)}\times P_{k-2}^{(-1/2,0)}(\tau,\gamma).$$

[9.6] $$Leg_k(\tau,\gamma) = \frac{2k-1}{k}\big(1-2\exp(-2\gamma\tau)\big)\times Leg_{k-1}(\tau,\gamma) - \frac{k-1}{k}Leg_{k-2}(\tau,\gamma).$$

[9.7] $$P_k^{(1/2,0)}(\tau,\gamma) = \frac{\Big((4k-3)(4k+1)\big(1-2\exp(-2\gamma\tau)\big)+1\Big)}{4k(4k-3)(2k+1)}\times$$
$$\times (4k-1)P_{k-1}^{(1/2,0)}(\tau,\gamma) - \frac{(k-1)(2k-1)(4k+1)}{k(2k+1)(4k-3)}\times P_{k-2}^{(1/2,0)}(\tau,\gamma).$$

[9.8] $$P_k^{(1,0)}(\tau,\gamma) = \frac{\Big((2k-1)(2k+1)\big(1-2\exp(-\gamma\tau)\big)+1\Big)}{(2k-1)(k+1)}\times P_{k-1}^{(1,0)}(\tau,\gamma) - \frac{(k-1)(2k+1)}{(2k-1)(k+1)}P_{k-2}^{(1,0)}(\tau,\gamma).$$

[9.9] $$P_k^{(2,0)}(\tau,\gamma) = \frac{\Big(k(k+1)\big(1-2\exp(-2\gamma\tau)\big)+1\Big)(2k+1)}{k^2(k+2)}\times P_{k-1}^{(2,0)}(\tau,\gamma) - \frac{(k-1)(k+1)^2}{k^2(k+2)}P_{k-2}^{(2,0)}(\tau,\gamma).$$

[9.10] $$P_k^{(\alpha,0)}(\tau,\gamma) = \frac{\Big((\alpha+2k)(\alpha+2k-2)\big(1-2\exp(-c\gamma\tau)\big)+\alpha^2\Big)}{2k(\alpha+2k-2)(\alpha+k)}\times(\alpha+2k-1)P_{k-1}^{(\alpha,0)}(\tau,\gamma) - \frac{(\alpha+k-1)(k-1)(\alpha+2k)}{k(\alpha+2k-2)(\alpha+k)}\times P_{k-2}^{(\alpha,0)}(\tau,\gamma).$$

[9.11] $$P_k^{(0,1)}(\tau,\gamma) = \frac{\Big((2k-1)(2k+1)\big(1-2\exp(-2\gamma\tau)\big)-1\Big)}{(k+1)(2k-1)}\times P_{k-1}^{(0,1)}(\tau,\gamma) - \frac{(k-1)(2k+1)}{(k+1)(2k-1)}P_{k-2}^{(0,1)}(\tau,\gamma).$$

[9.12] $$P_k^{(0,2)}(\tau,\gamma) = \frac{\Big(k(k+1)\big(1-2\exp(-2\gamma\tau)\big)-1\Big)(2k+1)}{k^2(k+2)}\times P_{k-1}^{(0,2)}(\tau,\gamma) - \frac{(k-1)(k+1)^2}{k^2(k+2)}P_{k-2}^{(0,2)}(\tau,\gamma).$$

[9.13] $$P_k^{(0,\beta)}(\tau,\gamma) = \frac{\Big((\beta+2k)(\beta+2k-2)\big(1-2\exp(-c\gamma\tau)\big)-\beta^2\Big)}{2k(\beta+2k-2)(\beta+k)}\times(\beta+2k-1)P_{k-1}^{(0,\beta)}(\tau,\gamma) - \frac{(\beta+k-1)(k-1)(\beta+2k)}{k(\beta+2k-2)(\beta+k)}\times P_{k-2}^{(0,\beta)}(\tau,\gamma).$$

9.2 Рекуррентные соотношения для производных ортогональных функций

[9.14] $$\frac{\partial P_k^{(-1/2,0)}(\tau,\gamma)}{\partial\tau} = \frac{\partial P_{k-2}^{(-1/2,0)}(\tau,\gamma)}{\partial\tau} - \gamma\left(\frac{(4k+1)}{2}P_k^{(-1/2,0)}(\tau,\gamma) - (4k-3)P_{k-1}^{(-1/2,0)}(\tau,\gamma) + \frac{(4k-7)}{2}P_{k-2}^{(-\frac{1}{2},0)}(\tau,\gamma)\right).$$

[9.15] $$\frac{\partial Leg_k(\tau,\gamma)}{\partial\tau} = \frac{\partial Leg_{k-2}(\tau,\gamma)}{\partial\tau} - \gamma\Big((2k+1)Leg_k(\tau,\gamma) - 2(2k-1)Leg_{k-1}(\tau,\gamma) + (2k-3)Leg_{k-2}(\tau,\gamma)\Big).$$

[9.16] $$\frac{\partial P_k^{(1/2,0)}(\tau,\gamma)}{\partial\tau} = \frac{\partial P_{k-2}^{(1/2,0)}(\tau,\gamma)}{\partial\tau} - \gamma\left(\frac{(4k+3)}{2}P_k^{(1/2,0)}(\tau,\gamma) - (4k-1)P_{k-1}^{(1/2,0)}(\tau,\gamma) + \frac{(4k-5)}{2}P_{k-2}^{(1/2,0)}(\tau,\gamma)\right).$$

[9.17] $$\frac{\partial P_k^{(1,0)}(\tau,\gamma)}{\partial\tau} = \frac{\partial P_{k-2}^{(1,0)}(\tau,\gamma)}{\partial\tau} - \gamma\Big((k+1)P_k^{(1,0)}(\tau,\gamma) - 2kP_{k-1}^{(1,0)}(\tau,\gamma) + (k-1)P_{k-2}^{(1,0)}(\tau,\gamma)\Big).$$

[9.18] $$\frac{\partial P_k^{(2,0)}(\tau,\gamma)}{\partial\tau} = \frac{\partial P_{k-2}^{(2,0)}(\tau,\gamma)}{\partial\tau} - \gamma\Big((2k+3)P_k^{(2,0)}(\tau,\gamma) - 2(2k+1)P_{k-1}^{(2,0)}(\tau,\gamma) + (2k-1)P_{k-2}^{(2,0)}(\tau,\gamma)\Big).$$

[9.19] $$\frac{\partial P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau} = \frac{\partial P_{k-2}^{(\alpha,0)}(\tau,\gamma)}{\partial\tau} - c\gamma/2\Big((\alpha+2k+1)P_k^{(\alpha,0)}(\tau,\gamma) - 2(\alpha+2k-1)\times P_{k-1}^{(\alpha,0)}(\tau,\gamma) + (\alpha+2k-3)P_{k-2}^{(\alpha,0)}(\tau,\gamma)\Big).$$

[9.20] $$\frac{\partial P_k^{(0,1)}(\tau,\gamma)}{\partial\tau} = \frac{(2k+1)}{(k+1)}\left(\frac{\partial Leg_k(\tau,\gamma)}{\partial\tau} + \frac{\partial Leg_{k-1}(\tau,\gamma)}{\partial\tau}\right) - \frac{\big((2k-1)(2k+1)+1\big)}{(k+1)(2k-1)}\frac{\partial P_{k-1}^{(0,1)}(\tau,\gamma)}{\partial\tau} - \frac{(k-1)(2k+1)}{(k+1)(2k-1)}\frac{\partial P_{k-2}^{(0,1)}(\tau,\gamma)}{\partial\tau}.$$

[9.21] $$\frac{\partial P_k^{(0,2)}(\tau,\gamma)}{\partial\tau} = \frac{2(k+1)}{k(k+2)}\left(k\frac{\partial P_k^{(0,1)}(\tau,\gamma)}{\partial\tau} + (k+1)\frac{\partial P_{k-1}^{(0,1)}(\tau,\gamma)}{\partial\tau}\right) - \frac{\big(k(k+1)+1\big)}{k^2(k+2)}(2k+1)\frac{\partial P_{k-1}^{(0,2)}(\tau,\gamma)}{\partial\tau} - \frac{(k-1)(k+1)^2}{k^2(k+2)}\frac{\partial P_{k-2}^{(0,2)}(\tau,\gamma)}{\partial\tau}.$$

[9.22] $$\frac{\partial P_k^{(0,\beta)}(\tau,\gamma)}{\partial\tau} = \frac{(\beta+2k)}{k(\beta+k)}\times \left(k\frac{\partial P_k^{(0,\beta-1)}(\tau,\gamma)}{\partial\tau} + (k+\beta-1)\frac{\partial P_{k-1}^{(0,\beta-1)}(\tau,\gamma)}{\partial\tau}\right) - \frac{\big((\beta+2k)(\beta+2k-2)+1\big)}{2k(\beta+2k-2)(\beta+k)}(\beta+2k-1)\frac{\partial P_{k-1}^{(0,\beta)}(\tau,\gamma)}{\partial\tau} - \frac{(\beta+k-1)(k-1)(\beta+2k)}{k(\beta+2k-2)(\beta+k)}\frac{\partial P_{k-2}^{(0,\beta)}(\tau,\gamma)}{\partial\tau}.$$

9.3 Рекуррентные соотношения для неопределенных интегралов от ортогональных функций

[9.23] $$\int L_k(\tau,\gamma)\mathrm{d}\tau = -2\sum_{\nu=0}^{k-1}\int L_\nu(\tau,\gamma)\mathrm{d}\tau - \frac{2}{\gamma}L_k(\tau,\gamma).$$

[9.24] $$\int \tau L_k(\tau,\gamma)\mathrm{d}\tau = -\frac{k+1}{\gamma}\int L_{k+1}(\tau,\gamma)\mathrm{d}\tau + \frac{2k+1}{\gamma}\int L_k(\tau,\gamma)\mathrm{d}\tau - \frac{k}{\gamma}\int L_{k-1}(\tau,\gamma)\mathrm{d}\tau.$$

[9.25] $$\int \tau L_k(\tau,\gamma)\mathrm{d}\tau = -\frac{4}{\gamma}\sum_{\nu=0}^{k-1}\Big(L_\nu(\tau,\gamma)\tau - \int L_\nu(\tau,\gamma)\mathrm{d}\tau\Big)(-1)^{k+\nu} - \frac{2}{\gamma}\Big(L_k(\tau,\gamma)\tau - \int L_k(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.26] $$\int L_k^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \int \tau L_k(\tau,\gamma)\mathrm{d}\tau + \int L_{k-1}^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau.$$

[9.27] $$\int L_k^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2}\int \tau L_{k+1}(\tau,\gamma)\mathrm{d}\tau + \frac{1}{\gamma}\int L_{k+1}(\tau,\gamma)\mathrm{d}\tau - \frac{1}{\gamma}\tau L_{k+1}(\tau,\gamma).$$

[9.28] $$\int \tau L_k^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = -\frac{k+1}{\gamma}\int L_{k+1}^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau + \frac{2(k+1)}{\gamma}\times \int L_k^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau - \frac{k+1}{\gamma}\int L_{k-1}^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau.$$

[9.29] $$\int L_k^{(2)}(\tau,\gamma)\mu^{\{L_k^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \int \tau L_k^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau + \int \tau L_{k-1}^{(2)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau.$$

[9.30] $$\int L_k^{(2)}(\tau,\gamma)\mu^{\{L_k^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2}\int \tau L_{k+1}^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau + \frac{2}{\gamma}\times \int L_{k+1}^{(1)}(\tau,\gamma)\mu^{\{L_k^{(1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau - \frac{1}{\gamma}\tau^2 L_{k+1}^{(1)}(\tau,\gamma).$$

[9.31] $$\int \tau L_k^{(2)}(\tau,\gamma)\mu^{\{L_k^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = -\frac{k+1}{\gamma}\int L_{k+1}^{(2)}(\tau,\gamma)\mu^{\{L_k^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau + \frac{2k+3}{\gamma}\times \int L_k^{(2)}(\tau,\gamma)\mu^{\{L_k^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau - \frac{k+2}{\gamma}\int L_{k-1}^{(2)}(\tau,\gamma)\mu^{\{L_k^{(2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau.$$

[9.32] $$\int L_k^{(\alpha)}(\tau,\gamma)\mu^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \int L_k^{(\alpha-1)}(\tau,\gamma)\mu^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau + \int L_{k-1}^{(\alpha)}(\tau,\gamma)\mu^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau.$$

[9.33] $$\int L_k^{(\alpha)}(\tau,\gamma)\mu^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = -\frac{1}{2}\int \tau L_{k+1}^{(\alpha-1)}(\tau,\gamma)\mu^{\{L_k^{(\alpha-1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau + \frac{\alpha}{\gamma}\times \int L_{k+1}^{(\alpha-1)}(\tau,\gamma)\mu^{\{L_k^{(\alpha-1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau - \frac{1}{\gamma}\tau^\alpha\times L_{k+1}^{(\alpha-1)}(\tau,\gamma).$$

[9.34] $$\int \tau L_k^{(\alpha)}(\tau,\gamma)\mu^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = -\frac{k+1}{\gamma}\int L_{k+1}^{(\alpha)}(\tau,\gamma)\mu^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau + \frac{2k+\alpha+1}{\gamma}\int L_k^{(\alpha)}(\tau,\gamma)\mu^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau - \frac{k+\alpha}{\gamma}\int L_{k-1}^{(\alpha)}(\tau,\gamma)\mu^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau.$$

[9.35] $$\int P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2(4k-3)}{4k+1}\times \int P_{k-1}^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{(4k-7)}{4k+1}\int P_{k-2}^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{2}{\gamma(4k+1)}\Big(P_k^{(-1/2,0)}(\tau,\gamma) - P_{k-2}^{(-1/2,0)}(\tau,\gamma)\Big).$$

[9.36] $$\int \tau P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2(4k-3)}{4k+1}\times \int \tau P_{k-1}^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{(4k-7)}{4k+1}\times \int \tau P_{k-2}^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{2}{\gamma(4k+1)}\times \left(\Big(P_k^{(-1/2,0)}(\tau,\gamma) - P_{k-2}^{(-1/2,0)}(\tau,\gamma)\Big)\tau - \int P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau + \int P_{k-2}^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau\right).$$

[9.37] $$\int \tau P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{4}{\gamma(4k+1)}\times \sum_{\nu=0}^{k-1}\Big(P_\nu^{(-1/2,0)}(\tau,\gamma)\tau - \int P_\nu^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau\Big) - \frac{2}{\gamma(4k+1)}\Big(P_k^{(-1/2,0)}(\tau,\gamma)\tau - \int P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.38] $$\int Leg_k(\tau,\gamma)\mathrm{d}\tau = \frac{2(2k-1)}{2k+1}\int Leg_{k-1}(\tau,\gamma)\mathrm{d}\tau - \frac{(2k-3)}{2k+1}\int Leg_{k-2}(\tau,\gamma)\mathrm{d}\tau - \frac{1}{\gamma(2k+1)}\times \Big(Leg_k(\tau,\gamma) - Leg_{k-2}(\tau,\gamma)\Big).$$

[9.39] $$\int \tau Leg_k(\tau,\gamma)\mathrm{d}\tau = \frac{2(2k-1)}{2k+1}\times \int \tau Leg_{k-1}(\tau,\gamma)\mathrm{d}\tau - \frac{(2k-3)}{2k+1}\int \tau Leg_{k-2}(\tau,\gamma)\mathrm{d}\tau - \frac{1}{\gamma(2k+1)}\left(\Big(Leg_k(\tau,\gamma) - Leg_{k-2}(\tau,\gamma)\Big)\tau - \int Leg_k(\tau,\gamma)\mathrm{d}\tau + \int Leg_{k-2}(\tau,\gamma)\mathrm{d}\tau\right).$$

[9.40] $$\int \tau Leg_k(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma(2k+1)}\times \sum_{\nu=0}^{k-1}\Big(Leg_\nu(\tau,\gamma)\tau - \int Leg_\nu(\tau,\gamma)\mathrm{d}\tau\Big) - \frac{1}{\gamma(2k+1)}\Big(Leg_k(\tau,\gamma)\tau - \int Leg_k(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.41] $$\int P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2(4k-1)}{4k+3}\times \int P_{k-1}^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{(4k-5)}{4k+3}\int P_{k-2}^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{2}{\gamma(4k+3)}\Big(P_k^{(1/2,0)}(\tau,\gamma) - P_{k-2}^{(1/2,0)}(\tau,\gamma)\Big).$$

[9.42] $$\int \tau P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2(4k-1)}{4k+3}\times \int \tau P_{k-1}^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{(4k-5)}{4k+3}\int \tau P_{k-2}^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{2}{\gamma(4k+3)}\left(\Big(P_k^{(1/2,0)}(\tau,\gamma) - P_{k-2}^{(1/2,0)}(\tau,\gamma)\Big)\tau - \int P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau + \int P_{k-2}^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau\right).$$

[9.43] $$\int \tau P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{4}{\gamma(4k+3)}\times \sum_{\nu=0}^{k-1}\Big(P_\nu^{(1/2,0)}(\tau,\gamma)\tau - \int P_\nu^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau\Big) - \frac{2}{\gamma(4k+3)}\Big(P_k^{(1/2,0)}(\tau,\gamma)\tau - \int P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.44] $$\int P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2k}{k+1}\int P_{k-1}^{(1,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{(k-1)}{k+1}\int P_{k-2}^{(1,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{1}{\gamma(k+1)}\times \Big(P_k^{(1,0)}(\tau,\gamma) - P_{k-2}^{(1,0)}(\tau,\gamma)\Big).$$

[9.45] $$\int \tau P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2k}{k+1}\int \tau P_{k-1}^{(1,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{k-1}{k+1}\int \tau P_{k-2}^{(2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{1}{\gamma(k+1)}\left(\Big(P_k^{(1,0)}(\tau,\gamma) - P_{k-2}^{(1,0)}(\tau,\gamma)\Big)\tau - \int P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau + \int P_{k-2}^{(1,0)}(\tau,\gamma)\mathrm{d}\tau\right).$$

[9.46] $$\int \tau P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma(k+1)}\times \sum_{\nu=0}^{k-1}\Big(P_\nu^{(1,0)}(\tau,\gamma)\tau - \int P_\nu^{(1,0)}(\tau,\gamma)\mathrm{d}\tau\Big) - \frac{1}{\gamma(k+1)}\Big(P_k^{(1,0)}(\tau,\gamma)\tau - \int P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.47] $$\int P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2(2k+1)}{2k+3}\int P_{k-1}^{(2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{(2k-1)}{2k+3}\int P_{k-2}^{(2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{1}{\gamma(2k+3)}\times \Big(P_k^{(2,0)}(\tau,\gamma) - P_{k-2}^{(2,0)}(\tau,\gamma)\Big).$$

[9.48] $$\int \tau P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2(2k+1)}{2k+3}\times \int \tau P_{k-1}^{(2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{2k-1}{2k+3}\int \tau P_{k-2}^{(2,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{1}{\gamma(2k+3)}\Big(\Big(P_k^{(2,0)}(\tau,\gamma) - P_{k-2}^{(2,0)}(\tau,\gamma)\Big)\tau - \int P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau + \int P_{k-2}^{(2,0)}(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.49] $$\int \tau P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma(2k+3)}\times \sum_{\nu=0}^{k-1}\Big(P_\nu^{(2,0)}(\tau,\gamma)\tau - \int P_\nu^{(2,0)}(\tau,\gamma)\mathrm{d}\tau\Big) - \frac{1}{\gamma(2k+3)}\Big(P_k^{(2,0)}(\tau,\gamma)\tau - \int P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.50] $$\int P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2(\alpha+2k-1)}{\alpha+2k+1}\times \int P_{k-1}^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{(\alpha+2k-3)}{\alpha+2k+1}\int P_{k-2}^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{2}{c\gamma(\alpha+2k+1)}\Big(P_k^{(\alpha,0)}(\tau,\gamma) - P_{k-2}^{(\alpha,0)}(\tau,\gamma)\Big).$$

[9.51] $$\int \tau P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = \frac{2(\alpha+2k-1)}{\alpha+2k+1}\times \int \tau P_{k-1}^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{\alpha+2k-3}{\alpha+2k+1}\int \tau P_{k-2}^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau - \frac{2}{c\gamma(\alpha+2k+1)}\Big(\Big(P_k^{(\alpha,0)}(\tau,\gamma) - P_{k-2}^{(\alpha,0)}(\tau,\gamma)\Big)\tau - \int P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau + \int P_{k-2}^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.52] $$\int \tau P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{4}{c\gamma(\alpha+2k+1)}\times \sum_{\nu=0}^{k-1}\Big(P_\nu^{(\alpha,0)}(\tau,\gamma)\tau - \int P_\nu^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau\Big) - \frac{2}{c\gamma(\alpha+2k+1)}\Big(P_k^{(\alpha,0)}(\tau,\gamma)\tau - \int P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.53] $$\int P_k^{(0,1)}(\tau,\gamma)\mu^{\{P_k^{(0,1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{2}\Big(\int Leg_{k+1}(\tau,\gamma)\mathrm{d}\tau + \int Leg_k(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.54] $$\int \tau P_k^{(0,1)}(\tau,\gamma)\mu^{\{P_k^{(0,1)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{2}\Big(\int \tau Leg_{k+1}(\tau,\gamma)\mathrm{d}\tau + \int \tau Leg_k(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.55] $$\int P_k^{(0,2)}(\tau,\gamma)\mu^{\{P_k^{(0,2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{2(2k+3)}\Big((k+1)\int Leg_{k+2}(\tau,\gamma)\mathrm{d}\tau + (2k+3)\times \int Leg_{k+1}(\tau,\gamma)\mathrm{d}\tau + (k+2)\int Leg_k(\tau,\gamma)\mathrm{d}\tau\Big).$$

[9.56] $$\int \tau P_k^{(0,2)}(\tau,\gamma)\mu^{\{P_k^{(0,2)}(\tau,\gamma)\}}(\tau,\gamma)\mathrm{d}\tau = \frac{1}{2(2k+3)}\Big((k+1)\int \tau Leg_{k+2}(\tau,\gamma)\mathrm{d}\tau + (2k+3)\times \int \tau Leg_{k+1}(\tau,\gamma)\mathrm{d}\tau + (k+2)\int \tau Leg_k(\tau,\gamma)\mathrm{d}\tau\Big).$$

9.4 Рекуррентные соотношения для преобразований Фурье

[9.57] $$W_k^{\{L_k^{(\alpha+1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma}\Big(1 - W_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)\big(j\omega - \gamma/2\big)\Big).$$

[9.58] $$W_k^{\{L_k^{(\alpha+1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{\gamma}\Big(1 + W_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)\frac{\gamma}{2\cos\varphi}\exp(-j\varphi)\Big), \quad \varphi = \arctan\frac{2\omega}{\gamma}.$$

[9.59] $$V_k^{\{L_k^{(\alpha+1)}(\tau,\gamma)\}}(j\omega) = \frac{\gamma}{(j\omega+\gamma/2)}V_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega).$$

[9.60] $$V_k^{\{L_k^{(\alpha+1)}(\tau,\gamma)\}}(j\omega) = 2V_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)\cos\varphi\exp(-j\varphi), \quad \varphi = \arctan\frac{2\omega}{\gamma}.$$

Глава 10

Соотношения взаимосвязи базисных функций

Определение.

Данные определения получены на основе представления функциональной характеристику k - того порядка $\vartheta_k(\tau,\gamma)$, связанной с $\psi_k(\tau,\gamma)$ в виде [7, 11, 5]

$$\vartheta_k(\tau,\gamma)=\sum_{\nu=0}^{\infty}\beta_{k,\nu}\psi_\nu(\tau,\gamma),$$

где

$$\beta_{k,\nu}=\frac{1}{\left\|\psi_\nu(\gamma)\right\|^2}\int_0^\infty \vartheta_k(\tau,\gamma)\psi_\nu(\tau,\gamma)\mu^{\{\psi_\nu(\tau,\gamma)\}}(\tau,\gamma)d\tau$$

– коэффициенты разложения ряда,
и понятии расширенного соотношения ортогональности [5].

[10.1] $$\int_0^\infty \vartheta_k(\tau,\gamma)\psi_\nu(\tau,\gamma)\mu^{\{\psi_\nu(\tau,\gamma)\}}(\tau,\gamma)d\tau=h_{k,\nu}(\gamma)\quad (k=0..K,\nu=0..K).$$

10.1 Соотношения взаимосвязи ортогональных функций

[10.2] $$L_k^{(1)}(\tau,\gamma)=\sum_{\nu=0}^{k}L_\nu(\tau,\gamma).$$

[10.3] $$L_k^{(2)}(\tau,\gamma)=\sum_{\nu=0}^{k}(k-\nu+1)L_\nu(\tau,\gamma).$$

[10.4] $$L_k(\tau,\gamma)=L_k^{(1)}(\tau,\gamma)-L_{k-1}^{(1)}(\tau,\gamma).$$

[10.5] $$L_k^{(1)}(\tau,\gamma)=L_k^{(2)}(\tau,\gamma)-L_{k-1}^{(2)}(\tau,\gamma).$$

[10.6] $$L_k^{(\alpha+1)}(\tau,\gamma)=\begin{cases}\sum\limits_{\nu=0}^{k}L_\nu(\tau,\gamma), & \text{если } \alpha=0;\\ \sum\limits_{\nu=0}^{k}L_\nu(\tau,\gamma)\prod\limits_{p=0}^{\alpha-1}\dfrac{k+p+1-\nu}{p+1}, & \text{если } \alpha>0,\end{cases}\qquad \alpha\in\mathbb{N}_0.$$

[10.7] $$P_k^{(0,1)}(\tau,\gamma)=\frac{1}{k+1}\sum_{\nu=0}^{k}(2\nu+1)(-1)^{k+\nu}Leg_\nu(\tau,\gamma)).$$

[10.8] $$P_k^{(0,2)}(\tau,\gamma)=\frac{1}{(k+1)(k+2)}\sum_{\nu=0}^{k}(2\nu+1)(-1)^{k+\nu}Leg_\nu(\tau,\gamma)\times\times\big((k+1)(k+2)-\nu(\nu+1)\big).$$

[10.9] $$Leg_k(\tau,\gamma)=\frac{(k+1)}{2k+1}P_k^{(0,1)}(\tau,\gamma)+ +\frac{k}{2k+1}P_{k-1}^{(0,1)}(\tau,\gamma).$$

[10.10] $$P_k^{(0,1)}(\tau,\gamma)=\frac{(k+2)}{2(k+1)}P_k^{(0,2)}(\tau,\gamma)+ +\frac{k}{2(k+1)}P_{k-1}^{(0,2)}(\tau,\gamma).$$

[10.11] $$P_k^{*(0,\beta+1)}(\tau,\gamma)=\frac{(-1)^k}{k+1}\times$$
$$\times\begin{cases}\sum\limits_{\nu=0}^{k}(2\nu+1)(-1)^{\nu}Leg_\nu(\tau,\gamma), & \text{если } \beta=0;\\ \sum\limits_{\nu=0}^{k}(2\nu+1)(-1)^{\nu}Leg_\nu(\tau,\gamma)\times \\ \times\prod\limits_{p=0}^{\beta-1}\frac{(k+p+1-\nu)(k+p+2+\nu)}{(p+1)(k+p+2)}, & \text{если } \beta>0,\end{cases}$$
$$\beta\in\mathbb{N}_0.$$

10.2 Соотношения взаимосвязи ортогональных функций и производных ортогональных функций

[10.12] $$\frac{\partial L_k(\tau,\gamma)}{\partial\tau}=-\gamma\sum_{\nu=0}^{k-1}L_\nu(\tau,\gamma)-\frac{\gamma}{2}L_k(\tau,\gamma).$$

[10.13] $$\frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial\tau}=-\gamma\sum_{\nu=0}^{k-1}L_\nu^{(1)}(\tau,\gamma)-\frac{\gamma}{2}L_k^{(1)}(\tau,\gamma).$$

[10.14] $$\frac{\partial L_k^{(1)}(\tau,\gamma)}{\partial\tau}=-\frac{\gamma}{2}\sum_{\nu=0}^{k-1}\big(2(k-\nu)+1\big)L_\nu(\tau,\gamma)- -\frac{\gamma}{2}L_k(\tau,\gamma).$$

[10.15] $$\frac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau}=-\gamma\sum_{\nu=0}^{k-1}L_\nu^{(2)}(\tau,\gamma)-\frac{\gamma}{2}L_k^{(2)}(\tau,\gamma).$$

[10.16] $$\frac{\partial L_k^{(2)}(\tau,\gamma)}{\partial\tau}=-\frac{\gamma}{2}\sum_{\nu=0}^{k-1}(k-\nu+1)^2L_\nu(\tau,\gamma)- -\frac{\gamma}{2}L_k(\tau,\gamma).$$

[10.17] $$\frac{\partial L_k^{(\alpha)}(\tau,\gamma)}{\partial\tau}=-\gamma\sum_{\nu=0}^{k-1}L_\nu^{(\alpha)}(\tau,\gamma)-\frac{\gamma}{2}L_k^{(\alpha)}(\tau,\gamma).$$

[10.18] $$\frac{\partial P_k^{(-1/2,0)}(\tau,\gamma)}{\partial\tau}=-\gamma\sum_{\nu=0}^{k-1}(4\nu+1)(-1)^{k+\nu}\times \times P_\nu^{(-1/2,0)}(\tau,\gamma)-\frac{\gamma(4k+1)}{2}P_k^{(-1/2,0)}(\tau,\gamma).$$

[10.19] $$\frac{\partial Leg_k(\tau,\gamma)}{\partial\tau}=-2\gamma\sum_{\nu=0}^{k-1}(2\nu+1)(-1)^{k+\nu}\times \times Leg_\nu(\tau,\gamma)-\gamma(2k+1)Leg_k(\tau,\gamma).$$

[10.20] $$\frac{\partial P_k^{(1/2,0)}(\tau,\gamma)}{\partial\tau}=-\gamma\sum_{\nu=0}^{k-1}(4\nu+3)(-1)^{k+\nu}\times \times P_\nu^{(1/2,0)}(\tau,\gamma)-\frac{\gamma(4k+3)}{2}P_k^{(1/2,0)}(\tau,\gamma).$$

[10.21] $$\frac{\partial P_k^{(1,0)}(\tau,\gamma)}{\partial\tau}=-2\gamma\sum_{\nu=0}^{k-1}(\nu+1)(-1)^{k+\nu}\times \times P_\nu^{(1,0)}(\tau,\gamma)-\gamma(k+1)P_k^{(1,0)}(\tau,\gamma).$$

[10.22] $$\frac{\partial P_k^{(2,0)}(\tau,\gamma)}{\partial\tau}=-2\gamma\sum_{\nu=0}^{k-1}(2\nu+3)(-1)^{k+\nu}\times \times P_\nu^{(2,0)}(\tau,\gamma)-\gamma(2k+3)P_k^{(2,0)}(\tau,\gamma).$$

[10.23] $$\frac{\partial P_k^{(\alpha,0)}(\tau,\gamma)}{\partial\tau}=-c\gamma\sum_{\nu=0}^{k-1}(\alpha+2\nu+1)(-1)^{k+\nu}\times \times P_\nu^{(\alpha,0)}(\tau,\gamma)-c\gamma/2(\alpha+2k+1)P_k^{(\alpha,0)}(\tau,\gamma).$$

[10.24] $$\frac{\partial P_k^{(0,1)}(\tau,\gamma)}{\partial\tau}=-\frac{4\gamma}{(k+1)}\sum_{\nu=0}^{k-1}(\nu+1)^2(-1)^{k+\nu}\times \times P_\nu^{(0,1)}(\tau,\gamma)-\gamma(2k+1)P_k^{(0,1)}(\tau,\gamma).$$

[10.25] $$\frac{\partial P_k^{(0,1)}(\tau,\gamma)}{\partial\tau}=-\frac{\gamma}{(k+1)}\sum_{\nu=0}^{k-1}\Big(2k(k+1)- -2\nu(\nu+1)+2k+1\Big)(-1)^{k+\nu}(2\nu+1)Leg_\nu(\tau,\gamma)- -\frac{\gamma(2k+1)^2}{(k+1)}Leg_k(\tau,\gamma).$$

[10.26] $$\frac{\partial P_k^{(0,2)}(\tau,\gamma)}{\partial\tau}=-\frac{2\gamma}{(k+1)(k+2)}\sum_{\nu=0}^{k-1}(\nu+1)\times \times(\nu+2)(-1)^{k+\nu}(2\nu+3)P_\nu^{(0,2)}(\tau,\gamma)- -\gamma(2k+1)P_k^{(0,2)}(\tau,\gamma).$$

[10.27] $$\frac{\partial P_k^{(0,2)}(\tau,\gamma)}{\partial\tau}=-\frac{\gamma}{(k+1)(k+2)}\times \times\sum_{\nu=0}^{k-1}\Big((k+1)(k+2)\big(k(k+3)+1\big)-\big(2k(k+3)+3\big)\times \times\nu(\nu+1)+\nu^2(\nu+1)^2\Big)(-1)^{k+\nu}(2\nu+1)Leg_\nu(\tau,\gamma)- -\frac{2\gamma(2k+1)^2}{(k+2)}Leg_k(\tau,\gamma).$$

10.3 Соотношения взаимосвязи ортогональных функций и неопределенных интегралов от ортогональных функций

[10.28] $$\int L_k(\tau,\gamma)\mathrm{d}\tau = -\frac{4}{\gamma}\sum_{\nu=0}^{k-1}(-1)^{k+\nu}L_\nu(\tau,\gamma) - \frac{2}{\gamma}L_k(\tau,\gamma).$$

[10.29] $$\int P_k^{(-1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{4}{\gamma(4k+1)}\times \sum_{\nu=0}^{k-1}P_\nu^{(-1/2,0)}(\tau,\gamma) - \frac{2}{\gamma(4k+1)}P_k^{(-1/2,0)}(\tau,\gamma).$$

[10.30] $$\int Leg_k(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma(2k+1)}\sum_{\nu=0}^{k-1}Leg_\nu(\tau,\gamma) - \frac{1}{\gamma(2k+1)}Leg_k(\tau,\gamma).$$

[10.31] $$\int P_k^{(1/2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{4}{\gamma(4k+3)}\times \sum_{\nu=0}^{k-1}P_\nu^{(1/2,0)}(\tau,\gamma) - \frac{2}{\gamma(4k+3)}P_k^{(1/2,0)}(\tau,\gamma).$$

[10.32] $$\int P_k^{(1,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma(k+1)}\sum_{\nu=0}^{k-1}P_\nu^{(1,0)}(\tau,\gamma) - \frac{1}{\gamma(k+1)}P_k^{(1,0)}(\tau,\gamma).$$

[10.33] $$\int P_k^{(2,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{2}{\gamma(2k+3)}\sum_{\nu=0}^{k-1}P_\nu^{(2,0)}(\tau,\gamma) - \frac{1}{\gamma(2k+3)}P_k^{(2,0)}(\tau,\gamma).$$

[10.34] $$\int P_k^{(\alpha,0)}(\tau,\gamma)\mathrm{d}\tau = -\frac{4}{c\gamma(\alpha+2k+1)}\times \sum_{\nu=0}^{k-1}P_\nu^{(\alpha,0)}(\tau,\gamma) - \frac{2}{c\gamma(\alpha+2k+1)}P_k^{(\alpha,0)}(\tau,\gamma).$$

10.4 Соотношения взаимосвязи преобразований Фурье

[10.35] $$W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega) = \sum_{\nu=0}^{k}W_\nu^{\{L_\nu(\tau,\gamma)\}}(j\omega).$$

[10.36] $$W_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\omega) = \sum_{\nu=0}^{k}(k-\nu+1)W_\nu^{\{L_\nu(\tau,\gamma)\}}(j\omega).$$

[10.37] $$W_k^{\{L_k^{(\alpha+1)}(\tau,\gamma)\}}(j\omega) = \begin{cases} \sum_{\nu=0}^{k}W_\nu^{\{L_\nu(\tau,\gamma)\}}(j\omega), & \text{если } \alpha=0; \\ \sum_{\nu=0}^{k}W_\nu^{\{L_\nu(\tau,\gamma)\}}(j\omega)\prod_{p=0}^{\alpha-1}\frac{k+p+1-\nu}{p+1}, & \text{если } \alpha>0, \end{cases}$$
$$\alpha \in \mathbb{N}_0.$$

[10.38] $$W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega) = \frac{1}{k+1}\sum_{\nu=0}^{k}(2\nu+1)\times (-1)^{k+\nu}W_\nu^{\{Leg_\nu(\tau,\gamma)\}}(j\omega).$$

[10.39] $$W_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega) = \frac{1}{(k+1)(k+2)}\times \sum_{\nu=0}^{k}(2\nu+1)(-1)^{k+\nu}\big((k+1)(k+2)-\nu(\nu+1)\big)\times W_\nu^{\{Leg_\nu(\tau,\gamma)\}}(j\omega).$$

[10.40] $$W_k^{\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega) = \frac{(-1)^k}{k+1}\times \begin{cases} \sum_{\nu=0}^{k}(2\nu+1)(-1)^\nu W_\nu^{\{Leg_\nu(\tau,\gamma)\}}(j\omega), & \text{если } \beta=0; \\ \sum_{\nu=0}^{k}(2\nu+1)(-1)^\nu W_\nu^{\{Leg_\nu(\tau,\gamma)\}}(j\omega)\times \\ \times\prod_{p=0}^{\beta-1}\frac{(k+p+1-\nu)(k+p+2+\nu)}{(p+1)(k+p+2)}, & \text{если } \beta>0, \end{cases}$$
$$\beta \in \mathbb{N}_0.$$

Глава 11

Обобщенные характеристики ортогональных функций

Определение.

По аналогии с известными определениями длительности и моментных характеристик введены следующие понятия:

– длительности ортогональных функций во временной области [6]

$$\tau_k^{(2)\{\psi_k(\tau,\gamma)\}} = \frac{\int\limits_0^\infty \psi_k(\tau,\gamma)\mathrm{d}\tau}{|\psi_k(0,\gamma)|};$$

$$\tau_k^{(4)\{\psi_k(\tau,\gamma)\}} = \frac{\int\limits_0^\infty \big(\psi_k(\tau,\gamma)\big)^2\mathrm{d}\tau}{\big(\psi_k(0,\gamma)\big)^2};$$

– длительности ортогональных функций в частотной области

$$\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\}} = \frac{\int\limits_0^\infty \mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega}{|\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(0)|};$$

$$\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\}} = \frac{\int\limits_0^\infty \big(\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\big)^2\mathrm{d}\omega}{\big(\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(0)\big)^2};$$

– моментные характеристики во временной области [6, 14]

$$\mu_k^{(n)[1]\{\psi_k(\tau,\gamma)\}} = \int\limits_0^\infty \tau^n\psi_k(\tau,\gamma)\mathrm{d}\tau;$$

$$\mu_k^{(n)[2]\{\psi_k(\tau,\gamma)\}} = j^n \left. \frac{d^n W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)}{d\omega^n} \right|_{j\omega=0};$$

– моментные характеристики в частотной области

$$\mu_k^{(n)[1]\{W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\}} = \int_0^\infty \omega^n W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega;$$

$$\mu_k^{(n)[2]\{W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi}{2}(-1)^n \left. \frac{\partial^n \psi_k(\tau,\gamma)}{\partial \tau^n} \right|_{\tau=0}.$$

11.1 Длительности ортогональных функций во временной области

[11.1] $\tau_k^{(2)\{L_k(\tau,\gamma)\}} = \frac{2(-1)^k}{\gamma}, \quad k = 0, 2, 4, ...2n, \quad n \in \mathbb{N}_0.$

[11.2] $\tau_k^{(4)\{L_k(\tau,\gamma)\}} = \frac{1}{\gamma}.$

[11.3] $\tau_k^{(2)\{L_k^(1)(\tau,\gamma)\}} = \frac{2\big((k+1) \mod 2\big)}{\gamma(k+1)}, \quad k = 0, 2, 4, ...2n, \quad n \in \mathbb{N}_0.$

[11.4] $\tau_k^{(4)\{L_k^(1)(\tau,\gamma)\}} = \frac{1}{\gamma(k+1)}.$

[11.5] $\tau_k^{(2)\{L_k^(2)(\tau,\gamma)\}} = \frac{4\big((k+2) \operatorname{div} 2\big)}{\gamma(k+1)(k+2)}.$

[11.6] $\tau_k^{(4)\{L_k^(2)(\tau,\gamma)\}} = \frac{2(2k+3)}{3\gamma(k+1)(k+2)}.$

[11.7] $\tau_k^{(2)\{P_k^{(-1/2,0)}(\tau,\gamma)\}} = \frac{2}{\gamma(4k+1)}.$

[11.8] $\tau_k^{(4)\{P_k^{(-1/2,0)}(\tau,\gamma)\}} = \frac{1}{\gamma(4k+1)}.$

[11.9] $\tau_k^{(2)\{Leg_k(\tau,\gamma)\}} = \frac{1}{\gamma(2k+1)}.$

[11.10] $\tau_k^{(4)\{Leg_k(\tau,\gamma)\}} = \frac{1}{2\gamma(2k+1)}.$

[11.11] $\tau_k^{(2)\{P_k^{(1/2,0)}(\tau,\gamma)\}} = \frac{2}{\gamma(4k+3)}.$

[11.12] $\tau_k^{(4)\{P_k^{(1/2,0)}(\tau,\gamma)\}} = \frac{1}{\gamma(4k+3)}.$

[11.13] $\tau_k^{(2)\{P_k^{(1,0)}(\tau,\gamma)\}} = \frac{1}{\gamma(k+1)}.$

[11.14] $\tau_k^{(4)\{P_k^{(1,0)}(\tau,\gamma)\}} = \frac{1}{2\gamma(k+1)}.$

[11.15] $\tau_k^{(2)\{P_k^{(2,0)}(\tau,\gamma)\}} = \frac{1}{\gamma(2k+3)}.$

[11.16] $\tau_k^{(4)\{P_k^{(2,0)}(\tau,\gamma)\}} = \frac{1}{2\gamma(2k+3)}.$

[11.17] $\tau_k^{(2)\{P_k^{(0,1)}(\tau,\gamma)\}} = \frac{\big((k+1) \mod 2\big)}{\gamma(k+1)^2}, \quad k = 0, 2, 4, ...2n, \quad n \in \mathbb{N}_0.$

[11.18] $\tau_k^{(4)\{P_k^{(0,1)}(\tau,\gamma)\}} = \frac{1}{2\gamma(k+1)^2}.$

[11.19] $\tau_k^{(2)\{P_k^{(0,2)}(\tau,\gamma)\}} = \frac{4(-1)^k\big((k+2) \operatorname{div} 2\big)^2}{\gamma(k+1)^2(k+2)^2}, \quad k = 0, 2, 4, ...2n, \quad n \in \mathbb{N}_0.$

[11.20] $\tau_k^{(4)\{P_k^{(0,2)}(\tau,\gamma)\}} = \frac{2\big((k+1)(k+2)+1\big)}{3\gamma(k+1)^2(k+2)^2}.$

11.2 Моментные характеристики ортогональных функций во временной области

[11.21] $\mu_k^{(0)\{L_k(\tau,\gamma)\}} = \frac{2(-1)^k}{\gamma}.$

[11.22] $\mu_k^{(1)\{L_k(\tau,\gamma)\}} = \frac{4(-1)^k(2k+1)}{\gamma^2}.$

[11.23] $\mu_k^{(2)\{L_k(\tau,\gamma)\}} = \frac{8(-1)^k\big((2k+1)^2+1\big)}{\gamma^3}.$

[11.24] $\mu_k^{(3)\{L_k(\tau,\gamma)\}} = \frac{16(-1)^k\big((2k+1)^3+5(2k+1)\big)}{\gamma^4}.$

[11.25] $\mu_k^{(n)\{L_k(\tau,\gamma)\}} = n!\left(\frac{2}{\gamma}\right)^{n+1} \sum_{s=0}^{k} \binom{k}{s}\binom{s+n}{s}(-2)^s.$

[11.26] $\mu_k^{(0)\{L_k^{(1)}(\tau,\gamma)\}} = \frac{2\big((k+1) \mod 2\big)}{\gamma}.$

[11.27] $\mu_k^{(1)\{L_k^{(1)}(\tau,\gamma)\}} = \frac{4(-1)^k(k+1)}{\gamma^2}.$

[11.28] $\mu_k^{(2)\{L_k^{(1)}(\tau,\gamma)\}} = \frac{16(-1)^k(k+1)^2}{\gamma^3}.$

[11.29] $\mu_k^{(3)\{L_k^{(1)}(\tau,\gamma)\}} =$
$$= \frac{16}{\gamma^4}\sum_{s=0}^{k}\binom{k+1}{k-s}(-2)^s(s+1)(s+2)(s+3).$$

[11.30] $\mu_k^{(n)\{L_k^{(1)}(\tau,\gamma)\}} =$
$$= n!\left(\frac{2}{\gamma}\right)^{n+1}\sum_{s=0}^{k}\binom{k+1}{k-s}\binom{s+n}{s}(-2)^s.$$

[11.31] $\mu_k^{(0)\{L_k^{(2)}(\tau,\gamma)\}} = \dfrac{2\big((k+2)\ \operatorname{div}\ 2\big)}{\gamma}.$

[11.32] $\mu_k^{(1)\{L_k^{(2)}(\tau,\gamma)\}} = \dfrac{4(-1)^k\big((k+2)\ \operatorname{div}\ 2\big)}{\gamma^2}.$

[11.33] $\mu_k^{(2)\{L_k^{(2)}(\tau,\gamma)\}} = \dfrac{8(-1)^k(k+1)(k+2)}{\gamma^3}.$

[11.34] $\mu_k^{(3)\{L_k^{(2)}(\tau,\gamma)\}} =$
$$= \frac{16}{\gamma^4}\sum_{s=0}^{k}\binom{k+2}{k-s}(s+1)(s+2)(s+3)(-2)^s.$$

[11.35] $\mu_k^{(n)\{L_k^{(2)}(\tau,\gamma)\}} =$
$$= n!\left(\frac{2}{\gamma}\right)^{n+1}\sum_{s=0}^{k}\binom{k+2}{k-s}\binom{s+n}{s}(-2)^s.$$

[11.36] $\mu_k^{(0)\{L_k^{(\alpha)}(\tau,\gamma)\}} = \dfrac{2}{\gamma}\displaystyle\sum_{s=0}^{k}\binom{k+\alpha}{k-s}(-2)^s.$

[11.37] $\mu_k^{(1)\{L_k^{(\alpha)}(\tau,\gamma)\}} = \dfrac{4}{\gamma^2}\displaystyle\sum_{s=0}^{k}\binom{k+\alpha}{k-s}(-2)^s(s+1).$

[11.38] $\mu_k^{(2)\{L_k^{(\alpha)}(\tau,\gamma)\}} =$
$$= \frac{8}{\gamma^3}\sum_{s=0}^{k}\binom{k+\alpha}{k-s}(-2)^s(s+1)(s+2).$$

[11.39] $\mu_k^{(3)\{L_k^{(\alpha)}(\tau,\gamma)\}} =$
$$= \frac{16}{\gamma^4}\sum_{s=0}^{k}\binom{k+\alpha}{k-s}(-2)^s(s+1)(s+2)(s+3).$$

[11.40] $\mu_k^{(n)\{L_k^{(\alpha)}(\tau,\gamma)\}} =$
$$= n!\left(\frac{2}{\gamma}\right)^{n+1}\sum_{s=0}^{k}\binom{k+\alpha}{k-s}\binom{s+n}{s}(-2)^s.$$

[11.41] $\mu_k^{(0)\{P_k^{(-1/2,0)}(\tau,\gamma)\}} = \dfrac{2}{\gamma(4k+1)}.$

[11.42] $\mu_k^{(1)\{P_k^{(-1/2,0)}(\tau,\gamma)\}} =$
$$= \frac{4}{\gamma^2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\frac{1}{(4s+1)^2}.$$

[11.43] $\mu_k^{(2)\{P_k^{(-1/2,0)}(\tau,\gamma)\}} =$
$$= \frac{16}{\gamma^3}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\frac{1}{(4s+1)^3}.$$

[11.44] $\mu_k^{(3)\{P_k^{(-1/2,0)}(\tau,\gamma)\}} =$
$$= \frac{96}{\gamma^4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\frac{1}{(4s+1)^4}.$$

[11.45] $\mu_k^{(n)\{P_k^{(-1/2,0)}(\tau,\gamma)\}} =$
$$= n!\left(\frac{2}{\gamma}\right)^{n+1}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s\frac{1}{(4s+1)^{n+1}}.$$

[11.46] $\mu_k^{(0)\{Leg_k(\tau,\gamma)\}} = \dfrac{1}{\gamma(2k+1)}.$

[11.47] $\mu_k^{(1)\{Leg_(\tau,\gamma)\}} =$
$$= \frac{1}{\gamma^2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{(2s+1)^2}.$$

[11.48] $\mu_k^{(2)\{Leg_(\tau,\gamma)\}} =$
$$= \frac{2}{\gamma^3}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{(2s+1)^3}.$$

[11.49] $\mu_k^{(3)\{Leg_(\tau,\gamma)\}} =$
$$= \frac{6}{\gamma^4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{(2s+1)^4}.$$

[11.50] $\mu_k^{(n)\{Leg_(\tau,\gamma)\}} =$
$$= \frac{n!}{\gamma^{n+1}}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s\frac{1}{(2s+1)^{n+1}}.$$

[11.51] $\mu_k^{(0)\{P_k^{(1/2,0)}(\tau,\gamma)\}} = \dfrac{2}{\gamma(4k+3)}.$

[11.52] $\mu_k^{(1)\{P_k^{(1/2,0)}(\tau,\gamma)\}} =$
$$= \frac{4}{\gamma^2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s\frac{1}{(4s+3)^2}.$$

[11.53] $\mu_k^{(2)\{P_k^{(1/2,0)}(\tau,\gamma)\}} =$
$$= \frac{16}{\gamma^3}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s\frac{1}{(4s+3)^3}.$$

[11.54] $$\mu_k^{(3)\{P_k^{(1/2,0)}(\tau,\gamma)\}} = \frac{96}{\gamma^4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s\frac{1}{(4s+3)^4}.$$

[11.55] $$\mu_k^{(n)\{P_k^{(1/2,0)}(\tau,\gamma)\}} = n!\left(\frac{2}{\gamma}\right)^{n+1}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s\frac{1}{(4s+3)^{n+1}}.$$

[11.56] $$\mu_k^{(0)\{P_k^{(1,0)}(\tau,\gamma)\}} = \frac{1}{\gamma(k+1)}.$$

[11.57] $$\mu_k^{(1)\{P_k^{(1,0)}(\tau,\gamma)\}} = \frac{1}{\gamma^2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s\frac{1}{(s+1)^2}.$$

[11.58] $$\mu_k^{(2)\{P_k^{(1,0)}(\tau,\gamma)\}} = \frac{2}{\gamma^3}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s\frac{1}{(s+1)^3}.$$

[11.59] $$\mu_k^{(3)\{P_k^{(1,0)}(\tau,\gamma)\}} = \frac{6}{\gamma^4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s\frac{1}{(s+1)^4}.$$

[11.60] $$\mu_k^{(n)\{P_k^{(1,0)}(\tau,\gamma)\}} = \frac{n!}{\gamma^{n+1}}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s\frac{1}{(s+1)^{n+1}}.$$

[11.61] $$\mu_k^{(0)\{P_k^{(2,0)}(\tau,\gamma)\}} = \frac{1}{\gamma(2k+3)}.$$

[11.62] $$\mu_k^{(1)\{P_k^{(2,0)}(\tau,\gamma)\}} = \frac{1}{\gamma^2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s\frac{1}{(2s+3)^2}.$$

[11.63] $$\mu_k^{(2)\{P_k^{(2,0)}(\tau,\gamma)\}} = \frac{2}{\gamma^3}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s\frac{1}{(2s+3)^3}.$$

[11.64] $$\mu_k^{(3)\{P_k^{(2,0)}(\tau,\gamma)\}} = \frac{6}{\gamma^4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s\frac{1}{(2s+3)^4}.$$

[11.65] $$\mu_k^{(n)\{P_k^{(2,0)}(\tau,\gamma)\}} = \frac{n!}{\gamma^{n+1}}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s\frac{1}{(2s+3)^{n+1}}.$$

[11.66] $$\mu_k^{(0)\{P_k^{(\alpha,0)}(\tau,\gamma)\}} = \frac{2}{c\gamma(2k+\alpha+1)}.$$

[11.67] $$\mu_k^{(1)\{P_k^{(\alpha,0)}(\tau,\gamma)\}} = \frac{4}{c^2\gamma^2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s\frac{1}{(2s+\alpha+1)^2}.$$

[11.68] $$\mu_k^{(2)\{P_k^{(\alpha,0)}(\tau,\gamma)\}} = \frac{16}{c^3\gamma^3}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s\frac{1}{(2s+\alpha+1)^3}.$$

[11.69] $$\mu_k^{(3)\{P_k^{(\alpha,0)}(\tau,\gamma)\}} = \frac{96}{c^4\gamma^4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s\frac{1}{(2s+\alpha+1)^4}.$$

[11.70] $$\mu_k^{(n)\{P_k^{(\alpha,0)}(\tau,\gamma)\}} = \frac{2^{n+1}n!}{(c\gamma)^{n+1}}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s\frac{1}{(2s+\alpha+1)^{n+1}}.$$

[11.71] $$\mu_k^{(0)\{P_k^{(0,1)}(\tau,\gamma)\}} = \frac{1}{\gamma}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s\frac{1}{(2s+1)}.$$

[11.72] $$\mu_k^{(1)\{P_k^{(0,1)}(\tau,\gamma)\}} = \frac{1}{\gamma^2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s\frac{1}{(2s+1)^2}.$$

[11.73] $$\mu_k^{(2)\{P_k^{(0,1)}(\tau,\gamma)\}} = \frac{2}{\gamma^3}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s\frac{1}{(2s+1)^3}.$$

[11.74] $$\mu_k^{(3)\{P_k^{(0,1)}(\tau,\gamma)\}} = \frac{6}{\gamma^4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s\frac{1}{(2s+1)^4}.$$

[11.75] $$\mu_k^{(n)\{P_k^{(0,1)}(\tau,\gamma)\}} = \frac{n!}{\gamma^{n+1}}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s\frac{1}{(2s+1)^{n+1}}.$$

[11.76] $$\mu_k^{(0)\{P_k^{(0,2)}(\tau,\gamma)\}} = \frac{1}{\gamma}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s\frac{1}{(2s+1)}.$$

[11.77] $$\mu_k^{(1)\{P_k^{(0,2)}(\tau,\gamma)\}} = \frac{1}{\gamma^2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s\frac{1}{(2s+1)^2}.$$

[11.78] $$\mu_k^{(2)\{P_k^{(0,2)}(\tau,\gamma)\}} = \frac{2}{\gamma^3}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s\frac{1}{(2s+1)^3}.$$

[11.79] $$\mu_k^{(3)\{P_k^{(0,2)}(\tau,\gamma)\}} = \frac{6}{\gamma^4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s\frac{1}{(2s+1)^4}.$$

[11.80] $$\mu_k^{(n)\{P_k^{(0,2)}(\tau,\gamma)\}} = \frac{n!}{\gamma^{n+1}}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s\frac{1}{(2s+1)^{n+1}}.$$

[11.81] $$\mu_k^{(0)\{P_k^{(0,\beta)}(\tau,\gamma)\}} = \frac{1}{\gamma}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}(-1)^s\frac{1}{(2s+1)}.$$

[11.82] $$\mu_k^{(1)\{P_k^{(0,\beta)}(\tau,\gamma)\}} = \frac{1}{\gamma^2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}(-1)^s\frac{1}{(2s+1)^2}.$$

[11.83] $$\mu_k^{(2)\{P_k^{(0,\beta)}(\tau,\gamma)\}} = \frac{2}{\gamma^3}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}(-1)^s\frac{1}{(2s+1)^3}.$$

[11.84] $$\mu_k^{(3)\{P_k^{(0,\beta)}(\tau,\gamma)\}} = \frac{6}{\gamma^4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}(-1)^s\frac{1}{(2s+1)^4}.$$

[11.85] $$\mu_k^{(n)\{P_k^{(0,\beta)}(\tau,\gamma)\}} = \frac{n!}{\gamma^{n+1}}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}(-1)^s\frac{1}{(2s+1)^{n+1}}.$$

11.3 Длительности ортогональных функций в частотной области

[11.86] $$\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(k+1)}{4\big((k+1) \mod 2\big)},$$ $$k=0,2,4,...2n,\quad n\in\mathbb{N}_0.$$

[11.87] $$\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(k+1)}{8\big((k+1) \mod 2\big)^2}, k=0,2,4,...2n,\quad n\in\mathbb{N}_0.$$

[11.88] $$\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(k+1)(k+2)}{8\big((k+2) \operatorname{div} 2\big)}.$$

[11.89] $$\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(2k+3)(k+1)(k+2)}{4\big((k+2) \operatorname{div} 2\big)^2}.$$

[11.90] $$\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(-1)^k(4k+1)}{4}, k=0,2,4,...2n,\quad n\in\mathbb{N}_0.$$

[11.91] $$\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(4k+1)}{8}.$$

[11.92] $$\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(-1)^k(2k+1)}{2},$$ $$k=0,2,4,...2n,\quad n\in\mathbb{N}_0.$$

[11.93] $$\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(2k+1)}{4}.$$

[11.94] $$\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(-1)^k(4k+3)}{4}, k=0,2,4,...2n,\quad n\in\mathbb{N}_0.$$

[11.95] $$\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(4k+3)}{8}.$$

[11.96] $$\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(-1)^k(k+1)}{2},$$ $$k=0,2,4,...2n,\quad n\in\mathbb{N}_0.$$

[11.97] $$\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(k+1)}{4}.$$

[11.98] $$\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma(-1)^k(2k+3)}{2}, k=0,2,4,...2n,\quad n\in\mathbb{N}_0.$$

[**11.99**] $\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi\gamma(2k+3)}{4}.$

[**11.100**] $\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma(k+1)^2}{2\big((k+1) \mod 2\big)}, k = 0, 2, 4, ...2n, \quad n \in \mathbb{N}_0.$$

[**11.101**] $\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma(k+1)^2}{4\big((k+1) \mod 2\big)^2}, k = 0, 2, 4, ...2n, \quad n \in \mathbb{N}_0.$$

[**11.102**] $\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma(-1)^k(k+1)^2(k+2)^2}{8\big((k+2) \;\mathrm{div}\; 2\big)^2}, \quad k = 0, 2, 4, ...2n, \quad n \in \mathbb{N}_0.$$

[**11.103**] $\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma\big((k+1)(k+2)+1\big)(k+1)^2(k+2)^2}{48\big((k+2) \;\mathrm{div}\; 2\big)^4}, \quad k = 0, 2, 4, ...2n, \quad n \in \mathbb{N}_0.$$

11.4 Моментные характеристики ортогональных функций в частотной области

[**11.104**] $\mu_k^{(0)\{W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi}{2}.$

[**11.105**] $\mu_k^{(1)\{W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi\gamma(2k+1)}{4}.$

[**11.106**] $\mu_k^{(2)\{W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi\gamma^2\big((2k+1)^2+1\big)}{16}.$

[**11.107**] $\mu_k^{(3)\{W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma^3\big((2k+1)^3+5(2k+1)\big)}{96}.$$

[**11.108**] $$\mu_k^{(n)\{W_k^{\{L_k(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^n}{2}\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \begin{cases}\binom{k}{n-j}, & \text{если } k-n+j \ge 0;\\ 0, & \text{иначе.}\end{cases}$$

[**11.109**] $\mu_k^{(0)\{W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi(k+1)}{2}.$

[**11.110**] $\mu_k^{(1)\{W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi\gamma(k+1)^2}{4}.$

[**11.111**] $\mu_k^{(2)\{W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma^2(k+1)(2k^2+4k+3)}{24}.$$

[**11.112**] $\mu_k^{(3)\{W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma^3(k+1)^2(k^2+2k+3)}{48}.$$

[**11.113**] $$\mu_k^{(n)\{W_k^{\{L_k^{(1)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^n}{2}\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \begin{cases}\binom{k+1}{n-j+1}, & \text{если } k-n+j \ge 0;\\ 0, & \text{иначе.}\end{cases}$$

[**11.114**] $\mu_k^{(0)\{W_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi(k+1)(k+2)}{4}.$

[**11.115**] $\mu_k^{(1)\{W_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma(k+1)(k+2)(2k+3)}{24}.$$

[**11.116**] $\mu_k^{(2)\{W_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma^2(k+1)(k+2)(k^2+3k+3)}{48}.$$

[**11.117**] $\mu_k^{(3)\{W_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)\}} =$
$$= \frac{\pi\gamma^3(k+1)(k+2)(2k+3)(k^2+3k+5)}{480}.$$

[**11.118**] $$\mu_k^{(n)\{W_k^{\{L_k^{(2)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^n}{2}\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \begin{cases}\binom{k+2}{n-j+2}, & \text{если } k-n+j \ge 0;\\ 0, & \text{иначе.}\end{cases}$$

[**11.119**] $\mu_k^{(0)\{W_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi}{4}\dbinom{k+\alpha}{k}.$

[**11.120**] $$\mu_k^{(1)\{W_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma}{2}\sum_{j=0}^{1}\binom{1}{j}2^{-j}\times \begin{cases}\binom{k+2}{3-j}, & \text{если } k-1+j \ge 0;\\ 0, & \text{иначе.}\end{cases}$$

[**11.121**] $$\mu_k^{(2)\{W_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^2}{2}\sum_{j=0}^{2}\binom{2}{j}2^{-j}\times \begin{cases}\binom{k+2}{4-j}, & \text{если } k-2+j \ge 0;\\ 0, & \text{иначе.}\end{cases}$$

[11.122] $\mu_k^{(3)\{W_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi\gamma^3}{2}\sum_{j=0}^{3}\binom{3}{j}2^{-j}\times \times\begin{cases}\dbinom{k+2}{5-j}, & \text{если } k-3+j\geq 0;\\ 0, & \text{иначе.}\end{cases}$

[11.123] $\mu_k^{(n)\{W_k^{\{L_k^{(\alpha)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi\gamma^n}{2}\sum_{j=0}^{n}\binom{n}{j}2^{-j}\times \times\begin{cases}\dbinom{k+2}{n-j+2}, & \text{если } k-n+j\geq 0;\\ 0, & \text{иначе.}\end{cases}$

[11.124] $\mu_k^{(0)\{W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi}{2}(-1)^k.$

[11.125] $\mu_k^{(1)\{W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma}{4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s(4s+1).$

[11.126] $\mu_k^{(2)\{W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma^2}{8}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s(4s+1)^2.$

[11.127] $\mu_k^{(3)\{W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma^3}{16}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s(4s+1)^3.$

[11.128] $\mu_k^{(n)\{W_k^{\{P_k^{(-1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi(\gamma/2)^n}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s-1/2}{s-1/2}(-1)^s(4s+1)^n.$

[11.129] $\mu_k^{(0)\{W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi}{2}(-1)^k.$

[11.130] $\mu_k^{(1)\{W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s(2s+1).$

[11.131] $\mu_k^{(2)\{W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma^2}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s(2s+1)^2.$

[11.132] $\mu_k^{(3)\{W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma^3}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s(2s+1)^3.$

[11.133] $\mu_k^{(n)\{W_k^{\{Leg_k(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma^n}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s}{s}(-1)^s(2s+1)^n.$

[11.134] $\mu_k^{(0)\{W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi}{2}(-1)^k.$

[11.135] $\mu_k^{(1)\{W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma}{4}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s(4s+3).$

[11.136] $\mu_k^{(2)\{W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma^2}{8}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s(4s+3)^2.$

[11.137] $\mu_k^{(3)\{W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma^3}{16}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s(4s+3)^3.$

[11.138] $\mu_k^{(n)\{W_k^{\{P_k^{(1/2,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi(\gamma/2)^n}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1/2}{s+1/2}(-1)^s(4s+3)^n.$

[11.139] $\mu_k^{(0)\{W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\}} = \dfrac{\pi}{2}(-1)^k.$

[11.140] $\mu_k^{(1)\{W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s(s+1).$

[11.141] $\mu_k^{(2)\{W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma^2}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s(s+1)^2.$

[11.142] $\mu_k^{(3)\{W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\}} = = \dfrac{\pi\gamma^3}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s(s+1)^3.$

[11.143] $\mu_k^{(n)\{W_k^{\{P_k^{(1,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^n}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s+1}(-1)^s(s+1)^n.$

[11.144] $\mu_k^{(0)\{W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi}{2}(-1)^k.$

[11.145] $\mu_k^{(1)\{W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s(2s+3).$

[11.146] $\mu_k^{(2)\{W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^2}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s(2s+3)^2.$

[11.147] $\mu_k^{(3)\{W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^3}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s(2s+3)^3.$

[11.148] $\mu_k^{(n)\{W_k^{\{P_k^{(2,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^n}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s+2}(-1)^s(2s+3)^n.$

[11.149] $\mu_k^{(0)\{W_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi}{2}(-1)^k.$

[11.150] $\mu_k^{(1)\{W_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi c\gamma}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s(2s+\alpha+1).$

[11.151] $\mu_k^{(2)\{W_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi c^2\gamma^2}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s(2s+\alpha+1)^2.$

[11.152] $\mu_k^{(3)\{W_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi c^3\gamma^3}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s(2s+\alpha+1)^3.$

[11.153] $\mu_k^{(n)\{W_k^{\{P_k^{(\alpha,0)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi c^n\gamma^n}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\alpha}{s+\alpha}(-1)^s(2s+\alpha+1)^n.$

[11.154] $\mu_k^{(0)\{W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi}{2}(-1)^k.$

[11.155] $\mu_k^{(1)\{W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s(2s+1).$

[11.156] $\mu_k^{(2)\{W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^2}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s(2s+1)^2.$

[11.157] $\mu_k^{(3)\{W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^3}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s(2s+1)^3.$

[11.158] $\mu_k^{(n)\{W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^n}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+1}{s}(-1)^s(2s+1)^n.$

[11.159] $\mu_k^{(0)\{W_k^{\{P_k^{(0,1)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi}{2}(-1)^k.$

[11.160] $\mu_k^{(1)\{W_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s(2s+1).$

[11.161] $\mu_k^{(2)\{W_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^2}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s(2s+1)^2.$

[11.162] $\mu_k^{(3)\{W_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^3}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s(2s+1)^3.$

[11.163] $\mu_k^{(n)\{W_k^{\{P_k^{(0,2)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi\gamma^n}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+2}{s}(-1)^s(2s+1)^n.$

[11.164] $\mu_k^{(0)\{W_k^{\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi}{2}(-1)^k.$

[11.165] $$\mu_k^{(1)\{W_k^{\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi c\gamma}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}(-1)^s(2s+1).$$

[11.166] $$\mu_k^{(2)\{W_k^{\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi c^2\gamma^2}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}(-1)^s(2s+1)^2.$$

[11.167] $$\mu_k^{(3)\{W_k^{\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi c^3\gamma^3}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}(-1)^s(2s+1)^3.$$

[11.168] $$\mu_k^{(n)\{W_k^{\{P_k^{(0,\beta)}(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi c^n\gamma^n}{2}\sum_{s=0}^{k}\binom{k}{s}\binom{k+s+\beta}{s}(-1)^s(2s+1)^n.$$

Глава 12

Соотношения неопределенности

Определение.

На основе понятий длительности ортогональных функций во временной и частотной областях, приведенных в Главе 11, получены соотношения неопределенности.

[12.1] $$\int\limits_0^\infty \big(\psi_k(\tau,\gamma)\big)^2 \mathrm{d}\tau = \frac{2}{\pi}\int\limits_0^\infty \big(\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\big)^2 \mathrm{d}\omega.$$

[12.2] $$\tau_k^{(2)\{\psi_k(\tau,\gamma)\}}\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\}} = \frac{\pi}{2}.$$

[12.3] $$\tau_k^{(4)\{\psi_k(\tau,\gamma)\}} \frac{\Big(\Delta\omega_k^{(2)\{\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\}}\Big)^2}{\Delta\omega_k^{(4)\{\mathrm{Re}W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\}}} = \frac{\pi}{2}.$$

[12.4] $$\int\limits_0^\infty \tau^n \psi_k(\tau,\gamma)\mathrm{d}\tau = j^n \left.\frac{d^n W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)}{d\omega^n}\right|_{j\omega=0}.$$

[12.5] $$\int\limits_0^\infty \omega^n W_k^{\{\psi_k(\tau,\gamma)\}}(j\omega)\mathrm{d}\omega = \frac{\pi}{2}(-1)^n \left.\frac{\partial^n \psi_k(\tau,\gamma)}{\partial\tau^n}\right|_{\tau=0}.$$

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ДЛЯ ЗАМЕТОК

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Прохоров Сергей Антонович – доктор технических наук, профессор, действительный член Международной академии информатизации, Международной общественной организации «Академия навигации и управления движением», академии телекоммуникаций и информатики, член-корреспондент Российской академии естественных наук, заслуженный работник высшей школы Российской Федерации, лауреат губернской премии в области науки и техники, областной премии Ленинского комсомола, конкурса на лучшую научную книгу 2005, 2007, 2009 годов среди преподавателей высших учебных заведений России, награжден медалями Келдыша М.В., Гагарина Ю.А., Королева С.П. федерации космонавтики РФ, изобретателя СССР, «За заслуги перед городом Самара», нагрудными знаками «Победитель социалистического соревнования - 1975» «Ветеран космодрома Плесецк», знаками РАЕ «Заслуженный деятель науки и образования», «Основатель научной школы», заведующий кафедрой информационных систем и технологий Самарского государственного аэрокосмического университета.

В качестве председателя Головного Совет Минвуза России по автоматизации научных исследований в период 1988-1996 г.г. руководил разработкой и выполнением шести научно-исследовательских программ и подпрограмм АН СССР, Минэлектронпрома СССР, Минвуза России.

Принимал участие в руководстве и выполнении 36 хоздоговорных и госбюджетных НИР.

Подготовил 4 докторов и 27 кандидатов технических наук, по 5 кандидатским диссертациям являлся научным консультантом, являлся руководителем 154 дипломных проектов и работ, 3 магистерских диссертаций. Являлся официальным оппонентом по 42 докторским и 57 кандидатским диссертациям.

Опубликовал 485 научных работ, в том числе, 23 монографии, 13 брошюр, 40 авторских свидетельств, 21 свидетельство о государственной регистрации программ для ЭВМ. Результаты работы обсуждались на 125 международных, Всесоюзных и республиканских конференциях и симпозиумах.

Куликовских Илона Марковна – к.т.н., доцент кафедры информационных систем и технологий Самарского государственного аэрокосмического университета имени академика С.П. Королева, научный сотрудник Загребского университета (Faculty of Electrical Engineering and Computing) и института Руджер Бошкович (Division of Electronics).

Обладатель грантов Президента РФ (2018-2019), РФФИ (2018-2019), стипендии правительства РФ (2010-2011), победитель программ У.М.Н.И.К. (2010-2011) и СТАРТ (2019-2020), лауреат губернской премии в области науки и техники для аспирантов (2009) и молодых ученых (2013), лауреат конкурса на лучшую научную книгу 2009 годов среди преподавателей высших учебных заведений России.

Являлась руководителем 25 дипломных работ и 13 магистерских диссертаций.

Опубликовала 106 научных работ, в том числе, 6 монографий, 12 свидетельств о государственной регистрации программ для ЭВМ. Результаты работы обсуждались на 27 международных и всероссийских конференциях.

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Часть I. Ортогональные функции экспоненциального типа

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