Scalogram-EMD distance for mobile ECGs

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***Abstract*—Now there are devices that are capable of recording ECGs. The distance between signals can be helpful in classification problems for finding ECGs like the given one to know the expected disease scenario. This paper proposes a new distance based on the wavelet decomposition of the signal and earth mover's distance with a new base distance function. It is shown that the introduced distance is a metric over the considered signal equivalence classes. In addition, a method for creating new signals based on the developed distance has been proposed, which can be used to augment data when training deep neural networks. Finally, an experimental study has demonstrated that the generated signals can improve classification quality.**

***Keywords*— ECG, Deep Learning, Augmentation, EMD, Wavelets.**

1. Introduction

Now there are devices that can record ECGs (electrocardiogram). Examples of such devices are AliveCor [1], CardioQvark [2].

The distance between such signals can be a helpful feature. For example, one of these tasks may be searching for an ECG signal like chosen to find out the expected scenario for a patient's disease case, comparing it with patients treated in the past. Also, this metric can be used together with the k nearest neighbors (knn) [3] method and other classification methods that use distance.

Existing distances have drawbacks: many are not theoretical metrics, for example, the dynamic time warping algorithm [4]. Others do not consider the time component of the ECG signal. For example, such a metric was proposed in the paper [5] for stationary signals based on the Fourier transform and the Wasserstein distance (WF distance). The ECG signal is not stationary, as it depends on the electrical impulses coming from the heart at any given time. Therefore, the distance based on the Fourier transformation is not entirely applicable for such signals.

The paper is organized as follows. Chapter two defines distance and gives a theorem about its being a metric and some facts about its efficient calculation. Chapter three discusses new signals generation based on the created distance. Chapter four is devoted to the experimental study of distance compared to WF and the new augmentation technique. In conclusion, findings from the research are presented.

2. Scalogram-EMD Distance

A. Distance Definition

In this paper, it is proposed to use a metric based on the Wavelet decomposition of the signal since the wavelet decomposition has the property that its coefficients allow to analyze frequencies distributed over time [6]. The signal is represented as its normalized scalogram:

$$I_x(t,a) = \frac{|W_a(t)|^2}{\sum_t \sum_a |W_a(t)|^2}, \qquad (1)$$

where W is the detail coefficients of wavelet decomposition, t is the time component of the wavelet decomposition, a is the scale component of the wavelet decomposition. The classical discrete wavelet transform was not used, but the stationary one [7] because of invariance to the time shift property.

Representing a signal as its normalized scalogram introduces an equivalence space [x]={x'|I_x' (t,a)=I_x (t,a)} on the set of signals. Thus, signals are considered equivalent if one is obtained from the other by multiplying by a coefficient. If not all levels are considered, the signals are considered to be equivalent to a certain frequency level contained in the first or last coefficients. Thus, the distance will be considered not over the signals themselves but the equivalence classes of these signals. In many machine learning problems can be assumed that signals that fall into the same equivalence class belong to the same class.

The Earth Movers Distance metric [8] with the base metric $L_{1\alpha}$ is used to determine the distance.

- Let the matrices P and Q have m rows and n columns and $N = m \times n$;
- The list of indexes for values is defined as $\mathcal{I} = \{(i,j): 1 \le i \le m, 1 \le j \le n\}$;
- The list of indexes for threads is defined as $\mathcal{J} = \{(i,j,k,l): (i,j) \in \mathcal{I}, (k,l) \in \mathcal{I}\}$;
- Matrices $P = \{p_{i,j}: (i,j) \in \mathcal{I}\}$ and $Q = \{q_{i,j}: (i,j) \in \mathcal{I}\}$ - matrices between which the distance is being searched.

$$EMD(P,Q) = \min_{F=\{f_{i,j,k,l}:(i,j,k,l)\in\mathcal{J}\}} \sum_{\mathcal{J}} f_{i,j,k,l}\, d_{i,j,k,l},$$

$$\begin{cases} \sum\limits_{(k,l)\in\mathcal{I}} f_{i,j,k,l} = p_{i,j} \quad \forall (i,j) \in \mathcal{I} \\ \sum\limits_{(i,j)\in\mathcal{I}} f_{i,j,k,l} = q_{k,l} \quad \forall (k,l) \in \mathcal{I} \\ f_{i,j,k,l} \ge= 0 \quad \forall (i,j,k,l) \in \mathcal{J} \end{cases}$$

where F is the flow from P to Q, and $f_{i,j,k,l}$ defines the flow from (i,j) to (k,l).

In this paper, it is proposed to define the base distance as:

$$L_{1\alpha}: d_{i,j;k,l} = |i-k| + \alpha|j-l|,\ \alpha \in (0,1].$$

This distance defines the distance between two points as a successive transfer in frequency and time. At the same time, it is proposed to introduce an additional coefficient α∈(0,1], which will show how much the transfer in time is less significant than the transfer in frequency.

Definition 1. Scalogram-EMD (SE) distance between two signal equivalence classes $[x]$ and $[y]$ is defined as:

$$SE([x],[y]) = EMD_{L_{1\alpha}}(I_x(t,a), I_y(t,a)).$$

Theorem 1. SE is a metric over signal equivalence classes.

Б. Efficient way to calculate $EMD_{L_{1\alpha}}$.

The article [9] proposes a method for quickly and accurately finding $EMD_{L_{1\alpha}}$ in the case of $\alpha = 1$. The number of unknowns in the equations for finding the minimum flow was reduced to $O(N)$ from $O(N^2)$ in matrices consisting of N points. Empirically, it was shown that the new algorithm has $O(N^2)$ time, while the classical algorithm shows $O(\log n N^3)$ complexity.

In this work, it was shown that the algorithm constructed for L_1 could be adapted to find $L_{1\alpha}$ without increasing the level of asymptotic complexity.

3. New Signals Generation with Scalogram-EMD Distance

The additional synthetic data can be used in the online and offline augmentation approaches to improve the model's quality on the original test cases. This paper proposes a new method for generating data, which uses the flow obtained by calculating the distance between two signals.

Two signals belonging to the same class are selected from the training set. Two matrices similar to formula (1) are built for each signal. One is based on the approximation coefficients, the other is based on the detail coefficients. In this case, both coefficients are needed for the inverse transformation from the wavelet decomposition to the signal. The matrices correspond to the signs of the approximation and detail matrices stored before they are squared.

Then a flow is built based on the EMD distance between each pair of matrices. The main idea of new signal generation is to apply only a part of the transformations from the entire flow that translates one signal into another. For example, suppose we use a random part of the flow from EMD distance. In that case, the resulting matrix may no longer correspond to the original class of matrices (it may contain negative values, and the sum of all its elements may no longer be equal to one).

The resulting flow is built according to the following principles. 1) For the resulting flow, single flows are selected from those points where there are no incoming flows. 2) As long as there are single flows not included in the resulting flow: points are selected to which there are no incoming flows, except for those in the resulting flow; flows from these points are added to the resulting flow.

Theorem 2. The flow built according to the above principle allows keeping the non-negativity of matrices and the sum of their elements equal to 1 when applying any starting part of the resulting flow.

Part of the transformed flow is applied to the original matrices. The root is extracted from the new matrices. The sign is determined as the sign of the closest original matrix in the point. The inverse wavelet transform is performed. Below is an example of the newly generated signal.

Table I. Dataset

Dataset	**Experiments**		
	Metric Name	***WF***	***SE***
Arithmia Detection	f-weighted	0.51	0.55
Tuberculosis	F1-score	0.32	0.38
Patient Detection	Patient Detection	0.01	0.06

4. Experiments and Results

Experiments were carried out on three different datasets: dataset for the presence of tuberculosis, identification of the patient by his ECG, and determination of arrhythmia type. For the first task, the f1 score was used. For the second task, accuracy was used, and for the last task f-weighted score was used. The knn method was used as a classifier. In the case of an extensive training sample, the WF distance selected 383 candidates. SE was used to choose the closest ones from them.

During the experiments, it was found that changing the α parameter positively affects improving the quality.

Also, synthetically generated signals from paragraph III were added to the training data from the article [10]. The same neural network and test set were used. The quality was improved from 0.934 to 0.952 ROC AUC.

5. Conclusion

On the selected test problems, the classification quality of the method using the WE distance shows a higher performance than the method using the WF distance. The α parameter improves the classification quality and can be used for tuning. The generated signals using the EMD distance improve the classification quality on the considered problem.

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